

Ambiguity and The Tradeoff Theory of Capital Structure

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Abstract

We examine the importance of ambiguity, or Knightian uncertainty, in the capital structure decision. A static tradeoff theory model is developed in which agents are both risk averse and ambiguity averse. The model confirms the usual idea that increased risk—the uncertainty over known possible outcomes—leads firms to decrease leverage. Conversely, greater ambiguity—the uncertainty over the probabilities associated with the outcomes—leads firms to increase leverage. Using a theoretically motivated measure of ambiguity, our empirical analysis provides results consistent with these predictions.

Keywords and Phrases: Capital Structure, Ambiguity measure, Ambiguity aversion.

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1 Introduction

Every significant financial decision is subject to ambiguity.¹ In an influential paper, Ellsberg (1961) demonstrates that individuals act as if they are averse to this aspect of uncertainty.² Nevertheless, the canonical results in finance have been developed using expected utility theory, a theory that can be interpreted as either ignoring ambiguity or assuming that individual preferences are neutral to it. If aversion to ambiguity plays a significant role in financial decision-making, ignoring this aspect of individual preferences implies that our characterizations of optimizing behavior are at best incomplete and at worst misleading.

In order to address this limitation and to develop a more complete understanding of the leverage decisions made by corporations, we incorporate ambiguity and ambiguity aversion into the static tradeoff theory of corporate capital structure. Developing an understanding of the way in which the ambiguity aversion of decision makers interacts with the uncertainty in their environments may provide new insights into capital structure decisions. While the nature of the financial market's pricing of uncertainty is commonly ignored in models of capital structure choice, the market's valuation of a firm's chosen mix of securities is, in fact, the focus of the capital structure question. Examining leverage decisions through the lens of a financial market with risk and ambiguity averse investors may serve to further enhance our understanding of decision making within corporations. Further, most empirical studies of leverage, including literature standards such as Titman and Wessels (1988), take no account of agent's aversion to ambiguity. We adapt recent empirical models of the static tradeoff theory of corporate capital structure in order to explore whether the degree of ambiguity is an important explanatory variable for leverage.

Modigliani and Miller's (1958) famous theorem asserts that, with perfect capital markets, two firms generating the same distribution of future cash flow will have the same market value regardless of their financing choices. Standard tradeoff theory frictions have been used to establish an interior optimum (e.g., Kraus and Litzenberger (1973)) for the financing decision. We extend these theories by introducing ambiguity and ambiguity aversion to the standard model. We show that in a perfect capital market, Modigliani and Miller's irrelevance proposition is maintained under ambiguity aversion. This can be seen using the familiar argument that buying the debt and the equity of a levered firm

¹*Risk* is defined as a condition in which the event to be realized is *a-priori* unknown, but the odds of all the possible events are perfectly known. *Ambiguity*, or *Knightian uncertainty*, refers to the case where not only is the event to be realized *a-priori* unknown, but also the odds of all possible events are either unknown or not uniquely assigned. Throughout the paper the term "uncertainty" is used to refer to the aggregation of risk and ambiguity.

²Ellsberg (1961) demonstrates that in the presence of ambiguity, individuals typically violate the independence (or "Sure-Thing Principle") axiom of expected utility theory.

results in the investor obtaining claim to cash flow that is equivalent to the equity cash flow of an unlevered firm. Stated in an alternative way, while the cost of debt and the cost of equity capital both increase with an increase in ambiguity, the weighted average cost of capital remains invariant to changes in leverage for a given level of ambiguity.

The increase in the cost of equity (a lower valuation for levered equity with a given face value of debt) that accompanies an increase in ambiguity can be used to highlight another standard concern in the capital structure literature: the over-investment problem. Agency problems have served as a cost of debt financing in a variety of tradeoff models. For a given debt level, increasing the risk of the firm's cash flow increases the value of the levered equity following the usual intuition from option pricing. This suggests that decision-makers who maximize the value of levered equity will trade off the (net present) value of an investment opportunity against its risk. This tradeoff has been argued to motivate negative net present value investment in projects with sufficient levels of risk. With an increase in ambiguity the opposite occurs, an increase in ambiguity reduces the value of levered equity. An increase in ambiguity corresponds to an increase in investor pessimism, which will serve to reduce the value of call options. Net present value would, therefore, be sacrificed by a decision-maker seeking to maximize the value of the existing equity only if the investment achieved a sufficient reduction in ambiguity (alternatively, investments that entail an increase in ambiguity may suffer from an under-investment problem). Investments that increase both risk and ambiguity may or may not be attractive to decision-makers seeking to maximize the value of a firm's levered equity. Ambiguity aversion may, therefore, mitigate over-investment incentives of managers acting in the interests of shareholders.

When taxes and bankruptcy costs are introduced into the model, the theory predicts that while an increase in the level of risk results in a reduction in the use of debt financing, an increase in the level of ambiguity is associated with an increase in the use of debt. The economic intuition explaining why higher risk is associated with less extensive use of leverage is maintained in the model with ambiguity aversion. At the optimal leverage the marginal cost of bankruptcy equals the marginal tax benefit to the use of debt. An increase in the level of risk associated with the firm's cash flows causes the marginal probability of bankruptcy to increase. Obtaining the new optimum, therefore, entails a reduction in the use of debt. The economic intuition for the result that increased ambiguity results in higher optimal leverage can be explained in a similar way. An increase in ambiguity results in ambiguity averse agents becoming more pessimistic (as if probability is shifted from high cash flow states to low cash flow states). This implies that the perceived marginal probability of bankruptcy decreases. An increase in leverage is therefore required to achieve the new optimum.

To examine the predictions of our model, we investigate whether a firm’s current leverage is related to the ambiguity and the risk of the firm. In order to account for the unobservable firm specific component of leverage identified by Lemmon *et al.* (2008), we also difference the data and consider the extent to which changes in ambiguity or risk are followed by a change in the use of leverage in the subsequent year. Lagged observations of a firms’ realized ambiguity and risk (computed directly from equity market data) as well as measures of expected ambiguity and expected risk as predicted by an ARMA model are employed as explanatory variables. Both the book leverage ratio and the market leverage ratio are examined as dependant variables. The empirical results are consistent with the predictions of the model and indicate that ambiguity is an important explanatory of observed leverage choices.

Previous studies of ambiguity have focused mainly on its theoretical aspects rather than on testing the models empirically.^{3,4} In general, these studies employ decision models that do not provide the appropriate separations between risk and ambiguity and between tastes and beliefs that are crucial for a straightforward measurement of ambiguity.⁵ The main idea of the decision-making model we employ, Izhakian (2017), is that preferences for ambiguity are applied directly to probabilities such that attitude toward ambiguity is defined as attitude toward mean-preserving spreads in probabilities—analogueous to the Rothschild-Stiglitz risk attitude toward mean-preserving spreads in outcomes. Izhakian’s model of ambiguity averse preferences therefore achieves the necessary separation between beliefs and tastes for risk and ambiguity. As such, the degree of ambiguity can be measured (by the variance of probabilities, just as the degree of risk can be measured by the variance of outcomes) independently of the degree

³Special attention has been given to the implications of ambiguity for the equity premium; e.g., Cao *et al.* (2005), Nau (2006), Epstein and Schneider (2008) and Izhakian and Benninga (2011). Other studies attempt to provide explanations for puzzling asset pricing phenomena by introducing ambiguity into conventional asset pricing models. For example, Pflug and Wozabal (2007) and Boyle *et al.* (2011) suggest extensions of the mean-variance approach to incorporate ambiguity. Additional papers have developed extensions of the capital asset pricing model that incorporate ambiguity, such as Chen and Epstein (2002) and Izhakian (2012).

⁴*Parameter uncertainty* (known unknowns) assumes that the set of events is known, the nature of the probability distribution is known, but the parameters governing the distribution are unknown and the decision maker maximizes utility using posterior parameters that generate a set of priors, which can be viewed as reflecting both information (beliefs) and tastes for ambiguity; e.g., Bawa *et al.* (1979), Barry and Brown (1984), Coles and Loewenstein (1988), and Coles *et al.* (1995). Therefore, parameter uncertainty may be viewed as special case of ambiguity, in which the nature of the probability distributions is known. In this view, *model uncertainty* is also a special case of ambiguity. This class of models assumes an uncertainty about the true probability law governing the realization of states, and a decision maker, with her concerns about misclassification, looks for a robust decision-making process; e.g., Hansen *et al.* (1999), and Hansen and Sargent (2001). Other studies take an empirical view of model uncertainty (or *model risk*). In this perspective, while estimating an empirical model, there is uncertainty about the true set of predictive variables. To account for such a model misspecification, a Bayesian (predictive distribution) approach may be taken by assigning each set of variables (or model) a posterior probability; e.g., Pastor (2000), Pastor and Stambaugh (2000), Avramov (2002), and Cremers (2002).

⁵Earlier literature dealt with this limitation either by model calibration (e.g., Drechsler (2012)) or by attributing the disagreement of professional forecasters to ambiguity (e.g., Anderson *et al.* (2009)).

of risk and independently of the tastes regarding ambiguity or risk.⁶ We employ the methodology used by Brenner and Izhakian (2017) and Izhakian and Yermack (2017) to estimate the degree of ambiguity from stock market data. The extensive tests conducted by Brenner and Izhakian (2017) rule out the concern that our ambiguity measure captures other well-known dimensions of uncertainty. Our own robustness tests document that the same is true in the present environment. We find that ambiguity and volatility measure distinctly separate aspects of financial uncertainty. We also show that previously used proxies for ambiguity (variance of mean, variance of variance, dispersion of analyst forecasts, etc.) are not significantly related to leverage.

