Dynamic Disclosures and the Secondary Market for Loan Sales

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Abstract

This paper predicts that prospective loan sales by banks create a positive option value for borrowing firms from deferring disclosures to a later period. When banks incur positive transaction costs in selling their loan assets, we show that borrowing firms’ initial firm and equity values can actually decrease when: (i) firms have multiple opportunities to disclose relative to when they can disclose only once; and (ii) banks engage in any positive level of monitoring relative to no monitoring at all. Further, even absent transaction costs, we predict that the likelihood of banks’ informationally motivated loan sales increases in each one of: firms’ leverage, firms’ forward-looking preference, the likelihood of banks not experiencing a liquidity shock, and the posterior default probability conditional on public news. We also show that the probability of borrowers’ disclosures can decrease in the intensity of bank monitoring and firms’ leverage.
1 Introduction

Banks constitute the single most important source of external finance for corporations around the world. Even if we count only syndicated loans, firms borrow more money from banks than they raise through public debt and equity issuance together\footnote{Between 1993 and 2003, industrial firms borrowed $13.2 trillion through syndicated bank loans, compared to $12.5 trillion from public capital markets (Drucker and Puri (2007)).}. The special role that banks play as providers of capital has long been investigated by a vast literature. Prominent examples include Diamond (1984) and Fama (1985). Further, Diamond (1984) emphasizes the critical role of monitoring by banks as information intermediaries. Fama (1985) claims that given the reserve burden borne by banks, the prominence of banks as a source of funding might appear at a first blush intriguing. He points out that banks routinely have access to inside information, whereas public debt holders rely mostly on publicly available information. Hence, signals from bank loans can reduce the information asymmetry and improve the efficiency of debt contracts.

The value of bank monitoring is also confirmed by several empirical studies. James (1987) and Lummer and McConnell (1989), followed by many others, document that bank loan announcements generate positive abnormal stock returns for the borrowing firm. Krishnaswami et al. (1999) find that firms with more growth options (and concomitant higher information asymmetry costs) benefit more from the monitoring associated with bank loans.

Earlier studies reflected the then-prevailing common practice of banks serving as lenders that make loans which they hold until maturity. However, with the rapid development and growth of the U.S. secondary markets for bank loans in recent years, banks’ loan sales reached a volume of $517.3 billion in 2013 from a mere $8.0 billion in 1991\footnote{See Reuters LPC Traders Survey 2014.}, allowing banks to diversify their loan portfolios and to recycle capital, which in turn is viewed as resulting in providing enhanced access to capital and benefiting a larger set of borrowers (Drucker and Puri (2009)).

While the presence of a secondary loan market can be socially desirable\footnote{Kamstra et al. (2014) show that even if the secondary loan market reduced monitoring intensity of banks, the net impact for the debt issuers is likely to be positive because they benefit from the enhanced liquidity.}, loan sales by banks could potentially have informational effects on the stock price of borrowers. Dahiya et al. (2003)
used the newsletter *Loan Market Week* to identify loan sale events from 1995 to 1998 and tested the effects of such sales on borrowers. They find a negative stock market reaction to loan sale announcements. Further, when banks have the opportunity to sell their loans in secondary markets, they could, in principle, exploit their potential informational advantage to sell loans that they privately know are likely to perform poorly.\(^4\)

The above studies suggest that the optimal ex ante design of a debt contract with a bank must anticipate prospective informationally motivated loan sales. Further, the client firm’s voluntary disclosure decisions are also likely to be influenced by its anticipation of the bank’s prospective loan sales, because of their information content. While a significant amount of the disclosure literature examines the impact of firm’s disclosures in the presence of pure equity financing (e.g., see [Verrecchia (2001)](#) and [Dye (2001)](#)), there is also a fair amount of literature that examines firms’ disclosure policies in the presence of limited liability, including the one entailed by debt contracts.\(^5\) However, none of this literature has analyzed the information content of loan sales by banks, an institutional feature which has become increasingly important in recent years, as we have pointed out above.\(^6\)

This paper studies a multi-period setting in which a firm borrows from a bank to partly finance a new project. We study a hierarchical information structure with two layers of information asymmetry: first, the owner of the firm is better informed about the firm’s prospects than the bank; and second, the bank’s information is superior to that of the market. The market learns about the firm’s cash flows from both the firm’s disclosures and the bank’s loan sale (or lack thereof), two decisions which our model endogenizes. In particular, we assume that the firm’s owner may or may not obtain private information about the firm’s terminal value. The bank relies on a monitor-

\(^4\)For example, in March 2014 the liquidation director of “tau.returns” filed a $300 million lawsuit against Leumi (the largest bank in Israel), claiming that, as a major debt holder, Leumi allegedly had access to private information available to management. It was argued that such an informational advantage allegedly enabled Leumi to regain most of the funds it had lent earlier to the firm. Other creditors have maintained that Leumi’s alleged informationally motivated action had the effect of shifting the bank’s share of losses to other creditors.

\(^5\)See, for instance, [Sridhar and Magee (1996)](#), [Fischer and Verrecchia (1997)](#), [Göx and Wagenhofer (2009)](#), and [Bertomeu et al. (2011)](#). To emphasize the preponderance of debt financing even outside the banking channel, the total U.S. corporate bond issuance in 2014 was $1.48 trillion compared with $174 billion in equities (www.federalreserve.gov).

\(^6\)Beyer et al. (2010) survey the empirical significance of voluntary disclosures.
ing mechanism to possibly learn a coarse partition of the owner’s private information. The owner maximizes a convex combination of the market value of equity at different points in time, where the market value of equity at each point in time is calculated rationally as the expected residual claim conditional on all publicly available information until that point in time. Since the bank is subject to occasional liquidity shocks, a loan sale by the bank could either be informationally motivated or prompted by its liquidity needs. Therefore, upon observing a loan sale subsequent to non-disclosure by the owner, the market is uncertain about whether the loan sale was liquidity motivated or informationally motivated.

We find that, in equilibrium, the bank sells its loan in the secondary market whenever it is hit by a liquidity shock or obtains adverse private information about the firm’s terminal value. The bank does not sell its loan when it fails to obtain adverse information and does not suffer a liquidity shock. In other words, a loan sale is bad news for equity, whereas no loan sale is good news. This creates a positive option value for the owner from delaying the disclosure until after the potential loan sale. The option value is positive because in case of bad news (that is, should a loan sale occur), the owner can always correct the price with a voluntary disclosure.

When transaction costs are present in the secondary loan market, our analysis predicts that the opportunity to make a second disclosure following the bank’s loan sale decision decreases the ex ante equity and firm values, compared to when the owner can disclose only once, before the potential loan sale. In this way, our paper predicts that greater disclosure opportunities can actually destroy value relative to when firms have limited opportunities to disclose.

In general, there are not many limitations on firm’s voluntary disclosures. One major exception is the instance of “quiet or waiting period” around a firm’s initial public offering (IPO). While there lacks a precise definition of quiet period in the securities laws, the U.S. Securities and Exchange Commission states that “a quiet period extends from the time a company files a registration statement with the SEC until SEC staff declare the registration statement ‘effective’”. During that period, the federal securities laws limit what information a company and related parties can re-
Billings and Cedergren (2015) finds that “quiet period rules prevent investors from learning useful information in a timely manner.” Our analysis identifies a setting in which efficiency gains can accrue to firms that do not have additional opportunities to make voluntary disclosures.

Our analysis moreover predicts that if the information endowment of the firm’s owner were common knowledge, or if she could commit not to make any disclosure in the first period, or if the bank were not to monitor, then the firm’s ex ante equity value would be maximized.

Even absent transaction costs associated with banks’ loan sales, our analysis demonstrates other significant real effects of firms’ dynamic disclosures in the presence of prospective loan sales by banks. First, we consider the arrival of public news about the cash flows (e.g., analysts’ reports) prior to the first opportunity for the firm to disclose. It is shown that the external news has an impact on the probability of loan sales via the firm’s propensity to disclose in the first period. In principle, it is not clear how public news would affect the probability of loan sales, as public news changes both the posterior distribution of cash flows and the disclosure threshold. Our analysis predicts that the probability of a loan sale increases if and only if the posterior default probability of the firm increases as a result of the public news. Our prediction stands in stark contrast with the ex ante irrelevance result of Acharya et al. (2011), who state conditions under which public news does not alter the disclosure probability. The crucial difference with Acharya et al. (2011) is our assumption of debt financing.

We next proceed to consider the case where the firm’s owner can affect the distribution of the cash flows through a personally costly action. Unobservability of the owner’s action, as one would predict, gives rise to a moral hazard problem. We provide a novel mechanism that connects the effectiveness of bank monitoring to the efficiency loss due to moral hazard.

The paper is organized as follows: Section 2 introduces the baseline model. In section 3, we establish and characterize the equilibrium and generate several predictions in the form of comparative statics results. Section 4 examines how the release of public news affects the probability of

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7See http://www.sec.gov/answers/quiet.htm
loan sales. Section 5 examines how the bank’s monitoring of the owner’s information endowment mitigates the moral hazard problem created by unobservability of her action. Section 6 generates and discusses empirical implications. Finally, Section 7 concludes the paper. All proofs are in the Appendix.

2 Model Setup

Consider a firm that lasts for two periods. At the beginning of the game (time $t = 0$), the owner of the firm faces a new investment opportunity that requires a fixed investment of $I > 0$. This investment gives rise to a stochastic cash flow $\tilde{x}$ at the end of the second period (time $t = 2$). The cash flow $\tilde{x}$ is continuously distributed with a strictly positive density $f(x)$ over $(-\infty, \infty)$. The cumulative distribution function (CDF) of $\tilde{x}$ is denoted by $F(x)$. The owner borrows $\gamma I$ from a bank, where $\gamma \in (0, 1]$ denotes the fraction of investment financed via debt. The debt has zero coupon rate, with a face value of $\delta$, and is due to be repaid at the end of the second period. The cash flow $\tilde{x}$ will be realized at $t = 2$, but after investing at the beginning of the first period, with probability $q_f \in (0, 1)$ the owner privately observes a perfect signal about the actual value of $\tilde{x}$. If informed, at $t = 1$ the owner has an opportunity to make a truthful voluntary public disclosure of $\tilde{x} = x$.

We wish to model the representative setting in which the bank sometimes obtains private information about its client firm’s performance – information which is superior relative to the markets’, albeit noisier than the firm’s private information, if any. We wish to examine this hierarchy of informational asymmetries across three different sets of players that is often empirically observed. To this end, we assume that conditional on the owner obtaining private information $\tilde{x}$ and

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8 We use the terms “firm” and firm’s “owner” interchangeably depending on the fit to a given context.
9 As in Dye (1985) and Jung and Kwon (1988), we assume that the owner’s information endowment is her private knowledge, that any disclosure by the owner must be truthful, and that the owner cannot credibly convey that she is uninformed.
10 That is, we model an information structure in which the firm’s owner potentially obtains the most precise information, followed by the bank occasionally observing noisier information than the firm and, finally, the market being uninformed, unless either the firm directly discloses or the bank’s loan sale partly conveys the bank’s noisy information.
withholding it at $t = 1$, with probability $q_b \in (0, 1)$, the bank’s monitoring system is effective at time $t = 2_{beg}$ in detecting that the owner was privately informed but chose to withhold her private information. This implies that if the bank is not successful in detecting the adverse information, then either the owner was uninformed (an event that occurs with probability $1 - q_f$) or the owner was informed but the bank’s monitoring was unsuccessful in detecting it (which event occurs with probability $q_f (1 - q_b)$). Throughout, we assume that the bank cannot credibly communicate to anyone the signal produced by its monitoring system or whether it suffered a liquidity shock.

At time $t = 2_{beg}$, with probability $\lambda \in (0, 1)$ the bank is hit by a liquidity shock, in which case the bank is forced to sell in the secondary market the indivisible loan it made to the borrowing firm at $t = 0$. With the remaining probability $1 - \lambda$, there is no liquidity shock. Regardless of the liquidity shock, the bank always has the option of selling its loan. We assume that the bank incurs transaction cost $k \in [0, \bar{k})$ whenever it sells the loan for some $\bar{k} > 0$. We need the upper bound $\bar{k}$ on the transaction cost to ensure that the transaction costs are not large enough to prevent the bank from selling its loan whenever the bank desires to do so.

At $t = 2$, after observing whether the bank sold the loan, the owner has a second opportunity to disclose her private information $\tilde{x} = x$ (if she was privately informed and chose not to disclose at

11In other words, even though the bank’s monitoring system is effective with probability $q_b < 1$ in detecting the firm having withheld its private information $\tilde{x}$, it is not powerful enough to detect the actual realized value of the owner’s signal. This information structure assures us that the firm’s own private information is superior to that of any information that the bank’s monitoring system is capable of producing, and is similar to the one analyzed by [Dye (1998)].

12We use the label “detecting the adverse information” as a short hand to mean that the bank’s monitoring system detected that the firm’s outcome was in a lower partition of the outcome space, i.e., $\tilde{x} \leq \hat{x}$, where $\hat{x}$ is the conjectured disclosure threshold as detailed below.

13To maintain our focus on the client firm’s dynamic disclosures, we abstract away from endogenizing the bank’s loan sale decision in the event of a liquidity shock. Even when the bank has a portfolio of multiple loans to a diverse set of client firms, it is possible to visualize the magnitude of liquidity shock as being stochastically spread over an interval such that for a sufficiently large liquidity shock, the bank is forced to sell the loan to the particular client firm which is the subject matter of our analysis.

14Among others, transaction costs may include significant legal costs in securitizing or otherwise selling the loans, finders’ fees in identifying buyers for such loans, administrative costs of collecting interest and otherwise managing loans, reporting costs, brokers’ commissions, and so forth. [Edwards et al. (2007)] find that secondary transaction costs in the corporate bond market increase in credit risk and decrease in issue size. Estimates range from three basis points (bps) to 150 bps.

15Further, it is reasonable to consider the upper limit for transaction costs, given that transaction costs associated with bond sales in the secondary market do not influence the economic behavior of market participants in a drastic manner, namely, by preventing them to transact when it would otherwise be optimal in the absence of transaction costs.
At every point in time $t$, the market prices equity and debt at $P_{E,t}(\cdot)$ and $P_{D,t}(\cdot)$, respectively, in Bayesian-rational manner conditional on all publicly available information till that point in time. Finally, at the end of the second period, the cash flow $\tilde{x}$ realizes.

We assume that $F(x)$ is log-concave, and that the firm’s project has a positive net present value (NPV). Further, we assume that all players in the game are risk neutral, and that the bank and market participants do not discount future consumption. Finally, the entire structure of the game is assumed to be common knowledge.