Lee (2014) considers a similar question to that posed here and finds that increased ambiguity leads to a reduction in leverage. Lee, however, uses the smooth model of ambiguity aversion (Klibanoff *et al.*, 2005) assuming a risk neutral utility function. Within such a model, ambiguity and ambiguity aversion have identical impacts on expected utility as do risk and risk aversion in a standard model, so it is not surprising that his result mirrors the standard result for an increase in risk. Furthermore, his model assumes that while the firm's manager is ambiguity averse, market participants are (and so market pricing is) ambiguity neutral. Lee's result, therefore, represents an agency cost rather than value maximizing behavior making it unclear how the model's predictions are expected to associate with observed firm behavior. Lee's empirical results are based on an event (the 1982 Voluntary Restraint Agreement on steel import quotas) that conflates risk and ambiguity, making interpretation of his estimates very difficult.⁷

The remainder of this paper is organized as follows. Section 2 presents a theoretical discussion of ambiguity and develops the model. Section 3 discusses the sample selection and empirical tests. Section 4 presents regression analysis of capital structure, and Section 5 concludes the paper. All proofs are provided in the Appendix.

⁶Some studies use proxies for ambiguity like disagreement among analysts; e.g., Anderson *et al.* (2009) and Antoniou *et al.* (2015). Most empirical (behavioral) studies about ambiguity use data collected in designed experiments and focus on individuals' attitudes toward ambiguity rather than on the implications of ambiguity itself. A very few studies use field data such as stock market trading data; e.g., Williams (2015), Ulrich (2013) and Thimmea and Volkertb (2015).

⁷Dahya *et al.*, (2017) use an approach that is very similar to ours to study the implications of ambiguity for dividend payout policy decisions.

2 The model

2.1 The decision theoretic framework

Ambiguity, or Knightian uncertainty, has provided the basis for a rich literature in decision theory.⁸ This literature takes a variety of approaches to modeling decision-making under ambiguity (i.e., the max-min expected utility (MEU) of Gilboa and Schmeidler (1989), the Choquet expected utility (CEU) of Schmeidler (1989), and many others). A fundamental issue these models have struggled to address is the need for a theoretical framework that allows for rigorous comparative statics that can also allow empirical explorations. The key to addressing this need is a complete separation between risk and ambiguity and between tastes and beliefs. Only when a complete separation of these components of utility is accomplished will we be able to measure the extent of the ambiguity surrounding a specific decision without confounding this measure with preferences over ambiguity, the amount of risk, and/or preferences regarding risk.

Earlier models of decision making under ambiguity have not achieved the separations which are critical for measuring ambiguity independently of risk and individuals' preferences regarding risk and ambiguity. For example, in the MEU (Gilboa and Schmeidler, 1989) the set of priors captures both beliefs and tastes regarding ambiguity. The capacities (sub-additive probabilities) used in the CEU (Schmeidler, 1989) and in the cumulative prospect theory (CPT) of Tversky and Kahneman (1992) also reflect both beliefs and tastes over ambiguity. Thus, earlier models cannot be used for empirical investigations of the implications of ambiguity. For this reason we employ the theoretical framework labeled expected utility with uncertain probabilities (EUUP), introduced in Izhakian (2017), to distinguish the concepts of risk and ambiguity. EUUP accomplishes a complete separation between risk and ambiguity and between beliefs and preferences, a separation that allows consideration of comparative static changes to each component of expected utility and also allows the development of a measure of ambiguity that may be employed in empirical investigations.

Under EUUP individuals are thought to act as if they solve a two-stage decision-making problem.

⁸Knight (1921) defines the concept of Knightian uncertainty as distinct from risk as conditions under which the set of events that may occur is *a priori* unknown, and the odds of these events are also either not unique or are unknown. Roughly speaking, this concept can be viewed as underpinning two branches of literature. The first is the “unawareness” literature, which assumes that decision makers may not be aware of a subset of events—formally the state space is updated consequentially to reflect learning about new states of nature (e.g., Karni and Vierø (2013)). The second is the ambiguity literature, which assumes that the set of events is perfectly known but their probabilities are either not unique or are unknown (e.g., Gilboa and Schmeidler (1989) and Schmeidler (1989)). These two literatures can be viewed as overlapping when dealing with monetary outcomes (real numbers). In these outcomes, the “uncertain”—risky and ambiguous—variable is defined by a measurable function from states into the real numbers such that there is no real monetary outcome that the decision maker is not aware of. It is possible that the decision maker is not aware of some events (the so-called black swans), which affects the uncertainty about the probabilities of some outcomes. But such uncertainty is taken into account by ambiguity—the uncertainty about the probabilities of outcomes.

In the first stage, the decision maker forms a representation of her perceived probabilities for each relevant event. The perceived probabilities are determined by the ambiguity that is present and the decision maker’s tastes regarding ambiguity. In the second stage of the process, the decision maker considers the expected utility associated with a set of possible outcomes, where the expectation is taken with respect to the perceived probabilities. This structure accomplishes the desired separation between risk and ambiguity and between tastes and beliefs and facilitates theoretical and empirical analyses.

The main idea of EUUP is that in the presence of ambiguity, i.e., when probabilities are uncertain, preferences concerning ambiguity are applied solely to the probabilities such that aversion to ambiguity is defined as aversion to mean-preserving spreads in probabilities. As such, the Rothschild and Stiglitz (1970) approach, which is typically applied to outcomes when examining risk, is applied to probabilities when examining ambiguity, independently of risk. As Izhakian (2016) shows, the degree of ambiguity can be measured by the volatility of probabilities—just as the degree of risk can be measured by the volatility of outcomes.⁹ Unlike other proposed measures of ambiguity, which are risk-dependent and consider only the variance of a single moment of the outcome distribution (i.e., the variance of mean or the variance of variance) the measure derived by Izhakian (2016) is risk-independent and conceptually accounts for the variance of all moments of the outcome distribution. The measure of ambiguity that is derived from EUUP can be employed in empirical studies and may be implemented using equity market data.

Formally, let \mathcal{S} be an infinite, nonempty state space, and let $X : \mathcal{S} \rightarrow \mathbb{R}$ be a measurable function describing consumption.¹⁰ An investor, who doesn’t know the precise probabilities of the relevant outcomes, holds a set \mathcal{P} of possible (additive) probability measures P on \mathcal{S} . In addition, she holds a second-order belief (a probability measure on \mathcal{P}), denoted ξ . Under EUUP, the expected utility of the investor can be represented by

$$V(X) = \int_{z \leq 0} \left[\Upsilon^{-1} \left(\int_{\mathcal{P}} \Upsilon (P_X (U(x) \geq z)) d\xi \right) - 1 \right] dz + \int_{z \geq 0} \Upsilon^{-1} \left(\int_{\mathcal{P}} \Upsilon (P_X (U(x) \geq z)) d\xi \right) dz, \quad (1)$$

where U is a (von Neumann-Morgenstern) utility function; and Υ is an *outlook function*, capturing

⁹Other models do not permit such a derivation since either ambiguity is not distinguished from aversion to ambiguity (e.g., Schmeidler (1989), and Gilboa and Schmeidler (1989)) or aversion to ambiguity is defined as aversion to mean-preserving spreads in certainty equivalent utilities, which are subject to risk and preferences for risk (e.g., Chew and Sagi (2008)).

¹⁰We consider an economy with an infinite state space and an infinite set of outcomes. All our results, however, may also be applied to an economy with a finite state space. Because the model’s intuition is illustrated more naturally in a finite state space setting, we introduce this presentation below and more completely in Appendix A.1.

attitude toward ambiguity. As usual, a concave U characterizes risk-averse behavior, and a convex U characterizes risk-loving behavior. Similarly, a concave Υ characterizes ambiguity-averse behavior, and a convex Υ characterizes ambiguity-loving behavior. The utility function is normalized such that $U(k) = 0$, for some reference point k . That is, any outcome $x \leq k$ is considered unfavorable (a loss) and any outcome $x \geq k$ is considered favorable (a gain). This aspect of utility is analogous to the reference point in CPT of Tversky and Kahneman (1992). EUUP allows the inclusion of the reference point k , reference dependent utility, in the analysis but does not require it. We incorporate the reference point in our analysis as it allows for somewhat richer results. Eliminating the reference dependent utility (so there is not explicit distinction between gains and losses, conceptually, setting $k = -\infty$ and making the expression for expected utility in Equation (1) contain only the integral over $z \geq 0$) would simplify the development and the statement of some of our results (see below). In this case, the probabilities of low outcomes (high cumulative probabilities) are underweighted by less than are the probabilities of the high outcomes (small cumulative probabilities), generating an effect that is similar to overweighting losses and underweighting gains as represented in Equation (1).

The EUUP model in Equation (1) can be thought of as extending CPT and CEU to provide a construction of the capacities, which are imposed exogenously in CPT and CEU. It is Izhakian's (2017) axiomatic development of the perceived probabilities that accomplishes the necessary separation between risk and ambiguity and between beliefs and tastes. It is well known that the MEU of Gilboa and Schmeidler (1989) is a special case of Schmeidler's (1989) CEU. EUUP can therefore be seen as an extension of these models. In this respect, EUUP also represents an extension of the rank dependent models proposed by Weymark (1981), Quiggin (1982), and Yaari (1987).