The timeline in Figure 1 depicts the sequence of events in the game:

![Figure 1: Timeline.](image)

There are three informational points in time: $t \in \{1, 2_{\text{beg}}, 2\}$. The set of date-1 private histories for the firm is $H_{f,1} \equiv \{(i_f,x), ni_f\}$ (the subscript $f$ indicates the ‘firm’), where $(i_f,x)$ stands for an informed owner who observes the realization $x$, and $ni_f$ denotes the owner not being informed. A strategy for the owner at $t = 1$ is, therefore, a function $\Sigma_{f,1}: H_{f,1} \rightarrow \{(d_1,x), nd_1\}$, where $d_1$ and $nd_1$ denote disclosure and non-disclosure at $t = 1$, respectively. It follows that the set of date-1...
public histories is $H_{p,1} \equiv \{(d_1,x),nd_1\}$ (the subscript $p$ stands for ‘public’). The set of the bank’s private histories at $t = 2_{beg}$ is $H_{b,2_{beg}} \equiv \{h_{p,1}\} \times \{i_b,ni_b\} \times \{ls\}$, that is, all tuples of the realized date-1 public history, $h_{p,1} \in H_{p,1}$, and the bank’s own private information: informed about the owner withholding her private information ($i_b$) or not informed ($ni_b$) given that the firm made no disclosure at $t = 1$, and whether the bank is hit by a liquidity shock ($ls = 1$) or not ($ls = 0$). A strategy for the bank at $t = 2_{beg}$ is a function $\Sigma_{b,2_{beg}} : H_{b,2_{beg}} \rightarrow \{s,ns\}$, where $s$ stands for a loan sale and $ns$ denotes no sale. The set of the public histories at $t = 2_{beg}$ is $H_{p,2_{beg}} \equiv \{h_{p,1}\} \times \{s,ns\}$, that is, all tuples containing a date-1 public history and the publicly observable loan sale, if any. The set of date-2 private histories for the owner of the firm is $H_{f,2} \equiv \{h_{f,1}\} \times \{h_{p,2_{beg}}\}$, that is, all tuples containing her date-1 private history (realization of $x$ or not informed) and the date-2$_{beg}$ public history (which in particular includes whether the bank sold its loan at $t = 2_{beg}$). To allow for the possibility of the owner retaining a choice to make a delayed disclosure, we define a strategy for the owner at $t = 2$ as a function $\Sigma_{f,2} : H_{f,2} \rightarrow \{(d_2,x),nd_2\}$, where $(d_2,x)$ stands for disclosure of the realization $x$ at $t = 2$ (given no disclosure at $t = 1$) and $nd_2$ stands for non-disclosure at $t = 2$. Finally, the set of public histories at $t = 2$ is $H_{p,2} \equiv \{h_{p,2_{beg}}\} \times \{(d_2,x),nd_2\}$, that is, all tuples containing a date-2$_{beg}$ public history (the firm’s disclosure or non-disclosure in the first period and whether the bank sold the loan), and the owner’s second-period response (disclosure or non-disclosure in the second period).

Last, debt and equity prices at any point $t \in \{1,2_{beg},2\}$ are functions $P_{r,t} : H_{p,t} \rightarrow \mathbb{R}$ for security $r \in \{D,E\}$, where $D$ stands for debt and $E$ for equity. Note that whenever the public history $h_{p,t}$ contains the disclosure of $\tilde{x} = x$ (i.e., $(d_1,x)$ or $(d_2,x)$), the prices of equity and debt are given by $P_{E,t}(h_{p,t}) = \max\{x - \delta,0\}$ and $P_{D,t}(h_{p,t}) = \min\{\max\{x,0\},\delta\}$, respectively. To minimize notational clutter, after histories of non-disclosures at both $t = 1$ and $t = 2$, asset prices are denoted by $P_{r,2}(s,nd_{1,2})$ and $P_{r,2}(ns,nd_{1,2})$, given loan sale and no sale by the bank, respectively, for security $r \in \{D,E\}$. We assume Bertrand competition among lenders for providing funds to the firm, which implies that in equilibrium the lending bank breaks even. In making her disclosure decisions, the
owner of the firm solves the following problem

\[ \max_{\Sigma f_1, \Sigma f_2} \mathbb{E}[\alpha P_{E,1} + (1 - \alpha) P_{E,2}] ; \]  

(1)

where the weight \( \alpha \in (0, 1) \) may be viewed as a measure of the owner’s myopia.\(^{17}\)

3 Analysis

We use the concept of Perfect Bayesian Equilibrium (PBE) to solve this game. A PBE in this game consists of disclosure strategies for the owner \( \{\Sigma f_1, \Sigma f_2\} \), a loan sale strategy for the bank \( \Sigma b_{2\text{beg}} \), and price functions \( \{P_{r,t} : r = D, E \text{ and } t = 1, 2_{\text{beg}}, 2\} \) such that: (i) each of the owner’s and the bank’s strategies is sequentially optimal given the other player’s strategies and the price functions; and (ii) prices at any time \( t \) are determined in a risk-neutral and Bayesian-rational manner conditional on all publicly available information available up to that point in time.

We solve the game backwards. However, in order to characterize the expectations that correspond to the date-2 equilibrium prices \( P_{E,2}(h_{p,2}) \), one requires knowledge of the circumstances under which the owner discloses and withholds information at \( t = 1 \), and of those under which the bank sells and retains the loan. We assert that if an equilibrium exists, then the owner’s date-1 disclosure strategy is upper-tailed, meaning that there exists some threshold value \( \tilde{x}_1 \) such that the owner discloses the realization \( \tilde{x} = x \) at \( t = 1 \) if and only if \( x > \tilde{x}_1 \).\(^{18,19}\) The rest of this section is organized as follows. In Section 3.1 below, we solve for the bank’s selling strategy given some conjectured date-1 disclosure threshold \( \tilde{x}_1 \). The bank’s decision problem can be analyzed separately from the date-2 continuation game, since at \( t = 2_{\text{beg}} \) the bank makes its last move. In Section 3.2

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\(^{17}\)Since the owner has another opportunity to disclose \( x \) at \( t = 2 \), the price \( P_{E,2\text{beg}} \) is immediately changed by the owner’s decision (disclosure or non-disclosure). Therefore, the owner’s utility function need not depend on \( P_{E,2\text{beg}} \). Moreover, since the disclosure decisions do not affect the terminal residual claim, omitting the latter from her objective function is without loss of generality.

\(^{18}\)To maintain brevity, we do not include the proof for this claim here, but the detailed proof is available from the authors upon request.

\(^{19}\)For convenience, we adopt the convention that, when indifferent, the owner withholds the private information. Since indifference occurs on a set with zero probability mass, the tie-breaker is immaterial.
we first solve the owner’s date-2 disclosure problem taking as given the conjectured threshold $\hat{x}_1$ and the bank’s equilibrium strategy established previously. Then, we analyze the date-1 disclosure problem and eventually find the actual value of disclosure threshold $\hat{x}_1$.

3.1 The Bank’s Loan Sale Strategy

We next proceed to establish the bank’s strategy at $t = 2_{\text{beg}}$ (assuming an equilibrium exists). We use $\tilde{\nu}_D$ to indicate the random variable that represents the cash flows from the firm to the debt holders.\(^{20}\) First, if the owner disclosed $x$ at $t = 1$, the bank has no informational advantage over the rest of the market: the value of the debt contract is the same for everyone and, therefore, the bank will not sell it.\(^{21}\) Second, if the bank faces a liquidity shock at $t = 2_{\text{beg}}$, then by assumption it must sell the loan regardless of its private information. Last, consider the case in which the bank is instead not hit by a liquidity shock. Here, the value of debt to the bank depends on its information set. If the bank’s monitoring detects adverse information, the posterior expected value of the loan from the bank’s perspective is $\mathbb{E}[\tilde{\nu}_D|nd_1,i_b] = \mathbb{E}[\tilde{\nu}_D|\tilde{x} < \hat{x}_1]$. If, on the other hand, the bank’s monitoring system does not detect anything, then the bank’s posterior value of the debt is $\mathbb{E}[\tilde{\nu}_D|nd_1,ni_b] > \mathbb{E}[\tilde{\nu}_D|\tilde{x} < \hat{x}_1]$. The last inequality holds because on the right-hand side the bank is certain that the realized $\tilde{x}$ is below the date-1 disclosure threshold, whereas on the left-hand side the possibility that $\tilde{x} > \hat{x}_1$ cannot be ruled out, as non-disclosure might have been due to the owner being uninformed.

Since the loan sale might be due to liquidity reasons unbeknownst to other market participants, adverse selection in the loan market does not lead to full unraveling. In other words, a loan sale does not indicate definitively that the bank possesses negative information about the firm’s performance. Hence, one can show that the equilibrium price of the debt conditional on the bank

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\(^{20}\)Observe that $\tilde{\nu}_D = \delta \cdot 1[\tilde{x} \geq \delta] + \tilde{x} \cdot 1[\tilde{x} \in (0,\delta)]$, where $1[\cdot]$ is the indicator function.

\(^{21}\)Specifically, when the firm discloses $x$, the bank would be indifferent between selling and not selling if transaction costs were absent. In such a case, we adopt the convention that the bank keeps the loan.
selling the loan, \( P_{D,2_{beg}}(nd_1,s) \), must satisfy the inequalities

\[
\mathbb{E}[\bar{\nu}_D|x < \bar{x}_1] < P_{D,2_{beg}}(nd_1,s) < \mathbb{E}[\bar{\nu}_D|nd_1,nib].
\]  \( (2) \)

An inspection of Equation \( (2) \) reveals that, in equilibrium, the bank sells the loan for liquidity reasons or if its monitoring system detects adverse information; the bank, when not subject to a liquidity shock, retains the loan if its monitoring reveals nothing.\textsuperscript{22}

Suppose the bank conjectures that the owner’s date-1 disclosure threshold is some \( \bar{x}_1 \). Based on the discussion above, the equilibrium face value of the debt \( \delta \) is given by the solution to the following break-even condition:

\[
\int_0^\delta xf(x) \, dx + \delta (1 - F(\delta)) - [\lambda + (1 - \lambda) q_f F(\bar{x}_1) q_b] k - \gamma I = 0.
\]  \( (3) \)

Note that the bank incurs the transaction cost in two scenarios: when facing a liquidity shock (which event occurs with probability \( \lambda \)); or in the absence of a liquidity shock, when its monitoring system detects adverse information which the firm withheld (with probability \( (1 - \lambda) q_f F(\bar{x}_1) q_b \)). When designing the debt contract specifying the face value \( \delta \), the bank anticipates the likelihood of non-disclosures at \( t = 1 \), which in turn affects its expected transaction costs from informationally motivated loan sales. Therefore, the competition among banks to offer a loan to the firm ensures that the face value of the loan \( \delta \) is determined such that the expected cash flows from the debt contract equal the amount lent at \( t = 0 \), as per condition \( (3) \).

### 3.2 The Owner’s Disclosure Strategy

We solve the disclosure game backwards. Clearly, at \( t = 2 \) the game is equivalent to a one-period disclosure model as in \textit{Dye (1985)} and \textit{Jung and Kwon (1988)}. The difference is that in our setting the date-2 threshold depends on the past history of play, since the history determines the market’s

\textsuperscript{22}Recall that we assume throughout our analysis that the transaction cost is not so prohibitive to prevent the bank to sell if it wishes to. In particular, we need that in equilibrium \( P_{D,2_{beg}}(nd_1,s) - k > \mathbb{E}[\bar{\nu}_D|\bar{x} < \bar{x}_1] \).
posterior beliefs at \( t = 2_{\text{beg}} \) and, hence, the equilibrium second-period disclosure threshold.

The following lemma formalizes the observation that regardless of the properties of the disclosure set, at any point in time the owner cannot derive any benefits from disclosing any realization of \( \bar{x} \) that is smaller than the face value of the debt, \( \delta \).

**Lemma 1** If an equilibrium exists, then the equilibrium set of disclosed values (at \( t = 1 \) or \( t = 2 \)) does not include any realization \( x \leq \delta \).

If the firm did not get any private information at \( t = 1 \), or if it made a disclosure of its private information \( \bar{x} \) at \( t = 1 \), then the owner has no decision to make at \( t = 2 \). Therefore, the only set of circumstances under which the owner has a non-trivial disclosure decision to make in the second period is when she was informed but chose to withhold information at \( t = 1 \). Hence, we can simplify the notation for the second-period disclosure threshold to \( \hat{x}_2(s) \) and \( \hat{x}_2(ns) \), depending on whether the bank has sold \((s)\) or not sold \((ns)\) the loan at \( t = 2_{\text{beg}} \), respectively.

Let the random variable \( \tilde{v}_E = \max\{\bar{x} - \delta, 0\} \) denote the residual claim to the owner. It is useful to define the function

\[
\Upsilon_E(z; B, C) = \frac{B \mathbb{E}[\tilde{v}_E] + C \times F(z) \mathbb{E}[\tilde{v}_E|\bar{x} < z]}{B + C \times F(z)},
\]

where the coefficients \( B \) and \( C \) will be determined by the public history \( h_{p,t} \). It is also useful to define \( \Psi_E(z; h_{p,t}) \) as the expected value of the firm’s equity given the public history \( h_{p,t} \), and given that \( z \) is the disclosure threshold as conjectured by the market at time \( t \in \{1, 2\} \).