To illustrate the intuition of Equation (1), it might be helpful to write it for a discrete state space \mathcal{S} and for finitely many probability measures in \mathcal{P} . In this case, the outcomes of X can be written $X = (x_1, \dots, x_n)$, where x_s stands for the outcome at state $s \in \mathcal{S}$ and x_1, \dots, x_n are listed in a non-decreasing order. The expected utility representation of the investor's preferences is then written

$$V(X) = \sum_{0 < s \leq k} Q_X(x_s) [U(x_s) - U(x_{s+1})] + \sum_{k < s \leq n} Q_X(x_s) [U(x_s) - U(x_{s-1})], \quad (2)$$

where

$$Q_X(x_s) = \Upsilon^{-1} \left(\sum_{P \in \mathcal{P}} \Upsilon(P_X(x_s)) \xi(\{P_X\}) \right) \quad (3)$$

and $P_X(x_s)$ stands for $P(X \geq x_s)$, i.e., the probability of the outcome being *greater* than x_s . The term $Q_X(x_s)$ is the discrete equivalent of $\Upsilon^{-1} \left(\int_{\mathcal{P}} \Upsilon(P_X(U(x) \geq z)) d\xi \right)$ in Equation (1) and represents the perceived probability that the outcome is greater than or equal to x_s . Intuitively, the *perceived*

probability is the unique certain probability value that the decision maker is willing to accept in exchange for the uncertain probability of a given event. The terms in square brackets in Equation (2) are equivalent to the dz terms in Equation (1), capturing the change in utility from one outcome to the next.

If the perceived probabilities were additive, Equation (2) could be written in the more familiar fashion as the perceived probability of state s times the utility of the outcome in state s summed over the n states. However, because the perceived probabilities ($Q_X(x_s)$) are in general non-additive, expected utility cannot commonly be written in this manner. However, this version of the preference representation makes it clear that EUUP contains expected utility theory as a special case. When Υ is linear the perceived probabilities become the (additive) expected probabilities (the linear reduction of compound lotteries). The apparent complexity of Equation (1) lies in the need to use Choquet integration (as in CEU) due to the non-additivity of the perceived probabilities.

Referring again to Equation (1), suppose that the outlook function is twice differentiable and satisfies

$$\left| \frac{\Upsilon''(\mathbb{E}[\mathbf{P}_X(x)])}{\Upsilon'(\mathbb{E}[\mathbf{P}_X(x)])} \right| \leq \frac{1}{\text{Var}[\varphi_X(x)]}, \quad (4)$$

for every x , where $\varphi_X(x)$ is the marginal probability function (density or mass function) of X associated with $\mathbf{P} \in \mathcal{P}$, and $\mathbb{E}[\cdot]$ and $\text{Var}[\cdot]$ are respectively the expectation and the variance of probabilities taken using the second order probability measure ξ ; specifically,

$$\mathbb{E}[\mathbf{P}_X(x)] = \int_{\mathcal{P}} \mathbf{P}_X(x) d\xi \quad \text{and} \quad \text{Var}[\varphi_X(x)] = \int_{\mathcal{P}} [\varphi_X(x) - \mathbb{E}[\varphi_X(x)]]^2 d\xi. \quad (5)$$

This assumption (Condition (4)) bounds the level of aversion to ambiguity (the concavity of Υ) and the level of love for ambiguity (the convexity of Υ) to assure that the approximated marginal perceived probabilities (derived below) are nonnegative and that the approximated perceived probability of each event is not smaller than the perceived probability of any of its sub-events. Izhakian (Theorem 2, 2016), proves that the dual representation of the expected utility in Equation (1), can then be approximated as¹¹

$$\begin{aligned} W(X) \approx & \int_{x \leq k} U(x) \mathbb{E}[\varphi_X(x)] \left(1 - \frac{\Upsilon''(\mathbb{E}[\mathbf{P}_X(x)])}{\Upsilon'(\mathbb{E}[\mathbf{P}_X(x)])} \text{Var}[\varphi_X(x)] \right) dx + \\ & \int_{x \geq k} U(x) \mathbb{E}[\varphi_X(x)] \left(1 + \frac{\Upsilon''(\mathbb{E}[\mathbf{P}_X(x)])}{\Upsilon'(\mathbb{E}[\mathbf{P}_X(x)])} \text{Var}[\varphi_X(x)] \right) dx. \end{aligned} \quad (6)$$

¹¹The remainder of this approximation is of order $o\left(\int \mathbb{E}[|\varphi(x) - \mathbb{E}[\varphi(x)]|^3] dx\right)$ as $\int |\varphi_x(x) - \mathbb{E}[\varphi_x(x)]| dx \rightarrow 0$. This is equivalent to a cubic approximation, $o(\mathbb{E}[|x - \mathbb{E}[x]|^3])$, in which the fourth and higher absolute central moments of outcomes X are of strictly smaller order than the third absolute central moment as $|x - \mathbb{E}[x]| \rightarrow 0$, and are therefore negligible.

To simplify notation, henceforth the subscript X , designating the random variable, is omitted. In addition, since X is a measurable function, we abuse the notation and write the outcome x in state s instead of state s itself. With this notation in place, the marginal perceived probability (density) of x can be written

$$\pi(x) = \begin{cases} \mathbb{E}[\varphi(x)] \left(1 + \eta(\mathbb{E}[P(x)]) \text{Var}[\varphi(x)] \right), & x \leq k \\ \mathbb{E}[\varphi(x)] \left(1 - \eta(\mathbb{E}[P(x)]) \text{Var}[\varphi(x)] \right), & x > k \end{cases}, \quad (7)$$

where $\eta(\cdot) = -\frac{\Upsilon''(\cdot)}{\Upsilon'(\cdot)}$ and, by Condition (4), $\pi(x)$ is always positive.

Based upon the functional form in Equation (6), the degree of ambiguity can be measured by the expected volatility of probabilities, across the relevant events. Formally, the measure of ambiguity is given by¹²

$$\mathcal{U}^2[X] = \int \mathbb{E}[\varphi(x)] \text{Var}[\varphi(x)] dx, \quad (8)$$

where x is assumed to be *symmetrically* distributed as defined by Izhakian (Definition 7, 2016).¹³ A major advantage of this measure is that it can be computed from data and can be employed in empirical tests. *Risk independence* is another major advantage of \mathcal{U}^2 (mho²); unlike risk measures, it does not depend upon the magnitudes of the associated consequences.

2.2 The asset pricing framework

The decision theoretic framework of the previous section enables the formalization of an ambiguous economy. We use a common market structure where the only variation in our analysis is the specification of probabilities. We assume markets are complete and an absence of arbitrage opportunities (the law of one price holds). There are two dates, 0 and 1. The development of the capital structure model in the next section is not dependent on the simple assumptions made regarding the market structure, and the two-period utility function employed here can be generalized to a dynamic structure following the approach taken in prior papers. The state at date 0 is known, and the states at date 1 are ordered from the lowest consumption level to the highest.

In order to develop the asset pricing framework as simply as possible we consider a representative investor with a twice differentiable utility function U , whose initial wealth is w_0 and whose wealth at time 1 is w_1 .¹⁴ The uncertain outcomes of the single product in the economy are defined by

¹²The measure \mathcal{U}^2 is also applicable in finite state space. In this case, $\mathcal{U}^2[X] = \sum_i \mathbb{E}[\varphi(x_i)] \text{Var}[\varphi(x_i)]$, where $\varphi(\cdot)$ is a probability mass function.

¹³Formally, outcomes are said to be symmetrically distributed around a point of symmetry l if for any $x, y \in X$ that satisfy $|x - l| = |y - l|$, both $\mathbb{E}[\varphi(x)] = \mathbb{E}[\varphi(y)]$ and $\text{Var}[\varphi(x)] = \text{Var}[\varphi(y)]$ hold.

¹⁴This assumption is made to focus the discussion on the capital structure choice problem, the extension to heterogeneous beliefs and preferences is straightforward. À la Constantinides (1982), a representative investor can be defined as

the set of outcomes X , where $x \in X$ is the time 1 outcome in some state $s \in \mathcal{S}$. Since, the asset pricing framework is used only to extract state prices, for simplicity, we assume a single product in the economy. However, the extension to multiple assets is straightforward.

The objective function of an individual agent is standard and can then be written

$$\max_{c_0, \theta} W(c_0) + W(c_1) \quad (9)$$

subject to the budget constraints

$$c_0 = w_0 - \theta \int q(x) x dx \quad \text{and} \quad c_1 = w_1 + \theta x,$$

where $q(x)$ is the date 0 price of the contingent claim on state s with consumption x and θ is the holding of the single asset X .¹⁵ Using the functional form of expected utility in Equation (6), the state prices can be extracted as follows.

Theorem 1 *Suppose a time-separable utility function and an outlook function Υ satisfying the conditions of Izhakian (Theorem 2, 2016).¹⁶ The state price of state x is then*

$$q(x) = \pi(x) \frac{\partial_x U}{\partial_0 U}, \quad (10)$$

where $q(x)$ is always positive.

The state price $q(x)$ is the price of a pure state contingent claim or Arrow security of state s with outcome x .¹⁷ In other words, it is the price of a claim to one unit of consumption contingent on the occurrence of state s . Since markets are complete and the law of one price holds, the payoff pricing functional assigns a unique price to each state contingent claim. The representation of state prices in Equation (10) illustrates the distinct impacts risk, ambiguity, and preferences over these aspects of uncertainty have on market pricing. Ambiguity and preferences over ambiguity impact state prices through their impact on perceived probabilities, $\pi(x)$. Risk and risk aversion impact the state price via the curvature of the utility function U and the magnitude of the outcomes x relative to consumption at time 0. When there is no ambiguity, i.e., probabilities are perfectly known, the state price $q(x)$ reduces to the conventional state price $q(x) = \varphi(x) \frac{\partial_x U}{\partial_0 U}$.¹⁸

one whose tastes and beliefs are such that, if all investors in the economy had identical tastes and beliefs, the equilibrium in the economy would remain unchanged.

¹⁵Formally, the consumptions, c_0 and c_1 , need not be restricted to positive values, since in the EUUP's setting, as in cumulative prospect theory (Tversky and Kahneman, 1992), the utility function can obtain negative values.

¹⁶As in Equation (4), Theorem 2 in Izhakian (2016) bounds the concavity and the convexity of Υ to assure that the approximated marginal perceived probabilities are always positive and that they satisfy set monotonicity.

¹⁷In a related study, Chapman and Polkovnichenko (2009) extract the state prices in a rank-dependent expect utility framework.

¹⁸Similarly, in the presence of ambiguity but with ambiguity neutral agents the state price $q(x)$ reduces to $q(x) =$

The following corollary is an immediate consequence of Theorem 1.

Corollary 1 *The risk neutral probability of state x is*

$$\pi^*(x) = \frac{q(x)}{\int q(x)dx} = \frac{\pi(x) \frac{\partial_x U}{\partial_0 U}}{\int \pi(x) \frac{\partial_x U}{\partial_0 U} dx}, \quad (11)$$

and the risk-free rate of return is

$$r_f = \frac{1}{\int q(x)dx} - 1 = \frac{1}{\int \pi(x) \frac{\partial_x U}{\partial_0 U} dx} - 1. \quad (12)$$

The definition of the risk-neutral probabilities and the risk-free rate of return in an ambiguous economy allows us to define the price of an asset as its discounted expected payoff. That is,

$$p = \int q(x)xdx = \frac{1}{1+r_f} \int \pi^*(x)xdx = \frac{1}{1+r_f} \mathbb{E}^*[x], \quad (13)$$

where \mathbb{E}^* is the expectation taken with respect to the risk neutral probabilities, π^* , defined in Corollary 1. Notice that the double-struck capital font is used to designate expectation or variance of *outcomes*, taken with respect to the expected probabilities or with respect to risk-neutral probabilities, while the regular straight font is used to designate expectation or variance of *probabilities*, taken with respect to the second-order probabilities, ξ .

2.3 The capital structure decision in a perfect capital market

The capital structure model used here is based on the canonical tradeoff theory model of Kraus and Litzenberger (1973). Consider a one-period project that requires a capital investment I at time 0. The payoff of this project, obtained at time $t = 1$, is given by the risky and ambiguous variable X , having a bounded support. As discussed above, it is assumed that the owner of the project is a representative investor, whose set of priors, the set of possible probability distributions \mathcal{P} for X , is identical to the set of priors of each investor in the economy, her belief (second-order probability distribution) over \mathcal{P} is identical to that of each investor, and her reference point is also identical to that of each investor. Therefore, all agents agree on the expected probability of any specific payoff x .

At time $t = 0$, a decision to invest in a project is made if and only if

$$\frac{1}{1+r_f} \mathbb{E}^*[X] > I.$$

Given the decision to invest in the project, its owner must decide at time $t = 0$ what mix of debt and equity financing the firm will use to establish the project. The objective of this decision is to maximize

$$\mathbb{E}[\varphi(x)] \frac{\partial_x U}{\partial_0 U}.$$

the value of the firm:

$$\begin{aligned} \max_F \quad & S_0(F) + D_0(F) \\ \text{s.t.} \quad & S_0 = \frac{1}{1+r_f} \mathbb{E}^* [\max(X - F, 0)] \\ & D_0 = \frac{1}{1+r_f} \mathbb{E}^* [\min(F, X)], \end{aligned}$$

where S_0 is the market value of the common shares; D_0 is the market value of debt and F is the debt's face value. Without loss of generality the debt is assumed to be a one-period zero-coupon bond. The optimization problem can be written more explicitly as

$$\begin{aligned} \max_F \quad & S_0(F) + D_0(F) \\ \text{s.t.} \quad & S_0 = \frac{1}{1+r_f} \int_F^\infty \pi^*(x) (x - F) dx \\ & D_0 = \frac{1}{1+r_f} \left(\int_0^k \pi^*(x) x dx + \int_k^F \pi^*(x) x dx + \int_F^\infty \pi^*(x) F dx \right). \end{aligned}$$

Any $x < F$ is considered a default state and the reference point reflected in state prices is assumed to satisfy $k < F$. In this maximization problem, the default states $\{x \mid k \leq x \leq F\}$ are considered favorable in the view of the representative investor. Assuming that $k < F$ is required for the analysis to be internally consistent, as otherwise, no rational decision maker would pay a positive amount for the firm's debt, since all possible payouts to the debt instrument would be considered losses. Alternatively, stating this assumption in terms of returns rather than payoffs, in order for a rational investor to be willing to hold a risk free asset, $k < r_f$ is required. Therefore, the corporation's cost of debt capital (r_D) being greater than the risk free rate, implies that $k < r_f < r_D$.¹⁹

Theorem 2 *Suppose that there are no taxes, no bankruptcy costs and no asymmetric information. Modigliani-Miller's capital structure irrelevance proposition is maintained under ambiguity.*

Proposition 1 in Modigliani and Miller (1958) shows that, in an economy without taxes, bankruptcy costs or asymmetric information, the capital-structure choice of a firm is irrelevant to its market value. Their proof is based on the idea the an investor can generate the return to holding the levered firm's equity by holding the equity of an equivalent unlevered firm and a loan in the same proportion as the leverage ratio of the levered firm. The same argument holds true in an ambiguous economy in which agents are ambiguity averse.

¹⁹As discussed above, the underlying decision theory need not include the reference point in the analysis. Eliminating the reference point from EUUP effectively sets $k = -\infty$, which ensures that this assumption holds. In this sense, explicitly considering the rank dependant utility as is done here, is more general and more reflective of observed decision making (*e.g.* see the discussion in Tversky and Kahneman (1992)).

2.4 The over-investment problem

In their seminal paper, Jensen and Meckling (1976) introduced the “over-investment” problem. This agency cost has been included as a cost of debt financing in a variety of tradeoff theory models of capital structure. For simplicity, we maintain our assumption of an absence of taxes and bankruptcy costs. Central to Jensen and Meckling’s argument is the simple notion that in the equity holders’ view, levered equity can be described as a call option, where the exercise price is the face value of the debt.²⁰ Formally, in the present model, the value of the equity is the expected value of the future equity payoff (using the risk neutral probabilities) discounted at the risk free rate,

$$S_0 = \frac{1}{1 + r_f} \int_F^\infty \pi^*(x) (x - F) dx. \quad (14)$$

This value is affected by ambiguity through $\pi^*(x)$ and by risk through the magnitude of x .

Assume a given face value of debt, F . The following proposition establishes that the standard relation between the value of levered equity and the level of risk holds in this setting. As in Rothschild and Stiglitz (1970), we consider an asset as being riskier if its payoffs at time t can be written as a mean-preserving spread of its outcomes at time $t - 1$.

Proposition 1 *For a given face value of debt, the higher is the degree of risk, the higher is the value of the levered equity.*

The fact that the value of levered equity is increasing in the risk of the underlying payoffs implies that levered equity holders have an incentive to increase the risk of the firm’s cash flows (levered equityholders benefit from the upside risk and are protected from downside risk). The over-investment problem follows from the recognition that levered equity holders will, therefore, benefit from an increase in risk even at the expense of a reduction in the value of the cash flows (levered equity holders will prefer to pursue a negative net present value investment if the investment provides a sufficient increase in the risk of the firm’s cash flows). The next proposition shows that this incentive becomes more complex in a setting with ambiguity and ambiguity averse agents. In Proposition 2 an increase in ambiguity occurs when the set of possible probabilities associated with a given outcome at time t can be written as a mean preserving spread of the possible probabilities at time $t - 1$

Proposition 2 *Assume that the reference point reflected in the state prices satisfies $k < F$. For a given face value of debt, the higher is the degree of ambiguity, the lower is the value of the levered equity.*

²⁰See, for example, Merton (1974).

The intuition from this result is equally straightforward. An increase in the level of ambiguity causes ambiguity averse agents to further underweight good outcomes (gains) and overweight poor outcomes (losses), reducing the value of the option like payoff represented by levered equity. Alternatively, as investors become more pessimistic, due to an increase in ambiguity, the value of a call option will fall.

Levered equity holders, therefore, see a loss in value from an increase in ambiguity. Ambiguity aversion implies there is a three way tradeoff between value, risk, and ambiguity. The incentive of levered equity holders regarding an investment that increases uncertainty (the combination of risk and ambiguity) is, therefore, unclear. An often cited example of the over-investment problem is the incentive of a firm's manager's to pursue firm value reducing M&A activities due to their potential to increase the risk of the firm's cash flow. However, it is plausible that while a given merger may increase the level of risk of a firm's cash flow, the merger will also increase the level of ambiguity associated with the firm's cash flow. In such a case, the incentive of a manager acting in the interests of the levered equity holders is unclear and the manager may actually avoid an uncertainty increasing merger, even if it increases firm value.