Based on the preceding step, in any PBE (if one exists), the bank sells the loan at \( t = 2_{\text{beg}} \) only when it suffers a liquidity shock, or when its monitoring system detects adverse information. Then, using (4) and

\[23\]To minimize the clutter, we drop the index to time \( t \) when we use the notation \( z \) in this expression unless such indexing is required.
the notation \( nd_{1,2} = (nd_1, nd_2) \), it follows that

\[
\begin{align*}
\Psi_E (z; nd_1) &= \Upsilon_E (z; (1 - q_f), q_f) ; \\
\Psi_E (z; s, nd_{1,2}) &= \Upsilon_E (z; (1 - q_f) \lambda, q_f [q_b + (1 - q_b) \lambda]) ; \text{ and} \\
\Psi_E (z; ns, nd_{1,2}) &= \Upsilon_E (z; (1 - q_f) (1 - \lambda), q_f (1 - \lambda) (1 - q_b)) \\
&= \Upsilon_E (z; (1 - q_f), q_f (1 - q_b)) \quad (5)
\end{align*}
\]

By Lemma 1, both \( \hat{x}_2 (s) \) and \( \hat{x}_2 (ns) \) are strictly greater than \( \delta \). Consequently, the price of the equity given disclosures (along the equilibrium path) at any point in time is \( x - \delta \). If \( \hat{x}_2 (\cdot) > \hat{x}_1 \), then an informed owner who observes \( \hat{x} > \hat{x}_1 \) has already disclosed \( x \) at \( t = 1 \), and hence, has no other disclosure decision to make at \( t = 2 \). In contrast, if \( \hat{x}_2 (\cdot) < \hat{x}_1 \), then after observing \( \hat{x} \in (\hat{x}_2 (\cdot), \hat{x}_1) \), an informed owner does not disclose at \( t = 1 \), but discloses at \( t = 2 \). By construction of the thresholds, types \( \bar{x} = \hat{x}_2 (s) \) and \( \bar{x} = \hat{x}_2 (ns) \) are indifferent between disclosing and not disclosing \( \bar{x} \) following sale and no-sale of the loan, respectively. That is,

\[
\begin{align*}
\hat{x}_2 (s) - \delta &= \Psi_E (\min \{\hat{x}_2 (s), \hat{x}_1\}; s, nd_{1,2}) ; \text{ and} \\
\hat{x}_2 (ns) - \delta &= \Psi_E (\min \{\hat{x}_2 (ns), \hat{x}_1\}; ns, nd_{1,2}) .
\end{align*}
\]

The left-hand sides of the two equations represent the equity market value from disclosing \( \bar{x} = \hat{x}_2 (s) \) and \( \bar{x} = \hat{x}_2 (ns) \), respectively. The right-hand sides represent the expected equity value from non-disclosure following loan sale and no sale, respectively. Recall that \( P_{E,2} (s, nd_{1,2}) \) and \( P_{E,2} (ns, nd_{1,2}) \) denote the market price of the equity given non-disclosure at both \( t = 1 \) and \( t = 2 \),

\[
\text{For instance, the last equation in (5) above states that the posterior expected value of the firm’s equity given non-disclosure by the firm both at } t = 1 \text{ and } t = 2, \text{ and given no loan sale, is equal to the expression in (4) with coefficients } B = (1 - q_f) (1 - \lambda) \text{ and } C = q_f (1 - \lambda) (1 - q_b). \]
and following sale and no-sale by the bank, respectively. Then, (6) and (7) yield

\[ P_{E,2}(s, nd_{1,2}) = \hat{x}_2(s) - \delta; \quad \text{and} \]
\[ P_{E,2}(ns, nd_{1,2}) = \bar{x}_2(ns) - \delta. \] (8)

The owner’s decision at date 1 is more involved, since she must anticipate the bank’s equilibrium loan sale strategy and her own future disclosure behavior as a function of the bank’s action. By Lemma 1, the owner’s (current and future) payoff given disclosure at \( t = 1 \) is \( x - \delta > 0 \). Since she can always disclose at date 2, the disclosure threshold \( \tilde{x} = \hat{x}_1 \) solves the following fixed-point equation

\[ \hat{x}_1 - \delta = \alpha \Psi_E(\hat{x}_1; nd_1) \]
\[ + (1 - \alpha) [q_b + (1 - q_b) \lambda] \max \{ \hat{x}_1 - \delta, P_{E,2}(s, nd_{1,2}) \} \]
\[ + (1 - \alpha) (1 - q_b) (1 - \lambda) \max \{ \hat{x}_1 - \delta, P_{E,2}(ns, nd_{1,2}) \} . \] (10)

The left hand side of (10) denotes the firm’s payoff from disclosing \( \tilde{x} = \hat{x}_1 \). The right-hand side represents the owner’s expected utility from not disclosing at \( t = 1 \). It correctly anticipates the likelihood of the bank’s sale at \( t = 2_{beg} \) and the firm’s own sequentially rational response at \( t = 2 \) to the bank’s sale decision. Here, note that the term \( \max \{ \hat{x}_1 - \delta, P_{E,2}(s, nd_{1,2}) \} \) reflects the firm’s sequentially rational response at \( t = 2 \) to the bank’s loan sale: if the firm decides not to disclose at \( t = 2 \), then the equity price will be \( P_{E,2}(s, nd_{1,2}) \) as derived in (8) and if it decides to disclose \( \tilde{x} = \hat{x}_1 \) at \( t = 2 \), then the second-period equity price would simply be \( \hat{x}_1 - \delta \). Similar observations apply to the expression \( \max \{ \hat{x}_1 - \delta, P_{E,2}(ns, nd_{1,2}) \} \) which reflects the firm’s sequentially rational disclosure decisions at \( t = 2 \) following no loan sale.

**Lemma 2** If an equilibrium characterized by the date-1 threshold \( \hat{x}_1 \) exists, then it must be true that

\[ P_{E,2}(s, nd_{1,2}) < \hat{x}_1 - \delta < P_{E,2}(ns, nd_{1,2}) . \] (11)
Lemma (2) establishes the informativeness of the bank’s sale decision. No loan sale signifies that the bank does not possess adverse information about $\tilde{x}$. On the other hand, a sale can be due to liquidity reasons or to the bank having adverse information. Thus, in terms of the equity value, a loan sale is bad news, whereas no loan sale is good news. This produces the ranking in (11).

By virtue of Lemma 2 and plugging in (7), we can rearrange (10) as

$$\tilde{x}_1 - \delta = \theta \Psi_E (\tilde{x}_1; nd_1) + (1 - \theta) \Psi_E (\tilde{x}_1; ns, nd_{1,2}),$$

(12)

where

$$\theta = \frac{\alpha}{1 - (1 - \alpha) [q_b + (1 - q_b) \lambda]}.$$

The equilibrium values of $\tilde{x}_1$ and $\delta$ are determined by the solution to the system of two equations in two unknowns given by (3) and (12). Equation (12) reveals that the date-1 disclosure decision is based on assigning the weight $\theta$ to the date-1 payoff. The weight $\theta$ not only depends on the owner’s time preference $\alpha$, but also on the probability that the bank will actually sell the loan given that the owner is privately informed (i.e., $q_b + (1 - q_b) \lambda$). As this probability increases, the owner assigns a greater weight to the date-1 payoff, because a loan sale constitutes bad news. The assigned weight $\theta$ is the ‘effective’ myopia of the owner, since it represents how effectively the owner prefers earlier payoffs given the probability of a loan sale.

Proposition 1 below incorporates all the aforementioned observations and results, and establishes the existence of a unique PBE in this game.

**Proposition 1** For any given transaction cost $k \in \left[0, \bar{k}\right)$, in this multi-period game there exists a unique PBE. In such a PBE:

(i) The owner adopts the date-1 disclosure threshold $\hat{x}_1$ given by (12) such that she will disclose $\tilde{x}$ if and only if $\tilde{x} > \hat{x}_1$;

(ii) The bank sells the loan in the secondary market at $t = 2_{beg}$ whenever it suffers a liquidity shock or if its monitoring system detects adverse information about $\tilde{x}$; and
(iii) The owner who did not disclose her private information $\bar{x}$ at $t = 1$ discloses $\bar{x}$ at $t = 2$ if and only if $\bar{x} \in (\hat{x}_2(s), \hat{x}_1)$, where $\hat{x}_2(s)$ is given by \[6\].

Proposition [1] demonstrates how the firm’s owner manages her dynamic disclosures in a strategic manner across multiple periods, given that she anticipates a loan sale by the bank with some positive probability. The resulting rank ordering of the equilibrium disclosure thresholds (i.e., $\hat{x}_2(s) < \hat{x}_1 < \hat{x}_2(ns)$) yields the result in part (iii), which states that the loan sale prompts an owner who withheld her private information at an earlier point in time ($t = 1$) to disclose it at a later point in time ($t = 2$), provided her private information is moderately unfavorable (i.e., $\bar{x} \in (\hat{x}_2(s), \hat{x}_1)$).

The bank benefits from selling the loan whenever its monitoring system detects adverse information because the market is unable to distinguish an informationally motivated loan sale from one due to liquidity reasons. Nevertheless, a loan sale diminishes the posterior expected equity value of the firm, thereby prompting owner types with moderately unfavorable private information to disclose at $t = 2$. If the bank does not sell the loan at $t = 2_{\text{beg}}$, the owner never discloses her private information at $t = 2$ given that she had decided not to disclose it at $t = 1$ because of the result that $\hat{x}_1 < \hat{x}_2(ns)$. This way, the potential loan sale generates a positive option value from deferring the disclosure to the later date. At $t = 1$, owner types in the intermediate range $(\hat{x}_2(s), \hat{x}_1)$ wait for the bank’s action: if the bank does not sell the loan (good news), then they keep silent; if instead the bank sells (bad news), then they intervene with a disclosure to distinguish themselves from types below $\hat{x}_2(s)$.

### 3.3 Impact of Multiple Disclosure Opportunities on Equity Value

To understand the role of dynamic disclosures on the firm’s ex ante expected equity (and firm) value, we examine the one-time disclosure regime as a benchmark setting in which the owner is always informed of the bank’s equilibrium action. The potential loan sale by the bank effectively plays the role of an endogenously determined public signal that is realized after the first disclosure opportunity. It is important to note that unlike in Acharya et al. (2011), where the distribution of the public signal is exogenously specified, in our setting the firm’s disclosure decision influences the information generated by the bank’s monitoring system. Therefore, the information content and the ex-ante distribution of the bank’s equilibrium action is influenced by the firm’s date-1 disclosure strategy.
allowed to disclose only once at $t = 1$. We then compare these benchmark results to our *two-time disclosure* regime, where the owner has opportunities to disclose at both $t = 1$ and $t = 2$.

Let $\hat{x}_{ot}$ refer to the equilibrium disclosure threshold in the one-time regime, where the subscript $ot$ stands for “one-time”. The following lemma establishes that with multiple opportunities to disclose, the first-period threshold is higher relative to when the firm has only one opportunity to disclose, keeping the face value of the debt fixed.

**Lemma 3** Fix a common $\delta$ in both the one-time and the two-time disclosure regimes. Then, in the one-time regime there exists a unique date-1 disclosure threshold $\hat{x}_{ot}$. Moreover, we have $\hat{x}_{ot} < \hat{x}_1$, where $\hat{x}_1$ is the date-1 threshold in the two-time regime.

Lemma is fairly intuitive. In the absence of a second opportunity to disclose, the owner loses the option value from being able to disclose at $t = 2$ after the bank’s (potential) loan sale. Anticipating this, the owner tends to disclose more often at $t = 1$ when she does not have a second opportunity to disclose.

While Lemma 3 is derived for a given value of the debt $\delta$ uniform across two different disclosure regimes, Theorem 1 below endogenizes the value of $\delta$ in each regime as a function of the transaction cost $k$. Thus, Theorem 1 is able to predict the impact of different disclosure regimes and the bank’s monitoring quality on the equilibrium ex ante equity value at $t = 0$.

**Theorem 1** For any given positive $k \in (0, \bar{k})$, in the unique PBE:

(i) For all $q_b > 0$, the ex ante equity value at $t = 0$ is greater in the one-time disclosure regime than in the two-time regime; and

(ii) The ex ante equity value at $t = 0$ in the two-time disclosure regime would be the highest if:

(a) The owner’s information endowment were public;

(b) The owner were able commit to non-disclosure at $t = 1$; or

---

26Theorem 1 concerns the equity value at $t = 0$. If we define firm value as the sum of market value of equity and market value of debt, then each of these results extend to the firm value at $t = 0$. Indeed, the market value of debt at $t = 0$ is always equal to $\gamma d$, by the breakeven condition. Hence, equity and firm values move in the same direction.
Theorem 1 underlines the link between the probability of disclosures and the ex ante value of equity. Generally speaking, greater opportunities to disclose are viewed as being beneficial. However, part (i) of Theorem 1 cautions that one must also understand how a firm’s first-period disclosure behavior changes when it anticipates another opportunity to disclose in future. The option to disclose at $t = 2$ actually destroys value for equity holders when compared to the one-time disclosure regime, thus demonstrating that greater disclosure opportunities do not always benefit equity holders. The intuition here is the following. In the two-time disclosure regime, the option value to possibly avoiding disclosures at $t = 2$ (following no loan sale by the bank) induces the owner to disclose less often at $t = 1$. Note that the bank and the participants in the secondary loan market are asymmetrically informed only in the event of non-disclosure at $t = 1$. Hence, greater information withholding at $t = 1$ allows the bank to exploit more often its informational advantage. In other words, the two-time regime induces a higher probability of informationally motivated loan sales, thereby increasing the bank’s expected transaction costs at the time of the debt contracting stage. In equilibrium, the rational bank expects to be compensated in the form of a higher face value $\delta$.

Part (ii) of Theorem 1 identifies conditions under which the likelihood of informationally motivated loans sales by the bank drops to zero, thereby minimizing expected transaction costs.\(^\text{27}\) This leads to the highest possible equity value at $t = 0$. Condition (a) states that if the owner’s information were public, the bank’s loan sale cannot be informative to the markets. Condition (b) is somewhat trickier: while the owner’s information is private, if the owner could commit never to disclose at $t = 1$, then the posterior expected debt value from the bank’s perspective conditional on detecting the owner’s receipt of information would be the same as its prior mean. Consequently, the bank’s loan sale cannot provide additional information to the market about $\tilde{x}$. Finally, when $q_b = 0$, condition (c) directly rules out the possibility of informationally motivated loan sales.

\(^{27}\)Expected transaction costs following liquidity-motivated sales cannot be avoided by the disclosure behavior of the owner and, hence, are not the focus of the present analysis.
3.4 Leverage and Timeliness of Disclosures

To simplify our analysis, henceforth we fix transaction costs $k = 0$. When $k = 0$, the bank’s participation constraint simplifies, by the law of iterated expectations, to obtain the equilibrium face value of the debt $\delta$ as the solution to

$$E[\bar{v}_D] - \gamma l = 0.$$  \hspace{1cm} (13)

We use the label “timeliness of disclosures” to refer to the probability of disclosures at date 1, which equals $q_f (1 - F(\bar{x}))$. This subsection is primarily interested in examining the influence of leverage on the owner’s disclosure policy and the likelihood of informationally motivated loan sales by the bank, where leverage is measured by the debt-to-equity ratio, $\ell \equiv \frac{\gamma}{(1-\gamma)\ell}$. From the expression for leverage, one sees that a firm is more levered as $\gamma$ increases. This subsection also generates several other testable predictions about significant economic factors in our setting.

**Corollary 1** Fix the transaction cost $k = 0$. Then, in the unique PBE:

(i) The timeliness of disclosures is increasing in each of $\alpha, q_f, \lambda$ and decreasing in leverage $\ell$;

(ii) The likelihood of informationally motivated loan sales is increasing in leverage $\ell$ and decreasing in each of $(\alpha, \lambda)$.

Corollary illustrates the dynamic nature of the model. The greater $\alpha$ is, the higher the weight the owner assigns to the first-period payoff. Consequently, the option to wait for the the second period – in the hope that the bank will not sell – becomes less valuable. This leads more owner types to disclose in the first period and to a higher timeliness of disclosures. The intuition for $q_f$ is as in the standard Dye (1985) and Jung and Kwon (1988) model: as the probability of information receipt by the owner increases, the more skeptical beliefs of rational investors force the owner to disclose more often in the first period.

As $\lambda$ increases, the bank faces liquidity shocks more often. This has two consequences. First, the owner expects a loan sale (bad news for equity) more often, which decreases the option value
to delay the disclosure. Second, the equity price drops less after loan sales, since these become less indicative of information withholding. The latter effect increases the option value. Overall, the former effect prevails, thereby leading to more disclosures at date 1.

For what concerns leverage, there are also two countervailing forces at work. A higher leverage needs to be compensated with a higher face value of the debt. As $\delta$ increases, both the disclosure payoff, $x - \delta$, and the non-disclosure payoff, $\Psi_E$, decrease. Disclosing $x$ yields a lower payoff, which reduces the incentive to disclose at $t = 1$. Corollary 1 reveals that the former effect dominates the latter. In other words, financing the project with a greater leverage leads to a higher date-1 disclosure threshold and less timely disclosures. Part (ii) of Corollary 1 follows from applying a similar logic.

We next proceed to analyze the impact of the bank’s monitoring technology on the timeliness of firm’s disclosures. In principle, the overall impact of a higher $q_b$ is not clear, since the effect of better monitoring on the option value from deferring the disclosure to a later period is ambiguous. Interestingly, there is a non-monotonic relation between the firm’s disclosure policy and the bank’s monitoring level. The timeliness of disclosures is minimized at a unique interior level of monitoring and is maximized at the boundaries. This is formally presented in the following theorem.

**Theorem 2** Fix the transaction cost $k = 0$. Then, in the unique PBE, there exists a monitoring level $q^*_b$, such that the timeliness of disclosures is decreasing in $q_b$ for $q_b \in [0, q^*_b)$ and increasing for $q_b \in (q^*_b, 1]$.

Theorem 2 characterizes the relation between the disclosure threshold in the first period and the level of bank monitoring. Since timeliness of disclosures is associated with high quality reports, one might, at a first blush, imagine that greater monitoring by the bank would induce the owner to disclose in a more timely fashion. Our result establishes that this is not necessarily true. First, a more intense monitoring produces a positive effect on the firm’s option value to defer its disclosure to $t = 2$. With a higher value of $q_b$, no loan sale by the bank is viewed as more likely to be

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28Levitt (1998) defines good accounting standards as those that “produce financial statements that report events in the periods in which they occur, not before, and not after.”
derived from an uninformed owner. This effect increases the firm’s option value from deferring its disclosure to the second period. At the same time, an increase in \( q_b \) also produces a second, negative effect which follows from a steeper reduction in the equity price because a loan sale is more likely to be informationally motivated. Besides, for a higher \( q_b \) the informed owner expects loan sales (bad news) more frequently, which decreases the expected value of the option to wait and consequently, the disclosure threshold. The combination of these effects produces the non-monotonic overall effect in Theorem 2. The positive effect is dominant when the informed owner is not likely to be monitored successfully (i.e., when \( q_b \) is sufficiently low), whereas the negative effect is dominant when the bank’s monitoring technology is likely to detect adverse information (i.e., for \( q_b \) sufficiently high). Figure 2 illustrates how the timeliness of disclosures and the date-1 threshold vary in the bank’s monitoring level. The figure was generated for normally distributed cash flows with \((\mu, \sigma) = (2000, \sqrt{100})\) and the following model parameters: \( I = 1000, \gamma = 0.3, q_f = 0.8, \lambda = 0.1, \) and \( \alpha = 0.2. \)

![Figure 2: Timeliness (left) and date-1 disclosure threshold (right) as functions of monitoring quality.](image)

At the extremes \( q_b = 0 \) and \( q_b = 1 \), the date-1 thresholds are identical and minimal. This obtains since in either case the option value is zero. In the case \( q_b = 0 \), the bank’s action is completely uninformative, as the bank itself is not privately informed. In the case \( q_b = 1 \), the bank’s action is
instead informative, but an informed owner knows that if she chooses to withhold, then for certain the bank will sell the loan, implying bad news for equity.

4 Public News and Loan Sales

The previous section examined, among other aspects, the role of private information acquired by the bank in its decision to sell its loan asset. The goal of this section is to understand how public information would influence the loan sales by the bank. We follow Acharya et al. (2011) and assume cash flows have the specific form \( \tilde{x} = \mu (\tilde{y}) + \sigma (\tilde{y}) \tilde{\omega} \), where \( \mu (\cdot) \) and \( \sigma (\cdot) \) are commonly known deterministic functions of the public signal \( \tilde{y} \), \( \sigma (y) > 0 \) for all \( y \), \( (\tilde{y}, \tilde{\omega}) \) are independent random variables, \( \mathbb{E}[\tilde{\omega}] = 0 \), and \( \mathbb{V}[\tilde{\omega}] = 1 \).\(^{29}\) Furthermore, for convenience we assume that \( \tilde{\omega} \) is continuously distributed with a strictly positive density over the real line and we denote its cumulative distribution function by \( F_{\tilde{\omega}} (\cdot) \). Suppose that after the owner signs the debt contract at \( t = 0 \), but prior to the first opportunity to disclose at \( t = 1 \), a signal \( \tilde{y} \) about \( x \) is publicly realized. Effectively, the public news \( \tilde{y} = y \) allows all agents to update their beliefs about the distribution of \( \tilde{x} \). The realization \( \mu (y) \) is the posterior estimate of the mean of \( \tilde{x} \), and \( \sigma (y) \) is the posterior estimate of its volatility. Observe that after the public news has been released, the posterior default probability is given by

\[
\Pr [\tilde{x} < \delta | y] = F_{\tilde{\omega}} \left( \frac{\delta - \mu (y)}{\sigma (y)} \right).
\]

Interestingly, and perhaps surprisingly, we find that the probability of loan sales is directly linked to the posterior default probability of the firm. The following Theorem identifies a necessary and sufficient condition under which the firm’s propensity to make a voluntary disclosure at \( t = 1 \) decreases, and the consequent likelihood of loan sales by the bank increases.

**Theorem 3** The bank is more likely to sell the loan in the secondary market if and only if the probability of default increases as a result of the public news \( y \).\(^{30}\)

\(^{29}\)The latter two assumptions on the distribution of \( \tilde{\omega} \) are not crucial for our analysis, but they allow us to interpret \( (\mu, \sigma) \) as the mean and standard deviation of \( \tilde{x} \).

\(^{30}\)Theorem 3 holds for any \( k \) sufficiently small.
Theorem 3 demonstrates that the arrival of public news prior to the opportunity to disclose has a particular impact on the probability of loan sales: the public news $\tilde{y}$ affects the probability of loan sales in the same direction that it affects the default probability. Since disclosure by the owner resolves all uncertainty, the bank’s information advantage from its monitoring manifests itself only in the event of non-disclosure at $t = 1$. It follows that the likelihood of informationally motivated sales is directly proportional to the likelihood of information withholding. As we show, the result in Theorem 3 follows because as the probability of default increases, the owner is more likely to withhold information at $t = 1$.

Acharya et al. (2011) point out that public news has a twofold effect on the probability of information withholding: a distribution effect and a threshold effect. The former effect consists in the fact that $y$ changes the posterior $\mu (y)$ and volatility $\sigma (y)$ of $\tilde{x}$, thereby changing the probability of the firm withholding its disclosure for a fixed threshold $\tilde{x}_1$. The latter is an indirect effect which occurs because a different posterior distribution of $\tilde{x}$ implies a different equilibrium threshold. In other words, $\tilde{x}_1 (y)$ itself is a function of the public news. In voluntary disclosure models such as Acharya et al. (2011), the two effects usually exactly offset each other. Here, on the contrary, the distribution effect dominates the threshold effect. The intuition for this phenomenon is that $\tilde{y}$ is a linear signal of the cash flow, $\tilde{x}$. Yet, the disclosure decision is taken to maximize the equity value, $\max \{\tilde{x} - \delta, 0\}$. From a statistical point of view, the residual claim to equity holders is a censored version of the random variable $\tilde{x}$. Therefore, equity prices respond less starkly to the public news than does the distribution of $\tilde{x}$.

A significant feature of the necessary and sufficient condition that Theorem 3 identifies is that it does not impose any monotonicity restrictions on posterior mean $\mu (y)$ and volatility $\sigma (y)$. The default probability encompasses both these posterior mean and variance effects on the disclosure probability into one popular metric. In this way, Theorem 3 provides one possible theoretical explanation for the empirical finding in Drucker and Puri (2009) of a negative association between the likelihood of a loan sale by banks and the distance-to-default of the client firm.
5 Informational Effects on Moral Hazard Problem

This section examines a moral hazard problem with respect to an unobservable action taken by the owner. The goal is to understand how the moral hazard problem is influenced by the bank’s monitoring system. We identify conditions under which a more effective bank monitoring is value-enhancing because it mitigates moral hazard. In order to investigate the informational effect of the bank monitoring, we revert to the original model in Section 2 with no public signal. Suppose the owner can affect the distribution of the firm’s cash flows $\tilde{x}$ through a costly private action $a \geq 0$. In particular, $\tilde{x} = a + \tilde{\omega}$, where $\tilde{\omega}$ is continuously distributed over the real line with a strictly positive density $f_{\omega}(\cdot)$. In other words, a higher action shifts upwards the distribution of the cash flows.

As is common in the literature, the owner bears a personal quadratic cost $\eta a^2 / 2$ from taking the action $a$, where $\eta > 0$ is a known parameter. Throughout the section, we assume that $E[\tilde{\omega}] > I$, so that the project has a positive NPV even if the owner takes no action.

For simplicity, we assume that the owner chooses her private action after the debt contract is signed, but before the realization of $\tilde{\omega}$. As a benchmark, we first examine a first-best setting where the owner’s action is assumed to be publicly observable to determine the extent of inefficiency occurring from the owner’s action being unobservable in the second best. Let $a^{FB}$ and $a^{SB}$ denote the equilibrium actions in the first and second best settings, respectively.

The first order condition reveals that $a^{FB}$ is independent of $q_b$, since when all players observe the owner’s action, the law of iterated expectations applies and only the ex ante distribution of the cash flows matters. In contrast, the second best action $a^{SB}$ depends on the effectiveness of the bank’s monitoring, $q_b$. We therefore write the second-best action as $a^{SB}(q_b)$.

Both in the first and second best, the face value of the debt is set such that the bank breaks even based on the bank’s anticipation of the action ($\hat{a}$) that the owner will subsequently take. Thus, the face value of the debt, $\delta(\hat{a})$, solves

$$\delta(\hat{a}) \left[ 1 - F_{\omega}(\delta(\hat{a}) - \hat{a}) \right] + \int_{-\hat{a}}^{\delta(\hat{a}) - \hat{a}} (\hat{a} + \omega) f_{\omega}(\omega) d\omega - \gamma I = 0. \quad (14)$$
Note that in this case, the owner can influence, through her action, the terminal value of the firm. Therefore, dropping the realized firm value from the owner’s utility function would entail a loss of generality. To prevent this, we assume that the owner takes the debt contract \( \delta (\hat{a}) \) as given and selects \( a \) to maximize

\[
\mathbb{E} \left[ \beta (\alpha P_1 + (1 - \alpha) P_2) + (1 - \beta) \max \left\{ a + \tilde{\omega} - \delta (\hat{a}), 0 \right\} \right] - \frac{\eta a^2}{2},
\]

for some \( \beta \in (0, 1) \).\(^{31}\) In the above expression, the prices \( P_1 (\cdot) \) and \( P_2 (\cdot) \) are computed based on the conjectured action \( \hat{a} \), whereas the owner’s expected payoff is computed using the actual action \( a \). The parameter \( \alpha \), as before, represents the owner’s time preference for the early equity price versus the price after the bank’s action; \( \beta \), in contrast, parametrizes the extent to which the owner is interested in the market’s perception of firm value as opposed to the fundamental.

In the first best scenario, \( (15) \) reduces to

\[
\int_{\delta (\hat{a}) - a}^{\infty} (a + \omega - \delta (\hat{a})) f_\omega (\omega) d\omega - \frac{\eta a^2}{2}.
\]

This follows from the law of iterated expectation. The explicit expression for \( (15) \) in the second best scenario is more intricate, and therefore, is included in our analysis in the Appendix.

Note that the ex ante total value of the firm, \( \mathbb{E} \left[ \max \{ a + \tilde{\omega}, 0 \} \right] \), is increasing in the equilibrium action \( a \). Since in equilibrium the bank breaks even, an increase in the equilibrium ex ante total value of the firm translates into an increase in the ex ante value of equity \( \mathbb{E} \left[ \max \{ a + \tilde{\omega} - \delta (a), 0 \} \right] \). Our efficiency criterion is represented by the ex ante total value of the firm. Based on this welfare metric, a higher action increases firm value.\(^{32}\)

The following theorem demonstrates the role of bank monitoring in determining the magnitude

\(^{31}\)Observe that, as noted before, including the weight \( (1 - \beta) \) on the residual value in the owner’s utility function would not have affected any of the results derived in the previous sections.

\(^{32}\)The results would remain unchanged if we were to take into account the personal cost \( \eta a^2 / 2 \). The action that maximizes the ex ante total welfare net of the owner’s personal cost, \( \mathbb{E} \left[ \max \{ a + \tilde{\omega}, 0 \} \right] - \eta a^2 / 2 \), solves \( 1 - F_\omega (-a) = \eta a \), and is strictly above the first-best \( a^{FB} \). As we establish, the second-best \( a^{SB} (q_b) \) is always below \( a^{FB} \). Hence, a higher second-best action is welfare-enhancing even if \( \mathbb{E} \left[ \max \{ a + \tilde{\omega}, 0 \} \right] - \eta a^2 / 2 \) were the welfare measure of interest.
of the inefficiency in the second best setting.

**Theorem 4** Suppose that cash flows are given by \( \tilde{x} = a + \tilde{\omega} \), that the owner bears a personal cost \( \eta a^2/2 \) from her action, and that \( f_\omega(\omega) < \eta \) for all \( \omega \). Then, for any given value of \( q_b \) and \( k = 0 \), there exists an equilibrium in each of the first-best and second-best settings. Further, in equilibrium:

(i) The owner’s second-best equilibrium action is less than her first-best action (i.e., \( a^{SB}(q_b) < a^{FB} \) for all \( q_b \in [0, 1] \));

(ii) There exists a \( q_b^* \in (0, 1) \) such that for \( q_b \geq q_b^* \) the owner’s second-best action \( a^{SB}(q_b) \) is increasing in \( q_b \)\(^{33} \) and

(iii) The inefficiency is minimal at \( q_b = 1 \).