2.5 Taxes and bankruptcy costs

Consider now an economy with corporate taxes and bankruptcy costs. Suppose that any state $x < F$ is considered a default state. Bankruptcy costs are assumed to be proportional to the firm's output in the event of default

$$B(x; F) = \begin{cases} \alpha x, & x < F \\ 0, & x \geq F \end{cases},$$

where $0 < \alpha < 1$. The tax benefit associated with the use of debt is, for simplicity, based on the entire debt service in non-default states,²¹

$$T(x; F) = \begin{cases} 0, & x < F \\ \tau F, & x \geq F \end{cases},$$

where τ is the corporate income tax rate. In such an economy, firm value is a function of the chosen capital structure, in particular the amount of debt F issued, and can be written

$$\begin{aligned} V(F) &= S_0(F) + D_0(F) \\ &= \frac{1}{1 + r_f} \left(\int_F^\infty \pi^*(x) (1 - \tau) (x - F) dx + \int_0^F \pi^*(x) (1 - \alpha) x dx + \int_k^F \pi^*(x) (1 - \alpha) x dx + \int_F^\infty \pi^*(x) F dx \right). \end{aligned} \tag{15}$$

²¹See for example, Kraus and Litzenberger (1973).

For a firm value maximizing decision maker the capital structure choice problem can be written

$$\begin{aligned} \max_F \quad & S_0(F) + D_0(F) \\ \text{s.t.} \quad & S_0 = \frac{1}{1+r_f} \mathbb{E}^* [\max((1-\tau)x - F + T(x;F), 0)] \\ & D_0 = \frac{1}{1+r_f} \mathbb{E}^* [\min(F, x - B(x;F))]. \end{aligned}$$

Notice that S_0 can be written explicitly

$$S_0 = \frac{1}{1+r_f} \mathbb{E}^* [\max((1-\tau)(x-F), 0)].$$

With the capital structure choice problem defined as above, the next theorem identifies the optimal level of leverage for the firm.

Theorem 3 *The optimal leverage satisfies*

$$F = \frac{\tau}{\pi^*(F)\alpha} \int_F^\infty \pi^*(x) dx. \quad (16)$$

Alternatively, optimal leverage can be written in terms of a risk neutral hazard rate for default. It can be immediately observed from Equation (16) that a higher tax rate, τ , or a lower bankruptcy cost, α , positively affect the use of leverage. Our focus is, however, on the implications of changes in the components of uncertainty (ambiguity and risk) for capital structure choice. Proposition 3 describes the consequence of a comparative static change in ambiguity.

Proposition 3 *Assume that the reference point reflected in the state prices satisfies $k < F$. The higher is the degree of ambiguity, the higher is the optimal leverage.*

As discussed above, the assumption that $k < F$ is necessary for the model to be internally consistent. However, mathematically, the proof of Proposition 3 suggests that the comparative static result will reverse if $k > F$. For this reason, we explore this question in the empirical analysis presented below. Similar to the comparative static concerning the effect of ambiguity on the optimal leverage, we can also study the effect of a change in risk.

Proposition 4 *The higher is the degree of risk, the lower is the optimal leverage.*

Propositions 3 and 4 demonstrate that increased ambiguity has a *positive* effect on the chosen level of leverage, while increased risk has a *negative* effect. The intuition for the result in Proposition 4 is standard: Greater volatility in the project payoff has a symmetric effect, pushing probability mass from the center of the distribution to its tails. A change in volatility has the effect of increasing the

perceived marginal cost of bankruptcy. This effect is illustrated in left panel of Figure 1. With a fixed cost of bankruptcy, the change in the marginal cost is reflected by the change in the marginal probability of default; the change in the density function at F . The increase in this value due to an increase in volatility illustrates the rise in the marginal cost of bankruptcy. The marginal benefit of the tax shield is decreased as the cumulative probability of the non-default states decreases. A reduction in the use of debt, to F' , is required to once again equate the cost and benefit of the use of debt financing at the margin. It can be tempting to suggest that this intuition can be applied to an increase in ambiguity as well; as more *uncertainty* regarding future cash flow may be thought to have the same effect. An increase in ambiguity, however, represents an increased uncertainty regarding probabilities. This increased uncertainty is independent of the outcomes (unlike an increase in risk) which implies the standard intuition cannot be applied. A more appropriate intuition derives from the effect of ambiguity and ambiguity aversion that is common across most models of ambiguity averse preferences; agents act as if they are more pessimistic, i.e., as if they put more weight on “unfavorable” outcomes and less on “favorable” outcomes. In the context of the present model, an increase in ambiguity has an asymmetric affect on the perceived probability distribution. This is illustrated in the right panel of Figure 1. Specifically, the perceived probabilities are reduced for outcomes above the reference point k and the perceived probabilities for outcomes below the reference point are increased. The fact that $k < F$ provides the intuition that an increase in ambiguity decreases the marginal perceived marginal probability of default at the original optimum. The marginal tax benefit associated with debt is reduced offsetting this change in the marginal cost of debt, however, the net result is that an increase in leverage is required to attain the new optimum.

[Figure 1]

3 Empirical design

We turn now to test the statements of Propositions 3 and 4 empirically. For consistency with the majority of the empirical capital structure literature, we examine how annual measures of levels and changes in firms’ leverage ratios are affected by annual measures of risk and ambiguity. Similar results are obtained using quarterly data.

3.1 Sample selection

We estimate the relation between ambiguity and the leverage ratio using a sample of all nonfinancial firm-year observations in the annual Compustat database between 1993-2016. The time window 1993-

2016 is dictated by the intraday stock data available on TAQ. This data is used for estimating the degree of ambiguity associated with each firm. After a process of data cleaning and filtering described below, we analyze 56,325 annual observations for 11,527 unique firms over the 23 years between 1993 and 2016.

To construct our sample, we begin with a download of all records from the Compustat fundamentals annual database. We drop all financial firms as well as all government entities. We also drop all observations missing one of the following descriptors: fiscal year end (FYR), total assets (AT), debt in current liabilities (DLC), long-term debt (DLTT) or stock price (PRCC). Finally, we drop all observations with negative values for sales, total assets or debt, and all observations with a negative leverage ratio or leverage ratio greater than 100% (book or market). This leaves us with 88,209 observations. In addition, we drop all duplicate records or records for which we are unable to match identifiers to the CRSP or TAQ stock price databases. All observations for which the annual degree of ambiguity or risk cannot be estimated are also dropped, which leaves us with 56,325 observations for regression analysis.

We estimate the annual degree of ambiguity and risk for each firm and each month as detailed below. We then use their average over the fiscal year as the annual risk and ambiguity associated with each firm-year observation. A similar method is used to estimate annual measures of *expected* ambiguity and *expected* risk. That is, for each firm and each month we estimate the expected values based on only the preceding 36 months, as detailed below, and then take the average of these values over the fiscal year. To check for robustness against potential biases that might be caused by outlier observations, we test all our hypotheses while winsorizing all variables at the lowest and the highest 5% level.

3.2 Estimating ambiguity

To assess the monthly ambiguity of a firm, we estimate the ambiguity of its equity using the model in Equation (8). This requires that the set of possible probability distributions governing outcomes be identified. As with standard measures of volatility certain assumptions must be made in order to implement such a measure. Defining returns to be the relevant outcomes, to provide an estimate of the set of possible distributions of returns requires that several choices are made. First, similar to measures of volatility, a period of time over which ambiguity is assumed to be stationary must be identified. Second, one must identify an interval over which to measure returns. Third, different estimates of probabilities of the realized returns must be associated with different (first-order) distributions. Finally, the second-order beliefs of the agent must be identified. Following Brenner and Izhakian

(2017) we assume that ambiguity is stationary over a calendar month. Equity returns, measured every five minutes of the trading day, are used as the relevant outcomes. Relative frequencies of observed outcomes are used to estimate probabilities, and the returns from each trading day in the month are assumed to represent a different distribution of returns. The set of these distributions estimated in a month are assumed to represent the agent’s set of priors for that month. Finally, we assume that the representative agent acts as if all priors for a given month are equally likely. Robustness of the measure to these specific choices has been examined in Brenner and Izhakian (2017). This process is detailed more completely below.

To estimate the probabilities of all possible returns from trading data, one must make assumptions about the nature of the distribution of returns. We assume that returns on assets are normally distributed; therefore, the degree of ambiguity of the return r_j on the equity of firm j can be measured by

$$\mathcal{U}^2[r_j] = \int \mathbb{E}[\phi(r_j; \mu_j, \sigma_j)] \text{Var}[\phi(r_j; \mu_j, \sigma_j)] dr_j, \quad (17)$$

where $\phi(r_j; \mu_j, \sigma_j)$ stands for the normal probability density function of r_j conditional upon the mean μ_j and the variance σ_j^2 .²² It is important to note that our empirical tests use a measure of the degree of ambiguity, defined by Equation (8), which is distinct from aversion to ambiguity. The former, which is a matter of beliefs (or information), is estimated from the data, while the latter, which is a matter of tastes, is endogenously determined by the empirical estimations.