The technical condition \( f_\omega(\omega) < \eta \) is sufficient to guarantee existence, and it is satisfied by a vast class of distributions\(^{34} \). Part (i) of Theorem 4 highlights the inefficiency as a consequence of the fact that in the second-best scenario the owner does not fully internalize the stock price benefit of a higher action. When the owner’s action is publicly observable, a higher action is incorporated directly into the non-disclosure prices. In contrast, in the second-best scenario the non-disclosure prices rely only on the conjectured \( \hat{a} \), which is not affected by the owner’s actual action. Therefore, the owner internalizes the stock price effects of her action only when the realized cash flows are subsequently disclosed in equilibrium, either at \( t = 1 \) or \( t = 2 \). This leads her to take a lower action than what she would have taken had the action been publicly observable.

For sufficiently large values of \( q_b \), part (ii) of Theorem 4 predicts that an increase in the bank’s monitoring quality \( q_b \) mitigates the ex ante moral hazard problem by encouraging the owner to take a higher action at \( t = 0 \) in anticipation of a prospective loan sale. The intuition for our finding

\(^{33}\text{If the equilibrium is not unique, this statement applies to the equilibria with the lowest and highest action (e.g., see Milgrom and Roberts (1994)).}

\(^{34}\text{If } \omega \text{ is normally distributed with any mean, this condition simply imposes a lower bound on the volatility, } \sigma > 1 / (\sqrt{2\pi \eta}) \text{. For instance, with } \eta = 0.2 \text{ one needs approximately } \sigma > 2 \text{, and with } \eta = 0.01 \text{ one needs } \sigma > 40.\)
in part (ii) is apparent from an examination of the relevant part of the first-order condition for the action choice. The owner’s willingness to take a higher action is determined by:

\[
1 - F(\tilde{x}_1(q_b)) + (1 - \alpha)(\lambda + (1 - \lambda)q_b)[F(\tilde{x}_1(q_b)) - F(\tilde{x}_2(s)(q_b))],
\]

which expression can be interpreted as a ‘discounted’ expected probability of disclosure from the owner’s perspective. 35 The inefficiency is mitigated whenever an increase in \(q_b\) is associated (for any given action \(a\)) with a higher (17). Part (ii) of Theorem 4 identifies a condition on \(q_b\) under which this occurs. When \(q_b \geq q_b^*\), a higher \(q_b\) induces greater expected probability of disclosure via three effects. First, by exploiting the hump shape of \(\tilde{x}_1(q_b)\) explained in Section 3.4, \(q_b\) is chosen so to ensure that the timeliness of disclosures is increasing in \(q_b \geq q_b^*\) for any fixed action \(a\). Second, a higher \(q_b\) increases the probability of the bank strategically selling the loan, which in turn increases the likelihood of disclosure at \(t = 2\) by owners that observe \(x \in (\tilde{x}_2(s), \tilde{x}_1)\). Finally, \(\tilde{x}_2(s)\) is monotonically decreasing in \(q_b\), since a higher \(q_b\) implies that it is more likely that the sale was triggered by an informed bank at \(t = 2\). Overall, the ‘discounted’ expected probability of disclosure in (17) increases, thereby increasing the equilibrium second-best action and mitigating the inefficiency.

Part (iii) is proven by means of the following argument. Both thresholds \(\tilde{x}_1\) and \(\tilde{x}_2(s)\) are minimal at \(q_b = 1\) for any given \(a\). The probability of a successful monitoring is the highest at \(q_b = 1\). Overall, for any given \(a\), the discounted expected probability of disclosure is maximized in the corner solution \(q_b = 1\). 36 This in turn implies that the second-best action is maximal at \(q_b = 1\).

Datta et al. (1999) find that the presence of bank debt lowers the at-issue yield spreads for first

35 The actual expected probability of disclosure in the dynamic game is

\[
1 - F(\tilde{x}_1(q_b)) + (\lambda + (1 - \lambda)q_b)[F(\tilde{x}_1(q_b)) - F(\tilde{x}_2(s)(q_b))],
\]

and does not include the preference parameter \((1 - \alpha)\), which effectively discounts the future disclosure probability by the weight that the owner assigns to the second-period price.

36 This can be seen by rewriting the discounted expected disclosure probability as

\[
1 - F(\tilde{x}_1)[1 - (1 - \alpha)(\lambda + (1 - \lambda)q_b)] - (1 - \alpha)(\lambda + (1 - \lambda)q_b)F(\tilde{x}_2(s)).
\]
public straight bond offers. They suggest that building reputation through bank loans reduces the firm’s cost of public debt. However, our analysis provides another possible explanation: to the extent that private debts such as bank loan entail generally a more effective monitoring by the lenders than public debt (with a widely disbursed set of bondholders), such a more effective monitoring can mitigate the firm’s investment moral hazard problem. Consistent with our result, Krishnaswami et al. (1999) find that firms with greater debt-related (potential) moral hazard problems use higher proportions of private debt.

6 Empirical Implications

Our analysis of the interactions between dynamic disclosure decisions and loan sales generates several testable predictions, a subset of which includes the following.

1. Our analysis predicts that higher leverage leads to a lower probability of voluntary disclosure.

2. We predict the likelihood of voluntary disclosures by firms to be higher after loan sales by banks than following no loan sales.

3. Drucker and Puri (2009) find that banks tend to sell loans of highly levered firms that contain additional covenants. Moreover, they find that greater distance-to-default is associated with a higher likelihood of a loan sale. This latter finding is consistent with our analysis. We suggest an intermediate mechanism which could drive this result, as well as that the timeliness of disclosures and the probability of loan sales (particularly, informationally motivated sales) are inversely related.

4. Public debt holders are usually widely dispersed and would mostly find it difficult to implement an as effective a monitoring of the borrower as banks do.\footnote{One third of large U.S. firms even have a banker on the board (see Kroszner and Strahan (2001)).} Even if public debt holders are able to replicate the same level of effective monitoring of banks (for instance,
through an intermediary such as a trust), any communication by the monitor to the widely-
dispersed set of public debt holders would unlikely remain private. Consequently, a debt sale
by public debt holders is unlikely to provide new information, contrary to a bank’s loan sale.
Therefore, a testable implication of our analysis is that stock prices of borrowing firms, on
average, are more likely to fall following the sale by a private debt holder, such as a bank,
than following the sale by a public debt holder.

5. This paper also predicts a non-monotonic effect of the banks’ monitoring technology on the
timeliness of disclosures by their client firms. Namely, a greater informational advantage
(higher $q_b$) leads firms to disclose more timely if $q_b$ is high enough, and to disclose less
timely if $q_b$ is low enough. By ranking firms according to the informational advantage of
their lenders (for example, by the level of private debt relative to public debt, or by the
level of analysts coverage), one should expect the voluntary disclosure probability to be U-
shaped. In addition, the non-monotonicity of the early disclosure threshold suggests that the
likelihood of a loan sale in the secondary market is potentially non-monotonic as well.

6. Banks face liquidity shocks for a variety of reasons (regulation requirements, repaying de-
positors, drop in asset values, non-performing assets, and so forth). Depending on the period,
some banks must sell their loans in the secondary market in order to comply with capital
adequacy requirements and other business contingencies. Our model predicts that, during
periods in which liquidity motives are more likely, the voluntary disclosure probability of
levered firms is higher.

7. Our model predicts that, beyond a certain level, better monitoring decreases the inefficiency
associated with moral hazard. This implies that, ceteris paribus, bank monitoring benefits
equity holders. An interesting, albeit challenging, empirical question is to disentangle the
effect we have identified from the reputation-building argument (Diamond (1991)).
7 Conclusion

This paper analyzes a multi-period voluntary disclosure game in which the presence of the secondary market for bank loans affects the disclosure decision of a levered firm. When the owner has two opportunities to disclose her private information, the potential loan sale by the bank creates an option value to delay the disclosure. We show that the value of the option to wait can increase or decrease in the monitoring level. Therefore, the timeliness of disclosures is non-monotonic in the monitoring effectiveness of the bank.

Further, the opportunity to disclose after the potential loan sale destroys value ex ante, relative to when the owner can only disclose once. This result follows from a greater likelihood of informationally motivated sales by the bank, which increases the expected transaction costs that the owner bears in the ex ante design of the debt contract.

We predict that the arrival of public news (e.g., as generated by analysts’ reports or concerning the macroeconomy) prior to the first opportunity for the firm to disclose has an impact on the timeliness of disclosures, which in turn affects the likelihood of an informationally motivated loan sale. Furthermore, it is shown that the probability of loan sales increases as a result of external public news if and only if the posterior default probability also increases. This finding provides one possible explanation for the empirically observed regularity of banks’ tendency to sell loans of firms which are closer to default (e.g., [Drucker and Puri (2009)]).

We also demonstrate that if the bank’s monitoring level is sufficiently high, a greater monitoring increases the overall expected probability of disclosure (in both periods), which in turn mitigates the agency costs.

Section generates several hypotheses for testing. To the best of our knowledge, the economic consequences of the interaction between financing decisions and strategic disclosures has not been addressed empirically. We believe that this line of research is essential as well as promising.
Proof of Lemma 1. Suppose, by contradiction, that in equilibrium the owner discloses a realization $x \leq \delta$ at date $t$. For such a realization, $P_{E,s} = 0$ for all dates $s \geq t$ and her payoff is zero. Since non-disclosure might be the result of an uninformed owner, the non-disclosure price must be strictly positive. Thus, non-disclosure of $x$ is a profitable deviation from the equilibrium path. 

Lemma 4 Let $z^*$ and $z^{**}$ solve, respectively, 

$$z^* - \delta = \Upsilon_E (z^*; B, C), \quad \text{and}$$
$$z^{**} - \delta = \Upsilon_E (z^{**}; B', C'),$$

where $\Upsilon_E (\cdot)$ is defined in (4). If $\forall z \in [\delta, \infty)$ the inequality 

$$\frac{B}{B + CF(z)} > \frac{B'}{B' + CF(z)}$$

(A1) holds, then $z^* > z^{**}$.

Proof of Lemma 4. It can be demonstrated that the Minimum Principle in Proposition 1 of Acharya et al. (2011) for the case of an all-equity firm continues to hold for the levered firm in our setting. Formally, if $z^*$ satisfies $z^* - \delta = \Upsilon_E (z^*; B, C)$, then, $z^*$ is the unique minimizer of the function $\Upsilon_E (\cdot; B, C)$ due to convexity. Recall that $\Upsilon_E (\cdot)$ is a convex combination of $E[\bar{v}_E]$ and $E[\bar{v}_E | \bar{x} < z]$. Since $E[\bar{v}_E] > E[\bar{v}_E | \bar{x} < z]$, condition (A1) implies that $\forall z < \infty$

$$\Upsilon_E (z; B, C) > \Upsilon_E (z; B', C').$$

(A2)

As a consequence, 

$$z^* - \delta = \Upsilon_E (z^*; B, C) > \Upsilon_E (z^*; B', C') > \Upsilon_E (z^{**}; B', C') = z^{**} - \delta,$$

where the first inequality follows directly from (A2) and the second inequality from the Minimum Principle. 

Proof of Lemma 2. Claim. $P_{E,2}(s, nd_{1,2}) \geq \tilde{x}_1 - \delta$ implies that $P_{E,2}(ns, nd_{1,2}) > \tilde{x}_1 - \delta$.

Proof of the Claim. Suppose by contradiction that $P_{E,2}(s, nd_{1,2}) \geq \tilde{x}_1 - \delta$ and $P_{E,2}(ns, nd_{1,2}) < \tilde{x}_1 - \delta$. The proof is derived by simple algebraic manipulation on $\Upsilon_E (z; B, C)$ and by showing that $\Upsilon_E (z; B, C) > z^* - \delta$, $\forall z \neq z^*$. Details are available from the authors.
\[ \hat{x}_1 - \delta \] (equivalently, that \( \hat{x}_2 (ns) < \hat{x}_1 \)). Then, (7) becomes

\[ \hat{x}_2 (ns) - \delta = \Psi_E (\hat{x}_2 (ns); ns, nd_{1,2}) = \Upsilon_E \left( \hat{x}_2 (ns); (1 - q_f), q_f (1 - q_b) \right). \]

However, \( \forall \delta \in [\delta, \infty) \) we have

\[ \frac{(1 - q_f)}{(1 - q_f) + q_f F(z) (1 - q_b)} > \frac{(1 - q_f) \lambda}{(1 - q_f) \lambda + q_f F(z) [q_b + (1 - q_b) \lambda]}, \]

which by Lemma 4 implies the contradiction \( x_2^* < \hat{x}_2 (ns) < \hat{x}_1 \).

**Part 1.** \( P_{E,2} (s, nd_{1,2}) < \hat{x}_1 - \delta \) (i.e., \( \hat{x}_2 (s) < \hat{x}_1 \)).

**Proof of Part 1.** Suppose, by contradiction, that \( P_{E,2} (s, nd_{1,2}) \geq \hat{x}_1 - \delta \) (i.e., that \( \hat{x}_2 (s) \geq \hat{x}_1 \)). Then, (6) boils down to

\[ \hat{x}_2 (s) - \delta = \Psi_E \left( \hat{x}_2; s, nd_{1,2} \right) = \Upsilon_E \left( \hat{x}_1; (1 - q_f) \lambda, q_f [q_b + (1 - q_b) \lambda] \right) \]

and, by the Minimum Principle, the solution to

\[ x_2^* - \delta = \Psi_E (x_2^*; s, nd_{1,2}) = \Upsilon_E \left( x_2^*; (1 - q_f) \lambda, q_f [q_b + (1 - q_b) \lambda] \right) \]

is such that \( x_2^* \in (\hat{x}_1, \hat{x}_2 (s)) \). Given \( P_{E,2} (s, nd_{1,2}) \geq \hat{x}_1 - \delta \) (by our initial supposition) and \( P_{E,2} (ns, nd_{1,2}) > \hat{x}_1 - \delta \) (implied by the Claim), we conclude that \( \hat{x}_1 \) solves the following fixed-point equation (see Equation (10)),

\[ \hat{x}_1 - \delta = \alpha \Upsilon_E \left( \hat{x}_1; (1 - q_f), q_f \right) + (1 - \alpha) [q_b + (1 - q_b) \lambda] P_{E,2} (s, nd_{1,2}) \]

\[ + (1 - \alpha) (1 - q_b) (1 - \lambda) P_{E,2} (ns, nd_{1,2}). \]

That is, \( \hat{x}_1 - \delta \) is a convex combination of three components, of which \( P_{E,2} (s, nd_{1,2}) = \Psi_E (\hat{x}_1; s, nd_{1,2}) \) is strictly the smallest. This implies that \( \hat{x}_1 > \hat{x}_2 (s) \), a contradiction to the initial supposition. Thus, it must be \( P_{E,2} (s, nd_{1,2}) < \hat{x}_1 - \delta \).

**Part 2.** \( P_{E,2} (ns, nd_{1,2}) > \hat{x}_1 - \delta \) (i.e., \( \hat{x}_2 (ns) > \hat{x}_1 \)).

**Proof of Part 2.** Suppose, by contradiction, that \( P_{E,2} (ns, nd_{1,2}) \leq \hat{x}_1 - \delta \) (i.e., that \( \hat{x}_2 (ns) \leq \hat{x}_1 \)). Using \( P_{E,2} (s, nd_{1,2}) < \hat{x}_1 - \delta \) from the first part of the lemma, (10) boils down to

\[ \hat{x}_1 - \delta = \Upsilon_E \left( \hat{x}_1; (1 - q_f), q_f \right). \]

If this is the case, Lemma 4 implies that \( \hat{x}_2 (ns) > \hat{x}_1 \), since

\[
\frac{(1-q_f)}{(1-q_f)+F(z)q_f(1-q_b)} > \frac{(1-q_f)}{(1-q_f)+F(z)q_f}
\]

holds \( \forall \delta \in [\delta, \infty) \). Again, we reach a contradiction. \( \blacksquare \)
Lemma 5 Suppose that multiple threshold equilibria exist. Let $\hat{x}_1$ and $\hat{x}_1'$ be the threshold of two such equilibria, and let $\hat{x}_2(s) - \delta$ and $\hat{x}_2'(s) - \delta$ be the respective date-2 non-disclosure prices after histories in which the bank has sold the debt. Then, $\hat{x}_2(s) = \hat{x}_2'(s)$.

Proof of Lemma 5. By Lemma 2, $\hat{x}_2(s) < \hat{x}_1$ and $\hat{x}_2'(s) < \hat{x}_1'$. It follows from Equation (6) that both $\hat{x}_2(s)$ and $\hat{x}_2'(s)$ are given by the unique solution to

$$\hat{x}_2(s) - \delta = \Psi_E(\hat{x}_2(s); (1 - q_f) \lambda, q_f [q_b + (1 - q_b) \lambda]),$$

whence $\hat{x}_2(s) = \hat{x}_2'(s)$. ■

Proof of Proposition 1. We structure the proof as follows. First (in Steps 1-3 below), we take $\delta$ as given and we prove existence of a unique tuple $(\hat{x}_1, \hat{x}_2(s), \hat{x}_2(ns))$ simultaneously satisfying (6), (7), and (12). Second (in Step 4), we argue that for $k$ small enough there exists a unique pair $(\hat{x}_1, \delta)$ that solves the system of equations given by (12) and the break-even condition (5).

Step 1. For a fixed $\delta$, there exists at least one solution $\hat{x}_1$ to Equation (12).

Proof of Step 1. As $\hat{x}_1 \downarrow \delta$, the left-hand side of (12) converges to zero, whereas its right-hand side converges to $E[\hat{v}_E]$. As $\hat{x}_1 \uparrow \infty$, the left-hand side goes to infinity, whereas the right-hand side converges to $E[\hat{v}_E]$. By continuity, a solution $\hat{x}_1 \in (\delta, \infty)$ exists.

Step 2. For a fixed $\delta$ and a fixed $\hat{x}_1$ from Step 1, there exists a unique $\hat{x}_2(s)$ that solves (6) and a unique $\hat{x}_2(ns)$ that solves (7). Moreover, these $\hat{x}_2(s)$ and $\hat{x}_2(ns)$ satisfy the ordering in (11).

Proof of Step 2. Observe that $\hat{x}_1$ is such that $\hat{x}_1 - \delta > \Psi_E(\hat{x}_1; nd_1)$ and $\hat{x}_1 - \delta < \Psi_E(\hat{x}_1; ns, nd_{1,2})$. Consider Equation (6). As $\hat{x}_2(s) \downarrow \delta$ the left-hand side tends to zero, whereas the right-hand side tends to a positive number. As $\hat{x}_2(s) \uparrow \hat{x}_1$ the left-hand side converges to $\hat{x}_1 - \delta$, whereas the right-hand side to $\Psi_E(\hat{x}_1; s, nd_{1,2})$. A solution $\hat{x}_2(s) \in (\delta, \hat{x}_1)$ exists because $\Psi_E(\hat{x}_1; s, nd_{1,2}) < \Psi_E(\hat{x}_1; nd_1) < \hat{x}_1 - \delta$. Uniqueness follows from standard arguments. Existence of a solution $\hat{x}_2(ns) > \hat{x}_1$ to Equation (7) is shown in a similar manner.

Step 3. For a fixed $\delta$, the tuple $(\hat{x}_1, \hat{x}_2(s), \hat{x}_2(ns))$ identified in Steps 1 and 2 is unique.

Proof of Step 3. We show the uniqueness of the solution by contradiction. The idea in this part is to show that two distinct equilibria lead to different ex ante payoffs, while by the law of iterated expectations the owner’s ex ante payoff must be $E[\hat{v}_E]$ in any equilibrium. Suppose that (12) admitted two distinct solutions $\hat{x}_1$ and $\hat{x}_1'$. Assume, without loss of generality, that $\hat{x}_1' > \hat{x}_1$. Owners that observe $x \geq \hat{x}_1'$ disclose in both equilibria, and thus obtain the same payoff. Informed owners that observe $x \in [\hat{x}_1, \hat{x}_1']$ must be weakly better off in the equilibrium that is characterized by $\hat{x}_1'$, since they could disclose and get $x \leq \hat{x}_1'$, the same payoff they get in the $\hat{x}_1$ equilibrium, and

39 Note that the Minimum Principle for the leveraged firm cannot be invoked here, because it does not apply to Equation (12). Indeed, the derivative with respect to $z$ of $\theta \Psi_E(z; nd_1) + (1 - \theta) \Psi_E(z; ns, nd_2)$ evaluated at the threshold $\hat{x}_1$ is strictly positive.
yet they choose non-disclosure. Informed owners with $x \in (\hat{x}_2(s), \hat{x}_1)$ are strictly better off in the equilibrium that is characterized by $\hat{x}_1^0$, as their (expected) payoff is

$$\alpha \Psi_E(\hat{x}_1^0; nd_1) + (1 - \alpha) [q_b + (1 - q_b) \lambda] \{ (1 - \alpha) [q_b + (1 - q_b) \lambda] \{ (\hat{x}_1^0 - \delta) + (1 - \alpha) [q_b + (1 - q_b) \lambda] \} (x - \delta)$$

$$= \{ 1 - (1 - \alpha) [q_b + (1 - q_b) \lambda] \} (\hat{x}_1^0 - \delta) + (1 - \alpha) [q_b + (1 - q_b) \lambda] \{ (\hat{x}_1^0 - \delta) + (1 - \alpha) [q_b + (1 - q_b) \lambda] \} (x - \delta),$$

where the equality exploits (12), which is satisfied by $\hat{x}_1^0$. Informed owners with $x \leq \hat{x}_2(s)$ are better off in the equilibrium with the higher threshold, as $P_{E,2}(s, nd_{1,2}) = \hat{x}_2(s) - \delta$ is the same in both equilibria (see Lemma 5) and their (expected) payoff in the equilibrium that is characterized by $\hat{x}_1^0$ is

$$\{ 1 - (1 - \alpha) [q_b + (1 - q_b) \lambda] \} (\hat{x}_1^0 - \delta) + (1 - \alpha) [q_b + (1 - q_b) \lambda] \{ (\hat{x}_1^0 - \delta) + (1 - \alpha) [q_b + (1 - q_b) \lambda] \} (\hat{x}_2(s) - \delta).$$

Finally, the (expected) payoff of uninformed owners in equilibrium with date-1 threshold $z$, $z \in \{\hat{x}_1, \hat{x}_1^0\}$ is

$$\alpha \Psi_E(\hat{x}_1^0; nd_1) + (1 - \alpha) \lambda P_{E,2}(s, nd_{1,2}) + (1 - \alpha) (1 - \lambda) P_{E,2}(ns, nd_{1,2})$$

$$= \{ 1 - (1 - \alpha) [q_b + (1 - q_b) \lambda] \} (z - \delta) + (1 - \alpha) \lambda P_{E,2}(s, nd_{1,2})$$

$$+ (1 - \alpha) (1 - \lambda) q_b \Psi_E(z; ns, nd_{1,2})$$

$$= \{ 1 - (1 - \alpha) [q_b + (1 - q_b) \lambda] \} (z - \delta) + (1 - \alpha) \lambda P_{E,2}(s, nd_{1,2})$$

$$+ (1 - \alpha) (1 - \lambda) q_b \frac{1}{1 - \theta} (z - \delta - \theta \Psi_E(z; nd_1)),$$

where we have used the equilibrium relationship (12). This payoff is increasing in $z$, since, by log-concavity of $F(x)$, the difference $z - \theta \Psi_E(z; nd_1)$ is strictly increasing in $z$. To see this, observe that the derivative of $\Psi_E(z; nd_1)$ with respect to $z$ is given by

$$\frac{\partial}{\partial z} (z - \theta \Psi_E(z; nd_1)) = 1 - \theta \frac{q_f_f(z)}{(1 - q_f) + q_f F(z)} [(z - \delta) - \Psi_E(z; nd_1)].$$

If $z - \delta \leq \Psi_E(z; nd_1)$, the derivative is positive. If $z - \delta > \Psi_E(z; nd_1)$, the derivative is minimal.
when \( q_f = 1 \) and therefore,

\[
1 - \theta \frac{q_j f(z)}{(1 - q_j) + q_j F(z)} [(z - \delta) - \Psi_E (z; nd_1)] \geq 1 - \theta \frac{f(z)}{F(z)} \left\{ (z - \delta) - \int_\delta^z (x - \delta) \frac{f(x)}{F(z)} \, dx \right\}
\]

\[
\geq 1 - \theta \frac{f(z)}{F(z)} \left\{ z - \int_0^z x \frac{f(x)}{F(z)} \, dx \right\} = 1 - \theta \frac{f(z)}{F(z)} \left\{ z - \mathbb{E} [\tilde{x} | \tilde{x} \in (0, z)] \right\}
\]

\[
\geq 1 - \theta \frac{f(z)}{F(z)} \left\{ z - \mathbb{E} [\tilde{x} | \tilde{x} < z] \right\} = 1 - \theta \frac{\partial \mathbb{E} [\tilde{x} | \tilde{x} < z]}{\partial z} > 0,
\]

since \( \frac{\partial \mathbb{E} [\tilde{x} | \tilde{x} < z]}{\partial z} < 1 \) by log-concavity of \( F(x) \) (e.g., see Theorem 5 in Bagnoli and Bergstrom (2005)). It follows that uninformed owners are also better off in the equilibrium with a higher date-1 threshold. To conclude: we have shown that all types are weakly better off, and a positive mass of types are strictly better off in the higher date-1 threshold equilibrium. Taking the average payoff across all types gives that ex ante owners are strictly better off in the equilibrium with a higher date-1 threshold, \( \tilde{x}_1^* \), which contradicts the fact that for any date-1 disclosure threshold the owner’s ex ante payoff is \( \mathbb{E} [\tilde{v}_E] \) (by the law of iterated expectations).

**Step 4.** For \( k \) sufficiently close to zero, there exists a unique \( (\tilde{x}_1, \delta) \) that simultaneously solves (12) and (3).

**Proof of Step 4.** Consider the \( \tilde{x}_1 \) from Step 1 as a function of \( \delta \), denoted \( \tilde{x}_1 (\delta) \). We can reduce the problem to a single equation in one unknown, \( \delta \), by plugging in the function \( \tilde{x}_1 (\delta) \) into (3), to obtain

\[
\int_0^\delta x f(x) \, dx + \delta \left( 1 - F(\delta) \right) - \left[ \lambda + (1 - \lambda) q_f F(\tilde{x}_1 (\delta)) q_b \right] k - \gamma I = 0. \tag{A3}
\]

When \( k = 0 \), (A3) admits a solution because the project has positive NPV. Further, this solution is unique because the left-hand side of (A3) is strictly increasing in \( \delta \). Uniqueness of a solution \( \delta (k) \) in a neighborhood of \( k = 0 \) follows from the implicit function theorem. 

**Lemma 6** If the owner has only one opportunity to disclose at date 1, then, for all \( z > \delta \),

\[
\Psi_E (z; nd_1) > \left[ q_b + (1 - q_b) \lambda \right] \Psi_E (z; s, nd_{1,2}) + (1 - q_b) (1 - \lambda) \Psi_E (z; ns, nd_{1,2}) \tag{A4}
\]

**Proof of Lemma 6** If the owner can only disclose at \( t = 1 \), and the conjectured threshold is \( z > \delta \), by the law of iterated expectations the date-1 price given non-disclosure is

\[
\Psi_E (z; nd_1) = \left( \frac{(1 - q_f) \lambda + q_f F(z) [q_b + (1 - q_b) \lambda]}{(1 - q_f) + q_f F(z)} \right) \Psi_E (z; s, nd_{1,2}) + \left( \frac{(1 - q_f) (1 - \lambda) + q_f F(z) (1 - q_b) (1 - \lambda)}{(1 - q_f) + q_f F(z)} \right) \Psi_E (z; ns, nd_{1,2}).
\]

35
Since $\Psi_E(z; s, nd_{1,2}) < \Psi_E(z; ns, nd_{1,2})$ and
\[
(1-q_f) \lambda + q_f F(z)[q_b + (1-q_b) \lambda] \left(1 - \frac{1-q_f}{q_f} + q_f F(z)\right) < q_b + (1-q_b) \lambda, \\
\]
the inequality (A4) follows. \hfill \blacksquare

**Lemma 7** Fix a common $\delta$ in both the one-time and the two-time disclosure regimes. Then, in the one-time regime there exists a unique date-1 disclosure threshold $\hat{x}_{ot}$. Moreover, $\hat{x}_{ot} < \hat{y}$, where $\hat{y} = \hat{x}_1 (\theta = 1) = \hat{x}_{1} (q_b = 0)$, i.e., $\hat{y}$ is the date-1 threshold in the two-time regime when $\theta = 1$ or $q_b = 0$. 

**Proof of Lemma 7.** The one-time disclosure threshold is given by $\hat{x}_{ot}$, the solution to
\[
\hat{x}_{ot} - \delta = \alpha \Psi_E(\hat{x}_{ot}; nd_{1}) + (1-\alpha) \left\{ \frac{[q_b + (1-q_b) \lambda] \Psi_E(\hat{x}_{ot}; s, nd_{1,2})}{(1-q_f) + q_f F(z)} + (1-q_b) (1-\lambda) \Psi_E(\hat{x}_{ot}; ns, nd_{1,2}) \right\}. \tag{A5}
\]
Existence follows from standard arguments of limits and continuity. Uniqueness is shown by contradiction. Assume $\hat{x}_{ot}$ and $\hat{x}'_{ot}$ are two distinct solutions to (A5), and $\hat{x}'_{ot} > \hat{x}_{ot}$ without loss of generality. Owners that observe $x \geq \hat{x}'_{ot}$ get $x - \delta$ in both equilibria. Owners that observe $x \in (\hat{x}_{ot}, \hat{x}'_{ot})$ are better off in the equilibrium with the higher threshold since non-disclosure yields $\hat{x}'_{ot} - \delta > x - \delta$. Finally, uninformed owners as well as owners that observe $x \leq \hat{x}_{ot}$ are better off in the equilibrium with the threshold $\hat{x}_{ot}$, as they do not disclose in both equilibria and the non-disclosure price $\hat{x}'_{ot} - \delta$ is larger than the non-disclosure price $\hat{x}_{ot} - \delta$. Taking the average payoff across all types of owners implies that the $\hat{x}'_{ot}$ equilibrium gives a higher ex ante payoff, which is a contradiction. Thus, Equation (A5) admits a unique solution. 