We employ the empirical method developed by Brenner and Izhakian (2017) to estimate the degree of ambiguity using intraday stock trading data from the TAQ database. We compute the degree of ambiguity, given in Equation (17), for each stock and for each month, by applying the following procedure. We sample the price of the stock every five minutes starting from 9:30 until 16:00. The decision to use five-minute time intervals is motivated in part by Andersen *et al.* (2001), who show this time interval is sufficient to eliminate microstructure effects. In cases for which there is no trade during a specific time interval, we take the volume-weighted average of the closest trading price. Using these prices we compute five-minute returns, 78 returns for each day at most. Note that this procedure implies that we ignore returns between each closing price and opening price on the following day, thereby eliminating the impact of overnight price changes and dividend distributions. For each stock, we drop all trading days that do not have at least 15 quotes representing different five minute time

²²Brenner and Izhakian (2017) and Augustin and Izhakian (2016) show that the measure \mathcal{U}^2 is robust to the assumption of normally distributed return. In particular, they show that computing \mathcal{U}^2 while assuming different elliptical distributions generate quantitatively similar results.

intervals, and we drop all trading months that do not have at least 15 days that satisfy this condition. Observations with extreme price changes (plus or minus 10 percent log returns) within five minutes are also dropped, because many of them are due to mistaken orders that were cancelled by the stock exchange.²³

For a given stock each day, we compute the normalized (by the number of intraday observations) mean and variance of the return, denoted μ_j and σ_j^2 respectively. As in French *et al.* (1987), the variance of the returns is computed by applying the adjustment for non-synchronous trading proposed by Scholes and Williams (1977).²⁴ Based upon the assumption that the intraday returns are normally distributed, for each stock j we construct the set of priors \mathcal{P}_j , where each prior P_j within the set \mathcal{P}_j is defined by a pair of μ_j and σ_j . It is important to mention that in our approach, the sets of priors that underlie ambiguity are endogenously derived.

The set \mathcal{P}_j of (normal) probability distributions of each stock j for a given month consists of 15 to 22 different probability distributions. To compute the monthly degree of ambiguity of a given asset, specified in Equation (17), we represent each daily return distribution by a histogram. To this end, we divide the range of daily returns, from -40% to 40% , into 160 intervals (bins), each of width 0.5% . For each day, we compute the probability of the return being in each bin. In addition, we compute the probability of the return being lower than -40% and the probability of the return being higher than $+40\%$. Using these probabilities, we compute the mean and the variance of probabilities for each of the 162 bins separately, where the histograms are assigned with equal weights, i.e., each probability distribution in the set \mathcal{P}_j is equally likely. This is equivalent to assuming that the daily ratios $\frac{\mu_j}{\sigma_j}$ are student's- t distributed.²⁵ Then, we estimate the degree of ambiguity of each stock j for each month by the discrete form

$$\mathfrak{U}^2[r_j] = \frac{1}{\pi} \left(\frac{0.01}{w} \right)^{1+\frac{1}{\pi^2}} \left(\begin{array}{l} \text{E}[\Phi(r_{j,0}; \mu_j, \sigma_j)] \text{Var}[\Phi(r_{j,0}; \mu_j, \sigma_j)] + \\ \sum_{i=1}^{160} \text{E}[\Phi(r_{j,i}; \mu_j, \sigma_j) - \Phi(r_{j,i-1}; \mu_j, \sigma_j)] \times \\ \text{Var}[\Phi(r_{j,i}; \mu_j, \sigma_j) - \Phi(r_{j,i-1}; \mu_j, \sigma_j)] + \\ \text{E}[1 - \Phi(r_{j,160}; \mu_j, \sigma_j)] \text{Var}[1 - \Phi(r_{j,160}; \mu_j, \sigma_j)] \end{array} \right), \quad (18)$$

²³Testing our hypotheses while including observations with extreme price changes shows that the effect of ambiguity is even more significant than while excluding these observations.

²⁴Scholes and Williams' (1977) adjustment for non-synchronous trading suggests that the volatility of returns takes the form $\sigma_t^2 = \frac{1}{N_t} \sum_{i=1}^{N_t} (r_{t,i} - \text{E}[r_{t,i}])^2 + 2 \frac{1}{N_t - 1} \sum_{i=2}^{N_t} (r_{t,i} - \text{E}[r_{t,i}]) (r_{t,i-1} - \text{E}[r_{t,i-1}])$. We also test our model without the Scholes-Williams correction for non-synchronous trading. The results are essentially the same.

²⁵When $\frac{\mu}{\sigma}$ is Student's t -distributed, cumulative probabilities are uniformly distributed. See, for example, Kendall and Alan (2010, page 21, Proposition 1.27). This is consistent with the idea that the representative investor does not have any information indicating which of the possible probability distributions is more likely, and thus he acts as if he assigns an equal weight to each possibility.

where $\Phi(\cdot)$ stands for the cumulative normal probability distribution, $r_0 = -0.40$, $w = r_i - r_{i-1} = 0.005$, and $\frac{1}{\pi} \left(\frac{0.01}{w}\right)^{1+\frac{1}{\pi^2}}$ scales the weighted-average volatilities of probabilities to the bins' size.²⁶ This scaling, which is analogous to Sheppard's correction, has been tested to verify that it minimizes the effect of the selected bin size on the values of \mathcal{U}^2 ; see Brenner and Izhakian (2017).

We don't investigate whether \mathcal{U}^2 captures other well-known uncertainty factors because this verification has been done in Brenner and Izhakian (2017). Brenner and Izhakian conduct an exhaustive set of tests to rule out the concern that the degree of ambiguity, measured by \mathcal{U}^2 , is related to other "uncertainty" factors including skewness, kurtosis, variance of variance, variance of mean, downside risk, mixed data sampling measure of forecasted volatility (MIDAS), investors' sentiment and others. They also conduct tests that rule out the concern that observed returns are generated by a single (additive) probability distribution. In our empirical exploration, data in Table 2 indicate that the correlation of ambiguity with risk is relatively low. Table 7 we examine the explanatory power of our measure of ambiguity relative to others that have been proposed.

3.3 Estimating expected ambiguity

Next, we estimate the expected ambiguity of every stock in every sample month. We assume that individuals form an expectation about future ambiguity by taking into account the evolution of a given stock's past ambiguity over time. The average monthly ambiguity \mathcal{U} (over firms) is positively skewed. To adjust for skewness and to avoid negative values of conditional ambiguity, we examine the natural logarithm of $\mathcal{U}_{j,t}$.

The estimated autocorrelations of $\mathcal{U}_{j,t}$ suggest using the autoregressive moving average (ARMA) model for estimating expected ambiguity. Using the formal link between realized and conditional volatilities, as provided by Andersen *et al.* (2003), we estimate the expected ambiguity (conditional volatility of probabilities) by substituting the realized ambiguity (realized volatility of probabilities), \mathcal{U} , for the latent monthly ambiguity.²⁷ To do so, for each company in each month, $\widehat{\ln \mathcal{U}_{j,t+1}}$ is computed using the coefficients estimated by the time-series ARMA(p, q) model

$$\ln \mathcal{U}_{j,t} = \psi_0 + \epsilon_{j,t} + \sum_{i=1}^p \psi_i \cdot \ln \mathcal{U}_{j,t-i} + \sum_{i=1}^q \theta_i \cdot \epsilon_{j,t-i} \quad (19)$$

²⁶We find that this scale improves the previous scale $\frac{1}{w \ln \frac{1}{w}}$ for the bin's size, used in Izhakian and Yermack (2017), in the sense that it reduces the sensitivity of the estimated \mathcal{U}^2 to the selection of the bin's size.

²⁷Andersen *et al.* (2003) provide the theoretical framework for integrating high-frequency intraday data into the measurement of daily volatility, and they show that long-memory Gaussian vector autoregression for the logarithmic daily realized volatilities performs admirably. We apply the same approach to ambiguity, since it uses probabilities that are estimated from intraday data.

with the minimal Akaike information criterion (AIC). The expected volatility is then calculated as

$$\left(\mathcal{U}_{j,t+1}^2\right)^E = \exp\left(2\ln\widehat{\mathcal{U}}_{j,t+1} + 2\text{Var}[u_{j,t+1}]\right). \quad (20)$$

where $\text{Var}[u_{j,t+1}]$ is the minimal predicted variance of the error term. For every month t , using the data of only the 36 preceding months, i.e., from month $t - 36$ to month $t - 1$, the time-series regression given by Equation (19) is estimated for each $p = 1, \dots, 10$ and $q = 1, \dots, 10$; in total $p \times q = 100$ combinations. The coefficients of the model that attains the minimal AIC (the highest-quality model) are then used for the estimation of the expected ambiguity, $\left(\mathcal{U}_{j,t+1}^2\right)^E$, by Equation (20). Note that these estimates of expected ambiguity are out-of-sample estimates.

3.4 Estimating expected volatility

Along with ambiguity, volatility serves as the most important explanatory variable in our analysis. We compute *monthly* volatility with standard methods, using daily log returns adjusted for dividends obtained from the CRSP database. Since probabilities are uncertain, volatilities can be viewed as computed using expected probabilities, as theoretically justified in Izhakian (2016). For each individual stock in a given month, we calculate the standard deviation, $\text{Std}_{j,t}$, of the stock's daily returns over that month, again applying the Scholes and Williams (1977) correction for non-synchronous trading and a correction for heteroscedasticity.²⁸ As with ambiguity, the average volatility measures $\text{Std}_{j,t}$ (over firms) are also positively skewed. To adjust for skewness and to avoid negative values of conditional volatility, we examine the natural logarithm of $\text{Std}_{j,t}$. Again we assume that investors form an expectation about future volatility based upon its evolution over time for a given stock. The average autocorrelations (over firms) of $\ln(\text{Std}_{j,t})$ decay beyond lag 6, suggesting that $\ln(\text{Std}_{j,t})$ follows an autoregressive process. Similarly to the procedure applied to ambiguity, for every month t , the expected volatility is also estimated with ARMA(p, q) for each equity j . Namely, using realized volatility, the parameter $\ln\widehat{\text{Std}}_{j,t+1}$ is estimated using the coefficients of the time-series model

$$\ln \text{Std}_{j,t} = \psi_0 + \epsilon_{j,t} + \sum_{i=1}^p \psi_i \cdot \ln \text{Std}_{j,t-i} + \sum_{i=1}^q \theta_i \cdot \epsilon_{j,t-i} \quad (21)$$

that attains the minimal AIC out of the $p \times q = 100$ combinations of coefficients obtained from this regression.²⁹ The expected volatility is then calculated as

$$\text{Var}_{j,t+1}^E = \exp\left(2\ln\widehat{\text{Std}}_{j,t+1} + 2\text{Var}[u_{j,t+1}]\right), \quad (22)$$

²⁸See, for example, French *et al.* (1987).