Let $\hat{y}$ be the solution to $\hat{y} - \delta = \Psi_E(\hat{y}; nd_{1})$. Lemma 6 implies that $\hat{x}_{ot} - \delta < \Psi_E(\hat{x}_{ot}; nd_{1})$ and, therefore, $\hat{x}_{ot} < \hat{y}$. \hfill \blacksquare

**Proof of Lemma 3.** From Lemma 7 we have $\hat{x}_{ot} < \hat{y}$. By Corollary 1, $\hat{x}_1$ is decreasing in $\alpha$. The claim then follows from having $\hat{x}_1 = \hat{y}$ when $\alpha = 1$. \hfill \blacksquare

**Proof of Theorem 1.** Proof of (i). Let $\hat{x}_{ot} (\delta)$ be the one-time disclosure threshold for some $q_b > 0$ and a given face value of the debt $\delta$. From Lemma 3 we know that $\hat{x}_{ot} (\delta) < \hat{x}_1 (\delta)$ for all $\delta > 0$. For $\tau \in \{0, 1\}$, define
\[
\Gamma(\delta, \tau) \equiv \{ \lambda + (1-\lambda) q_f [\tau F(\hat{x}_1 (\delta)) + (1-\tau) F(\hat{x}_{ot} (\delta))] q_b \} k \\
+ \gamma l - \int_{0}^{\delta} x f(x) dx - \delta (1-F(\delta)).
\]
\( \Gamma(\delta, \tau) = 0 \) is the break-even condition for the bank: if \( \tau = 0 \), one obtains the one-time disclosure model; whereas if \( \tau = 1 \), one obtains the two-time disclosure model. First, note that \( \Gamma(\delta, 1) > \Gamma(\delta, 0) \). Second, as \( \delta \downarrow 0 \), \( \int_0^\delta xf(x)dx + \delta (1 - F(\delta)) \downarrow 0 \), and therefore there exists a \( \bar{\delta} > 0 \), such that \( \int_0^\delta xf(x)dx + \delta (1 - F(\delta)) < \gamma I \) for all \( \delta < \bar{\delta} \). Consequently, \( \Gamma(\delta, \tau) > 0 \) for all \( \delta < \bar{\delta} \) and \( \tau \in \{0, 1\} \). Third, as \( \delta \uparrow \infty \), \( \int_0^\delta xf(x)dx + \delta (1 - F(\delta)) \uparrow \mathbb{E}[\max\{\tilde{x}, 0\}] \). Since \( \mathbb{E}[\max\{\tilde{x}, 0\}] > [\lambda + (1 - \lambda) q_f q_b] \tilde{k} + \gamma I \), there exists a \( \ddot{\delta} \in (0, \infty) \) such that \( \int_0^{\ddot{\delta}} xf(x)dx + \delta (1 - F(\delta)) > [\lambda + (1 - \lambda) q_f q_b] \tilde{k} + \gamma I \) for all \( \delta > \ddot{\delta} \). Therefore, \( \Gamma(\delta, \tau) < 0 \) for all \( \delta > \ddot{\delta} \) and \( \tau \in \{0, 1\} \). By continuity of \( \Gamma(\delta, \tau) \), we can write that \( \Gamma(\ddot{\delta}, \tau) \geq 0 \) and \( \Gamma(\dddot{\delta}, \tau) \leq 0 \), which implies that the solutions to \( \Gamma(\delta, 0) = 0 \) and \( \Gamma(\delta, 1) = 0 \) lie in the compact interval \( [\delta, \dddot{\delta}] \). The function \( \Gamma(\delta, \tau) \) defined for \( (\delta, \tau) \in [\delta, \dddot{\delta}] \times \{0, 1\} \) thus satisfies the assumptions of Theorem 1 in Milgrom and Roberts (1994). Hence, \( \delta \tau = 1 > \delta \tau = 0 \) and \( \mathbb{E}[\tilde{v}_E] \) is higher in the one-time disclosure model.

**Proof of (ii).** The case where the owner can credibly commit to non-disclosure is mathematically identical to the case where the owner mandatorily discloses \( x \), whenever she is informed. The reason is that in both cases, the bank sells only if it faces a liquidity shock. Moreover, regardless of the disclosure regime, when \( q_f = 0 \) the loan is also sold with probability \( \lambda \). This implies that in all three cases, the ex ante probability of the loan sale is \( \lambda \), and therefore the face value of the debt \( \delta \) in these cases is the smallest regardless of the disclosure regime.

**Proof of Corollary 1.** Let \( \xi \in \{q_f, q_b, \alpha, \lambda, \ell\} \) be one of these parameters. For ease of exposition, we denote \( \Psi_1(\tilde{x}_1) \equiv \Psi_E(\tilde{x}_1; nd_1) \) and \( \Psi_2(\tilde{x}_1) \equiv \Psi_E(\tilde{x}_1; ns, nd_1, 2) \). By applying the implicit function theorem to Equation (12), one obtains

\[
\tilde{x}_1'(\xi) = -\frac{\frac{\partial \bar{\theta}}{\partial \xi} (\Psi_1(\tilde{x}_1) - \Psi_2(\tilde{x}_1)) + \bar{\theta} \frac{\partial \Psi_1(\tilde{x}_1)}{\partial \xi} + (1 - \theta) \frac{\partial \Psi_2(\tilde{x}_1)}{\partial \xi} + \delta'(\xi)}{\theta \frac{\partial \Psi_1(\tilde{x}_1)}{\partial z} + (1 - \theta) \frac{\partial \Psi_2(\tilde{x}_1)}{\partial z} - 1}.
\]  
(A6)

The differences \( z - \Psi_1(\tilde{x}_1) \) and \( z - \Psi_2(\tilde{x}_1) \) are increasing in \( z \) by log-concavity of \( F(x) \). This implies that the denominator of (A6) is strictly negative and, therefore, the sign of \( \tilde{x}_1'(\xi) \) is the sign of the numerator. Note that when \( k = 0 \), we have \( \delta'(\xi) = 0 \) for \( \xi \in \{q_f, q_b, \alpha, \lambda\} \), as none of these parameters appear in Equation (13). Specializing (A6) to each parameter \( \xi \in \{q_f, q_b, \alpha, \lambda\} \) yields the following.

**Case \( \xi = \alpha \).** The numerator of (A6) is given by

\[
\frac{\partial \bar{\theta}}{\partial \alpha} (\Psi_1(\tilde{x}_1) - \Psi_2(\tilde{x}_1)) < 0,
\]

\(^{40}\)See the proof of Proposition 1 for more details.
since $\frac{\partial \theta}{\partial \alpha} > 0$ and $\Psi_2(\tilde{x}_1) > \Psi_1(\tilde{x}_1)$. Hence, $\tilde{x}'_1(\alpha) < 0$ and probability of disclosures at date 1 is increasing in $\alpha$.

Case $\xi = q_f$. The numerator of (A6) is given by

$$\theta \frac{\partial \Psi_1(\tilde{x}_1)}{\partial q_f} + (1 - \theta) \frac{\partial \Psi_2(\tilde{x}_1)}{\partial q_f} < 0,$$

since $\frac{\partial \Psi_1(\tilde{x}_1)}{\partial q_f}, \frac{\partial \Psi_2(\tilde{x}_1)}{\partial q_f} < 0$. Hence, $\tilde{x}'_1(q_f) < 0$.

Case $\xi = \lambda$. The numerator of (A6) is given by

$$\frac{\partial \theta}{\partial \lambda} (\Psi_1(\tilde{x}_1) - \Psi_2(\tilde{x}_1)) < 0,$$

since $\frac{\partial \theta}{\partial \lambda} > 0$ and $\Psi_2(\tilde{x}_1) > \Psi_1(\tilde{x}_1)$. Hence, $\tilde{x}'_1(\lambda) < 0$.

Case $\xi = \ell$. Since $\ell$ is strictly increasing in $\gamma$, we simply perform comparative statics with respect to $\gamma$. The equilibrium $\delta$ is determined by the bank’s break-even condition as given in (13). Totally differentiate (13) with respect to $\gamma$ to obtain

$$\delta'(\gamma) = \frac{l}{1 - F(\delta)} > 0. \quad (A7)$$

For $\xi = \delta (A6)$ becomes

$$\tilde{x}'_1(\delta) = -\left[ \theta \frac{\partial \Psi_1(\tilde{x}_1)}{\partial x} + (1 - \theta) \frac{\partial \Psi_2(\tilde{x}_1)}{\partial x} \right] + \left( 1 - \theta \right) (\Psi_1(\tilde{x}_1) - \Psi_2(\tilde{x}_1)) > 0, \quad (A8)$$

as $\frac{\partial \Psi_1(\tilde{x}_1)}{\partial x}, \frac{\partial \Psi_2(\tilde{x}_1)}{\partial x} > -1$ and $\frac{\partial \Psi_1(\tilde{x}_1)}{\partial z}, \frac{\partial \Psi_2(\tilde{x}_1)}{\partial z} < 1$ by log-concavity of $F(x)$. Hence, we have $\tilde{x}'_1(\gamma) = \tilde{x}'_1(\delta) \cdot \delta'(\gamma) > 0$. ■

Proof of Theorem 2. For $\xi = q_b$, the numerator of (A6) is given by

$$\frac{\partial \theta}{\partial q_b} \left[ \Psi_1(\tilde{x}_1(q_b)) - \Psi_2(\tilde{x}_1(q_b)) \right] + \left( 1 - \theta \right) \frac{\partial \Psi_2(\tilde{x}_1(q_b))}{\partial q_b}. \quad (A9)$$

Since $\frac{\partial \theta}{\partial q_b} > 0$ and $\frac{\partial \Psi_2(\tilde{x}_1(q_b))}{\partial q_b} > 0$, the sign of $\tilde{x}'_1(q_b)$ is ambiguous. Take the limit of (A9) as $q_b \downarrow 0$ and $q_b \uparrow 1$. At $q_b = 0$, we have that (A9) equals

$$\left( 1 - \frac{\alpha}{1 - (1 - \alpha) \lambda} \right) \frac{\partial \Psi_2(\tilde{x}_1(0))}{\partial q_b} > 0. \quad (A41)$$

The function in (A9) can be shown to be continuous in $q_b$ on the compact interval $[0, 1]$. Besides, observe that the denominator of (A6) is non-zero (and negative) at the boundaries.
At \( q_b = 1 \), the expression in \((A9)\) equals

\[
(1 - \alpha) (1 - \lambda) [\Psi_1 (\hat{\xi}_1 (1)) - \Psi_2 (\hat{\xi}_1 (1))] < 0.
\]

Therefore, \( \hat{\xi}_1 (q_b) \) is increasing is \( q_b \) at \( q_b = 0 \) and decreasing at \( q_b = 1 \).

Further, the sign of \( \hat{\xi}'_1 (q_b) \) is the sign of

\[
\frac{\partial \theta}{\partial q_b} [\Psi_1 (\hat{\xi}_1) - \Psi_2 (\hat{\xi}_1)] + (1 - \theta) \frac{\partial \Psi_2 (\hat{\xi}_1)}{\partial q_b} = \frac{\theta (1 - \alpha) (1 - \lambda) [\Psi_1 (\hat{\xi}_1) - \Psi_2 (\hat{\xi}_1)]}{1 - (1 - \alpha) [q_b + (1 - q_b) \lambda]} + \frac{(1 - \theta) q_f \left[ F (\hat{\xi}_1) \Psi_2 (\hat{\xi}_1) - \int_{\hat{\delta}}^{\hat{\xi}_1} (x - \delta) f (x) \, dx \right]}{(1 - q_f) + q_f (1 - q_b) F (\hat{\xi}_1)}.
\]

Since

\[
\theta [\Psi_1 (\hat{\xi}_1) - \Psi_2 (\hat{\xi}_1)] = (\hat{\xi}_1 - \delta) - \Psi_2 (\hat{\xi}_1)
\]
\[
= \begin{cases} 
(\hat{\xi}_1 - \delta) \left[ (1 - q_f) + q_f (1 - q_b) F (\hat{\xi}_1) \right] \\
- (1 - q_f) \int_{\delta}^{\hat{\xi}_1} (x - \delta) f (x) \, dx \\
- q_f (1 - q_b) \int_{\delta}^{\hat{\xi}_1} (x - \delta) f (x) \, dx \\
(1 - q_f) + q_f (1 - q_b) F (\hat{\xi}_1)
\end{cases}
\]

the sign of \( \hat{\xi}'_1 (q_b) \) is determined by

\[
F (q_b) \equiv (\hat{\xi}_1 - \delta) \left[ (1 - q_f) + q_f (1 - q_b) F (\hat{\xi}_1) \right] - (1 - q_f) \int_{\delta}^{\hat{\xi}_1} (x - \delta) f (x) \, dx - q_f (1 - q_b) \int_{\delta}^{\hat{\xi}_1} (x - \delta) f (x) \, dx + q_f (1 - q_b) \left[ F (\hat{\xi}_1) \Psi_2 (\hat{\xi}_1) - \int_{\delta}^{\hat{\xi}_1} (x - \delta) f (x) \, dx \right].
\] \hspace{1cm} (A10)

Observe that \( F' (q_b) \) is of the form

\[
F' (q_b) = \hat{\xi}_1 \cdot \Xi (q_b) - q_f (\hat{\xi}_1 - \delta) F (\hat{\xi}_1) + q_f \int_{\delta}^{\hat{\xi}_1} (x - \delta) f (x) \, dx - q_f \left[ F (\hat{\xi}_1) \Psi_E (\hat{\xi}_1; ns, nd_{1,2}) - \int_{\delta}^{\hat{\xi}_1} (x - \delta) f (x) \, dx \right] + q_f (1 - q_b) F (\hat{\xi}_1) \frac{\partial \Psi_E (\hat{\xi}_1; ns, nd_{1,2})}{\partial q_b},
\]

where \( \Xi (q_b) \) is a function of \( q_b \).

\[\text{Note that at } q_b = 0 \text{ the functions } \Psi_1 (z) \text{ and } \Psi_2 (z) \text{ are identical pointwise.}\]
The first-order condition $\dot{x}_1 (q_b) = 0$ implies that $F' (q_b) < 0$, as for a $q_b$ satisfying it we have

\[
F' (q_b) = -q_f (\chi_1 - \delta) F (\chi_1) + q_f \int_{\delta}^{\hat{x}_1} (x - \delta) f (x) dx - q_f \left[ F (\chi_1) \Psi_2 (\chi_1) - \int_{\delta}^{\hat{x}_1} (x - \delta) f (x) dx \right] \\
+ q_f (1 - q_b) F (\chi_1) \left[ 1 - \frac{q_f (1 - q_b) F (\chi_1)}{(1 - q_f) + q_f (1 - q_b) F (\chi_1)} \right] < 0,
\]

where the first equality uses integration by parts and the inequality follows from

\[
\Psi_2 (\chi_1) > \frac{1}{F (\chi_1)} \int_{\delta}^{\hat{x}_1} (x - \delta) f (x) dx.
\]

The facts $\chi_1 (0) > 0$ and $\chi_1 (1) < 0$ imply that there exists some $q_b^o \in (0, 1)$ at which $\chi_1 (q_b^o) = 0$. Let $q_b^*$ denote the lowest such $q_b^o$. We prove that if $\chi_1 (q_b^o) = 0$, then $q_b^*$ is a local maximum. Then, we conclude that $q_b^*$ must be the unique solution to the first-order condition, since if there were another $q_b^o > q_b^*$ solving it, the fact that both $q_b^*$ and $q_b^o$ are local maxima implies the existence of a local minimum in $(q_b^*, q_b^o)$. Such a minimum cannot exist, since we have argued that only maxima can satisfy the first-order condition.