²⁹Hansen and Lunde (2005) also estimate the expected volatilities by substituting the realized volatilities for the latent volatilities.

where $\text{Var}[u_{j,t+1}]$ is the minimal predicted variance of the error term. As with expected ambiguity, the obtained estimates of expected volatility are out-of-sample.

3.5 Leverage and firm specific characteristics

The annual leverage ratio of each firm is the main dependent variable in our empirical tests, and we employ both the book value and the market value versions of this ratio. Book leverage is computed as “debt in current liabilities” plus “long-term debt” divided by the total book value of assets. Market leverage is computed as “debt in current liabilities” plus “long-term debt” divided by the total market value of assets, where the latter is the market value of equity at the end of the fiscal year plus “debt in current liabilities” plus “long-term debt”.

In addition to the leverage ratios, we obtain a set of annual firm characteristics using variables that are standard in the empirical capital structure literature. Firm size is measured by the log of sales normalized by gross domestic product (GDP).³⁰ Firm profitability is measured by operating income before depreciation divided by book assets. Asset tangibility is measured by property, plant and equipment divided by book assets. The market to book ratio is measured by the market value of equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is measured by R&D expenses relative to sales normalized by 10,000. In addition, to explore the firms’ leverage ratios relative to their industry, we also include industry median annual leverage, where firms are classified by their four-digit SIC codes. Expected marginal tax rates are obtained from John Graham’s website. When the expected marginal tax rate is missing, we use the industry median annual marginal tax rate if available or the market-wide median annual marginal tax rate if necessary. Firm age is measured relative to the first year the stock price is reported in Compustat (Hadlock and Pierce, 2010).

Table 1 presents summary statistics for the key variables used in the paper. Based on the statistics presented in the table, our sample seems broadly consistent with those from other studies.

[Table 1]

Table 2 presents a correlation matrix for the variables. The two measures of uncertainty, risk (volatility) and ambiguity, are only weakly negatively related to one another when measured at a monthly frequency, with an average estimated monthly correlation of -0.0857 (not tabulated). This suggests that the two variables capture economically distinct aspects of financial uncertainty, a conclusion that coincides with the comprehensive tests of Brenner and Izhakian (2017). The process

³⁰Alternatively, firm size can be measured by the log of book assets normalized to the annual GDP level. We have also tested our hypotheses using this alternate measure of firm size and the results are virtually identical.

of averaging monthly variables that are weakly related to provide an annual measure will of course reduce randomness (noise) and so as expected, annual measures of risk and ambiguity, have a stronger estimated correlation, -0.404 in Table 2.

The correlations between ambiguity (or expected ambiguity) and the firm characteristics such as size (sales), age, and profitability are important to consider as they may appear counter-intuitive. Use of the max-min model has established a common intuition for the level of ambiguity as related to the probability distribution that generates the worst-case scenario. However, rather than provide a useful framework for evaluating the degree of ambiguity this actually points out a limitation of the max-min model for evaluating the extent of ambiguity.³¹ The degree of ambiguity within the EUUP model is measured as the variance of the possible probabilities for each event, averaged across the possible events. Consider a set of possible outcomes defined by the unit interval, with a set of possible (continuous) unimodal probability density functions that all have the same precision but differ by the location of their modes. For concreteness, let the possible distributions be distinguished by having modes of 0.1, 0.2, ..., 0.9. If the density functions in question have very high precision (i.e., are steeply sloped) then the variance of the probabilities of each outcome, averaged across outcomes will be high (a high degree of ambiguity). In this case, each possible outcome has a wide variety of potential probabilities associated with it. In contrast, if the density functions have very low precision (e.g., are close to being uniform) then the average variance of the probabilities will be very small (a low degree of ambiguity). Each outcome has as many possible probabilities as in the previous case, however, in this setting all the possible probabilities are relatively similar. Further, notice that density functions with relatively low precision imply relatively high risk.

It is intuitive to suggest that there is greater uncertainty over the possible distributions (a “larger” set of possible distributions) for small firms compared to large firms, young firms compared to old, or unprofitable as compared to profitable firms. In doing so, one thinks in terms of the number of possible distributions. However, the intuition described above can help explain the correlations in Table 2 between ambiguity and firm size, age, and profitability despite this observation. The explanation is based upon the nature of the possible distributions for younger, smaller, and less profitable firms. While small, young, and unprofitable firms may have a “larger” set of possible distributions that may

³¹As an extreme example, consider an Ellsberg urn experiment in which the decision maker is faced with an urn from which to draw a ball. A black ball results in a payoff of \$1 and a red ball a payoff of \$0. The decision maker is told that the urn has 100 total balls, all of which are black or red, and the number of black balls is either 20 or 80. Contrast that experiment with one in which the decision maker is told that the number of black balls may be 20 or 21. The decision maker clearly faces more uncertainty over probability distributions (greater ambiguity) in the first experiment, however the max-min model views the two scenarios identically.

govern their outcomes, intuition would also suggest that these possible distributions also have lower precision (are less steeply sloped and therefore more risky) than are the possible density functions for older, larger, and more profitable firms. Considering not only the “number” of potential distributions but also the nature of the distributions in the set, therefore, leads to the intuition that larger (older, more profitable) firms are expected to display a greater degree of ambiguity than smaller (younger, less profitable) firms. Note also that in all tests, firm size, firm age, and profitability are included as control variables so that the estimated relation between leverage and ambiguity holds size, age, and profitability constant.

[Table 2]

3.6 Regression tests

To test the statements of Propositions 3 and 4, we use two main empirical models. The first model explores firm-level variation in the leverage ratio L . This model uses the time series/cross sectional regression that is standard in the empirical capital structure literature.

$$L_{j,t} = \alpha + \beta_1 \cdot \mathcal{U}_{j,t-1}^2 + \beta_2 \cdot \mathbb{V}\text{ar}_{j,t-1} + \gamma \cdot Z_{j,t-1} + \varepsilon_{j,t}, \quad (23)$$

where j designates firm and t designates year. The vector Z consists of the standard controls in the literature for firm characteristics as described in Section 3.5. The main goals of this regression test are to examine the explanatory power (beyond the standard control variables) provided by ambiguity for the leverage ratio and to provide a benchmark for comparison to the existing literature.

The second model explores the effect of firm-level change in ambiguity on the change in firm leverage; i.e., we difference the data. To this end, we estimate

$$\Delta L_{i,t} = \alpha + \beta_1 \cdot \Delta \mathcal{U}_{j,t-1}^2 + \beta_2 \cdot \Delta \mathbb{V}\text{ar}_{j,t-1} + \gamma \cdot \Delta Z_{j,t-1} + \varepsilon_{j,t}, \quad (24)$$

where $\Delta L_{i,t}$ is the change in variable L between time $t - 1$ and t and $\Delta Z_{i,t-1}$ is the change in the vector of variables Z between time $t - 2$ and time $t - 1$. This test is introduced in order to address the omitted variables problem inherent in the standard levels regression (as identified by Lemmon *et al.* (2008)). Lemmon *et al.* employ firm fixed effects to control for the omitted variables bias. They, however, note that this correction “sweeps out all of the cross sectional variation so that the model cannot identify what is responsible for the majority of the variation in leverage ratios.” Because we expect that cross sectional and time series variation in ambiguity will be important for explaining leverage, we focus on the use of first differences of the variables rather than fixed effects.

The two models are tested using both book leverage and market leverage. They are also tested using both lagged realized ambiguity and risk as well as measures of expected ambiguity and expected risk, estimated as detailed above. All reported standard errors are clustered by year and by firm.

4 Empirical findings

4.1 Main findings

First, we test the basic model in Equation (23). That is, we test whether the level of the leverage ratio is affected by the level of ambiguity and risk in the previous year. Table 3 presents the findings of these regression tests. Panel A shows estimates for the book leverage ratio, while Panel B shows estimates for the market leverage ratio. In both panels, the actual values of ambiguity and risk are used in the first four columns, while the expected values, as defined earlier, are used in the right four columns. In each case, we present estimates for leverage as a function of ambiguity alone, then as a function of risk alone, then in a model with both the risk and ambiguity measures included, and finally in a model that also includes a full range of additional control variables.

Table 3 shows a relatively consistent pattern of results. When ambiguity, expected ambiguity, risk, or expected risk are included in the model alone or in combination the estimated coefficients are consistent with the hypothesized relationships between these variables and leverage. That is, in each of these models ambiguity or expected ambiguity has an estimated coefficient that is positive and significant while risk has an estimated coefficient that is negative and (except for one case) significant. However, when the usual control variables are added to the regression explaining book leverage (columns 4 and 8), the estimated coefficients on the measures of risk become positive and significant. In the regressions explaining market leverage, the coefficient estimates for both the ambiguity measures and the risk measures see a reversal of the sign when full slate of controls are included. The coefficient estimates on the measures of ambiguity become negative and significant while the coefficient estimates on the measures of risk become positive and significant.