Suppose, by contradiction, that $q_b^o$ satisfies $\chi_1 (q_b^o) = 0$. By \((A10)\), $F (q_b^o) = 0$. Then, $F' (q_b^o) < 0$ implies that for all $q_b$ in a sufficiently small neighborhood to the right (resp., left) of $q_b^o$ we have $F (q_b) < 0$ (resp., $F (q_b) > 0$), and therefore $\chi_1 (q_b) < 0$ (resp., $\chi_1 (q_b) > 0$). Hence, $q_b^o$ must be a local maximum. \(\blacksquare\)

**Proof of Theorem**\(\text{[3]}\) Observe that the public news does not alter the face value of the debt, since $\delta$ is determined before $y$ is observed by any player. For a given $\mu, \sigma$, the probability of a loan sale is

\[
\lambda + q_f F (\chi_1 - \mu) (1 - \lambda) q_b.
\]

Note that all other parameters do not vary in $\mu, \sigma$. Consequently, the probability of a loan sale solely depends on the ratio $\chi_1 - \mu / \sigma$. In addition, observe that the probability of default is higher if and only if

\[
\mu' - \frac{\mu - \delta}{\sigma} \sigma' < 0,
\]

40
whereas the probability of a loan sale is higher if and only if

$$\left( \frac{\partial \hat{x}_1}{\partial \mu} - 1 \right) \mu' + \left( \frac{\partial \hat{x}_1}{\partial \sigma} - \frac{\hat{x}_1 - \mu}{\sigma} \right) \sigma' > 0.$$  

From equation (A6),

$$\frac{\partial \hat{x}_1}{\partial \mu} - 1 = \frac{\theta \left( \frac{\partial \Psi_1(\hat{x}_1)}{\partial \mu} + \frac{\partial \Psi_1(\hat{x}_1)}{\partial z} - 1 \right)}{1 - \left[ \theta \frac{\partial \Psi_1(\hat{x}_1)}{\partial z} + (1 - \theta) \frac{\partial \Psi_2(\hat{x}_1)}{\partial z} \right]} + \frac{(1 - \theta) \left( \frac{\partial \Psi_2(\hat{x}_1)}{\partial \mu} + \frac{\partial \Psi_2(\hat{x}_1)}{\partial z} - 1 \right)}{1 - \left[ \theta \frac{\partial \Psi_1(\hat{x}_1)}{\partial z} + (1 - \theta) \frac{\partial \Psi_2(\hat{x}_1)}{\partial z} \right]}.$$  

Observe that

$$Y_E(z; B, C) = \frac{B \int_{\sigma - \delta}^\infty (\mu + \sigma \omega - \delta) f_\omega(\omega) d\omega + C \int_{\sigma - \delta}^\infty (\mu + \sigma \omega - \delta) f_\omega(\omega) d\omega}{B + C \cdot F_\omega \left( \frac{z - \mu}{\sigma} \right)}.$$  

It follows that the derivative of $\hat{x}_1 - \mu$ with respect to $\mu$ is

$$\frac{\partial \hat{x}_1}{\partial \mu} - 1 = \frac{-\chi \cdot F_\omega \left( \frac{\delta - \mu}{\sigma} \right)}{1 - \left[ \theta \frac{\partial \Psi_1(\hat{x}_1)}{\partial z} + (1 - \theta) \frac{\partial \Psi_2(\hat{x}_1)}{\partial z} \right]},$$

where

$$\chi \equiv \theta \frac{(1 - q_f) + q_f}{(1 - q_f) + q_f F_\omega \left( \frac{\hat{x}_1 - \mu}{\sigma} \right)} + (1 - \theta) \frac{(1 - q_f) + q_f (1 - q_b)}{(1 - q_f) + q_f (1 - q_b) F_\omega \left( \frac{\hat{x}_1 - \mu}{\sigma} \right)}.$$  

Differentiating $\frac{\hat{x}_1 - \mu}{\sigma}$ with respect to $\sigma$, we have

$$\frac{\partial \hat{x}_1}{\partial \sigma} - \frac{\hat{x}_1 - \mu}{\sigma} = \frac{\theta \left[ \frac{\partial \Psi_1(\hat{x}_1)}{\partial \sigma} - \left( 1 - \frac{\partial \Psi_1(\hat{x}_1)}{\partial z} \right) \frac{\hat{x}_1 - \mu}{\sigma} \right] + (1 - \theta) \left[ \frac{\partial \Psi_2(\hat{x}_1)}{\partial \sigma} - \left( 1 - \frac{\partial \Psi_2(\hat{x}_1)}{\partial z} \right) \frac{\hat{x}_1 - \mu}{\sigma} \right]}{1 - \left( \theta \frac{\partial \Psi_1(\hat{x}_1)}{\partial z} + (1 - \theta) \frac{\partial \Psi_2(\hat{x}_1)}{\partial z} \right)}.$$  

Moreover,

$$\frac{\partial Y_E(\hat{x}_1; B, C)}{\partial \sigma} = \left( 1 - \frac{\partial Y_E(\hat{x}_1; B, C)}{\partial z} \right) \frac{\hat{x}_1 - \mu}{\sigma} = \frac{B \int_{\sigma - \delta}^\infty \omega f_\omega(\omega) d\omega + C \int_{\sigma - \delta}^\infty \omega f_\omega(\omega) d\omega}{B + C \cdot F_\omega \left( \frac{\hat{x}_1 - \mu}{\sigma} \right)} - \frac{\hat{x}_1 - \mu}{\sigma},$$
and therefore
\[
\frac{\partial \hat{x}_1}{\partial \sigma} - \frac{\hat{x}_1 - \mu}{\sigma} = \frac{\frac{\mu - \delta}{\sigma} \chi F_\omega \left( \frac{\delta - \mu}{\sigma} \right)}{1 - \left( \theta \frac{\partial \Psi_1(\hat{x}_1)}{\partial z} + (1 - \theta) \frac{\partial \Psi_2(\hat{x}_1)}{\partial z} \right)} .
\]

Hence, we have
\[
\left( \frac{\partial \hat{x}_1}{\partial \mu} - 1 \right) \mu' + \left( \frac{\partial \hat{x}_1}{\partial \sigma} - \frac{\hat{x}_1 - \mu}{\sigma} \right) \sigma' = \frac{-\chi F_\omega \left( \frac{\delta - \mu}{\sigma} \right)}{1 - \left( \theta \frac{\partial \Psi_1(\hat{x}_1)}{\partial z} + (1 - \theta) \frac{\partial \Psi_2(\hat{x}_1)}{\partial z} \right)} \left( \mu' - \frac{\mu - \delta}{\sigma} \sigma' \right) .
\]

Inspection reveals that the left-hand side is positive if and only if \( \mu' - \frac{\mu - \delta}{\sigma} \sigma' < 0 \).

**Proof of Theorem 4**

Proof of (i). Taking the first-order condition of (16), and imposing the equilibrium condition \( \hat{a} = a^{FB} \) (whence \( \delta (\hat{a}) = \delta (a^{FB}) \)), one sees that the first-best action solves
\[
1 - F_\omega \left( \delta (a^{FB}) - a^{FB} \right) - \eta a^{FB} = 0 .
\]

As for the second best, note that (15) can be rewritten as
\[
\begin{align*}
-\eta \frac{a^2}{2} + (1 - \beta) & \int_{\delta(\hat{a}) - a}^{\infty} (a + \omega - \delta (\hat{a})) f_\omega (\omega) d\omega + \beta q_f \int_{\hat{x}_1 - a}^{\infty} (a + \omega - \delta (\hat{a})) f_\omega (\omega) d\omega \\
+ \beta q_f \int_{\hat{x}_2(s) - a}^{\hat{x}_1 - a} \left\{ \alpha \Psi_1 (\hat{x}_1) + (1 - \alpha) \left[ \frac{\lambda + (1 - \lambda) q_b (a + \omega - \delta (\hat{a}))}{+ (1 - \lambda) (1 - q_b) (\hat{x}_2(ns) - \delta (\hat{a}))} \right] \right\} f_\omega (\omega) d\omega \\
+ \beta q_f \int_{-\infty}^{\hat{x}_2(s) - a} \left\{ \alpha \Psi_1 (\hat{x}_1) + (1 - \alpha) \left[ \frac{\lambda + (1 - \lambda) q_b (\hat{x}_2(s) - \delta (\hat{a}))}{+ (1 - \lambda) (1 - q_b) (\hat{x}_2(ns) - \delta (\hat{a}))} \right] \right\} f_\omega (\omega) d\omega \\
+ \beta (1 - q_f) & \int_{-\infty}^{\hat{x}_1 - a} \left\{ \alpha \Psi_1 (\hat{x}_1) + (1 - \alpha) \left[ \frac{\lambda (\hat{x}_2(s) - \delta (\hat{a}))}{+ (1 - \lambda) (\hat{x}_2(ns) - \delta (\hat{a}))} \right] \right\} f_\omega (\omega) d\omega .
\end{align*}
\]

where the threshold \( \hat{x}_1 (\hat{a}) \), as well as the prices \( \Psi_1 (\hat{x}_1), \hat{x}_2(s)(\hat{a}) - \delta (\hat{a}) \) and \( \hat{x}_2(ns)(\hat{a}) - \delta (\hat{a}) \) are determined based on the conjecture \( \hat{a} \) and cannot be affected by the owner. Taking the first-order condition of (A12) with respect to \( a \), and imposing \( \hat{a} = a^{SB} \), gives
\[
0 = \beta q_f \left\{ 1 - F_\omega \left( \hat{x}_1 (a^{SB}) - a^{SB} \right) + (1 - \alpha) (\lambda + (1 - \lambda) q_b) \left[ F_\omega \left( \hat{x}_1 (a^{SB}) - a^{SB} \right) - F_\omega \left( \hat{x}_2(s) (a^{SB}) - a^{SB} \right) \right] \right\} + (1 - \beta) \left[ 1 - F_\omega \left( \delta (a^{SB}) - a^{SB} \right) \right] - \eta a^{SB} .
\]

(A13)
Since
\[ q_f \{ 1 - F_\omega (\tilde{x}_1 (a) - a) + (1 - \alpha) (\lambda + (1 - \lambda) q_b) [F_\omega (\tilde{x}_1 (a) - a) - F_\omega (\tilde{x}_2 (s) (a) - a)] \} \]
\[ < 1 - F_\omega (\delta (a) - a) \]
for all \( a \), we conclude that \( a^{\text{SB}} < a^{\text{FB}} \).

**Proof of (ii).** The proof is outlined in the main text. Here, we show how to select a suitable \( q_b \). Recall from Theorem 2 that, for any given action \( a \), \( \tilde{x}_1 (q_b) \) is hump-shaped and that \( q_b^* \) denotes the monitoring level at which \( \tilde{x}_1 (q_b) \) is maximized. The point \( q_b^* \) is ultimately a function of \( a \), and thus we write \( q_b^* (a) \). If we take \( q_b = \max \{ q_b^* (a) : a \in [0, 1/\eta] \} \), we ensure that \( \tilde{x}_1 (q_b) \) is decreasing for \( q_b \geq q_b \) uniformly across all actions in \([0, 1/\eta]\) (i.e., those that can be effectively chosen in equilibrium).

**Proof of (iii).** In the main text.

Existence and Uniqueness. We begin by considering the first-best scenario. The expression in (A11) is strictly positive when \( a = 0 \) and strictly negative when \( a = 1/\eta \). By continuity, a solution \( a^{\text{FB}} \) in the open interval \((0, 1/\eta)\) exists. There remains to check that such an \( a^{\text{FB}} \) is a global maximizer for the owner’s problem. Consider the owner’s first-order condition \( 1 - F_\omega (\delta (\tilde{a}) - a) - \eta a = 0 \). The second derivative with respect to \( a \) is \( f_\omega (\delta (\tilde{a}) - a) - \eta \), and it can be ensured to be strictly negative under the condition \( f_\omega (\omega) < \eta \) for all \( \omega \). In this case, the owner’s objective function is strictly concave and the first-order condition is also sufficient for a global maximum.

We now turn to the second best. Equation (A13) also admits a solution \( a^{\text{SB}} \) in the open interval \((0, 1/\eta)\). Strict concavity of the owner’s objective function requires
\[ 0 > \beta q_f \{ [1 - (1 - \alpha) (\lambda + (1 - \lambda) q_b)] f_\omega (\tilde{x}_1 (\tilde{a}) - a) \]
\[ + (1 - \alpha) (\lambda + (1 - \lambda) q_b) f_\omega (\tilde{x}_2 (s) (a) - a) \} + (1 - \beta) f_\omega (\delta (\tilde{a}) - a) - \eta. \]

A sufficient condition is again \( f_\omega (\omega) < \eta \).

\[43\] Here, we have simplified the expression exploiting the definition of \( \tilde{x}_1 \) from (12).

\[(\tilde{x}_1 - \delta) [1 - (1 - \alpha) (\lambda + (1 - \lambda) q_b)] = \alpha \Psi_E (\tilde{x}_1) + (1 - \alpha) (1 - q_b) (1 - \lambda) (\tilde{x}_2 (ns) - \delta) .\]
References