[Table 3]

Next, we present the results of an empirical model designed to control the endogeneity problem inherent in the standard levels regression by differencing the data and testing whether the change in leverage is explained by changes to the extent of ambiguity and/or to risk. That is, we test the model in Equation (24), which suggests that a change in the extent of ambiguity or a change in the level of risk will be followed by a change in the use of leverage in the subsequent year. As with the level

regressions, we test both book leverage ratio (Panel A) and market leverage ratio (Panel B). Estimates are arranged identically to those in Table 3. In these versions of the hypothesis tests, we find that in every model, the ambiguity (or expected ambiguity) variable has a positive and highly significant coefficient estimate while the coefficient estimates for risk (or expected risk) are uniformly negative and significant.³²

Control variables in Table 4 generally have estimates in line with prior research. Leverage is higher in larger firms while more profitable firms use less leverage. Firms with higher asset tangibility borrow more (perhaps due to the availability of collateral when assets in place are large) while leverage is lower when the market-to-book ratio is high (which may be a reflection of investment opportunities versus assets in place). One surprising result in our model is that the estimated relation between research & development spending and leverage is essentially zero. Most studies, such as Mackie-Mason (1990) and Berger *et al.* (1997), have found a negative relation, in line with the Myers (1977) prediction that growth opportunities will be financed mostly by equity.

[Table 4]

Note that, when we estimate the regression using changes, the estimates for the ambiguity variable—the central focus of our study—are positive and significant regardless of the controls used or the inclusion or exclusion of the risk variable. For all specifications of the regressions the estimates for the effect of ambiguity upon leverage appear to be more consistent and more reliably significant than the estimates for the risk variable. Using firm and year fixed effects, rather than first differences to control for the firm specific unobserved heterogeneity in leverage, provides similar results. Specifically, the (unreported) estimated coefficients on ambiguity or expected ambiguity are positive and significant while the estimated coefficients on risk or expected risk are negative but not always statistically significant. In order to confirm that the results are not driven by the smallest firms in our sample we estimated that regressions in Table 4 on a sample that excludes the firm year observations for the smallest decile of firms in each year. The (unreported) results are very similar to those reported in Table 4. In all the models the estimated coefficients for ambiguity or expected ambiguity are positive and significant. All the estimated coefficients for risk or expected risk are negative, however, a couple of the estimated coefficients for expected risk are not statistically significant. This same pattern is remarkably robust across a variety of restrictions placed on the sample of firms included in the tests. If we trim rather than windsorize the data, require consecutive data points to ensure a time series

³²Note that firm age is not included as a control variable in the changes regression for the obvious reason.

component to the estimates, use quarterly rather than annual data, or measure firm size using the natural log of book assets, the ambiguity measures are always positively and significantly related to leverage while the risk measures are negatively but not always significantly related to leverage.³³ The somewhat erratic significance of the risk variable can be seen as consistent with Frank and Goyal (2009), who find that risk, measured as the variance of stock returns, is not one of the core factors determining leverage, although their analysis is based on the incremental contribution to R-squared rather than the magnitude or significance of coefficient estimates.

A further robustness test involves creating unlevered versions of the equity returns for each firm and using this data to measure ambiguity and risk. It has long been recognized that leverage magnifies measures of volatility, creating cross-sectional variation in volatility that is related to leverage. Similarly, leverage can affect the measure of ambiguity, but the nature of this impact is uncertain. Differencing the data removes some of the cross-sectional explanatory power of the right-hand side variables and so we do not predict that this will have a material impact on the reported results. Nevertheless, for completeness, we estimate the regressions in Tables 3 and 4 using measures of ambiguity and risk based on proxies for firm level (unlevered) returns. This is done by combining the book value of total debt and the market value of equity to represent firm value for each relevant interval (every 5 minutes for measuring ambiguity and every day for measuring volatility).³⁴ The resulting periodic firm values are used to estimate “unlevered” or firm level returns for each interval. The measures for ambiguity and volatility are calculated (as described above) using these returns and the regressions in Tables 3 and 4 are re-estimated. The results are very similar to those reported in Tables 3 and 4 and are omitted in the interest of space. Results for the regressions in Table 3 are more strongly consistent with the hypothesized relationships. In the re-estimated Table 4 a few of the estimated coefficients weaken in significance.³⁵ Importantly, ambiguity remains positively and significantly related to leverage in all the regressions while risk remains negatively related to leverage in all the regressions (a few of the coefficient estimates for risk lose statistical significance).

The development of the underlying theory indicates that for internal consistency in the model it is necessary that the reference point k is such that $k < F$. If this condition holds, Proposition 3 predicts a positive relation between the level of ambiguity and leverage. If (contrary to this restriction) the reference point was instead sufficiently larger than optimal leverage, the proof of Proposition 3

³³These are all treatments of the data that have been used in prior studies of leverage.

³⁴We considered three different frequencies of estimating the market value of equity: updating the market value of equity every 5 min, every day and every month.

³⁵This is not unexpected given that the estimate of unlevered returns lowers the variation of the estimated firm returns relative to actual equity returns.

would provide the opposite result, that the level of ambiguity and the level of leverage are negatively related. This may be interpreted as suggesting that the level of ambiguity and the level of leverage are negatively related for firms with low leverage while the level of ambiguity and the level of leverage are positively related for higher leverage firms. In order to examine this possibility Table 5 re-estimates the model from Table 4 on samples of low and high leverage firms. Specifically, for each event year in the sample, we first remove all firms with zero leverage. This is done to prevent censoring issues since zero leverage firms can only increase leverage. We separate the remaining firms into low and high leverage subsamples depending on whether each firm is below or above the median leverage for the non-zero leverage firms in that year. This is done for each sample year and we then estimate the empirical model presented in Table 4 on the low and high leverage samples using both lagged and our estimate of expected ambiguity and risk as explanatory variables for leverage.

The estimates presented in Table 5 are consistent with the model's requirement that $k < F$. Specifically, the coefficient estimates for ambiguity or expected ambiguity in both the high and low leverage sample are all positive and statistically significant. For the case of book leverage the point estimates of the coefficients on ambiguity and expected ambiguity are very close in size for the low and high leverage firms. In each regression, the estimated coefficient on risk is negative and significant. The estimated coefficient on expected risk is significantly negative for the high leverage firms and negative but insignificant for the low leverage firms. The coefficients on the control variables are roughly consistent with the results presented in Table 4. One interesting comparison is that Table 5 indicates that the measure of firm size (the log of sales) is more strongly related to leverage for high leverage firms than it is for low leverage firms.

[Table 5]

To investigate the economic significance of ambiguity in explaining capital structure decisions, Table 6 presents measures of the economic significance of the different independent variables we employ. The table is constructed by multiplying the coefficient estimate of each explanatory variable from Table 4 (Panels A and B), shown in the first column, by its standard deviation, shown in the second column, with the product of these two quantities displayed in the third column under the heading significance. The table reveals several patterns. First, ambiguity and expected ambiguity have greater levels of economic significance (though of opposite signs) than risk or expected risk in explaining leverage choice. Second, consistent with theoretical results, the measure of expected future ambiguity appears to have a much larger impact on leverage than does the simple lagged realized value of

ambiguity. Finally, a few of the standard control variables used in empirical capital structure studies have roughly equivalent levels of economic significance than ambiguity or expected ambiguity. When explaining book leverage, the tangibility of the firm's assets and firm size ($\log(1 + sales)$) exhibit greater economic significance than ambiguity and roughly the same level of significance as expected ambiguity. However, both ambiguity and expected ambiguity display greater economic significance than the other standard control variables (profitability, market to book ratio, R&D, industry median leverage and the measure of the marginal tax rate). When explaining market leverage only firm size displays greater economic significance than ambiguity, while expected ambiguity and firm size have roughly equivalent levels of economic significance. In the market leverage regressions, ambiguity and expected ambiguity display greater levels of economic significance than all of the other control variables. Overall, relative to a set of firm specific characteristics commonly used in the empirical capital structure literature, the decision maker's expectation for the future level of ambiguity faced by the firm appears to have a highly significant impact on the leverage decision.

[Table 6]

Table 7 reports robustness tests using alternative proxies for ambiguity that have appeared previously in the literature. We test each of the following four factors as a substitute for our ambiguity measure as well as a complementary factor alongside our measure. The variance of the mean (Var mean) is the variance of daily mean returns (computed from 5-minute returns) over the month. The variance of the variance (Var var) is the variance of daily variance returns (computed from 5-minute returns) over the month. Bid-ask spread is the effective bid-ask spread. Analysts dispersion is the variance among analyst forecasts of the stock price.

We run our regression tests using changes in book and market leverage, as explained by year lagged changes in the ambiguity proxies. All regressions include an intercept and changes in the standard control variables that are included in Table 4. Panel A presents the results using book leverage and panel B presents results using market leverage. The results display a fairly consistent pattern. When the alternative proxies for ambiguity are included in the regressions explaining book leverage (instead of our measure of ambiguity) none of the alternative proxies show a statistically significant relation with leverage (columns 2 - 5 in panel A). When the alternative measures of ambiguity are included in the regressions explaining market leverage (columns 2 - 5 in panel B) only the bid-ask spread is a significant explanatory variable. The bid-ask spread, however, has a significantly negative relation with market leverage rather than the positive relation predicted by the theory. This may simply be

due to the fact that the bid-ask spread is likely to proxy for a variety of other forces related to market leverage; asymmetric information, liquidity, etc.

The final four columns in panels A and B in Table 7 include both our measure of ambiguity and the previously used proxies in regressions that also control for risk and the other control variables. Importantly, our measure of ambiguity remains positive and statistically significant in all the regressions for both book and market leverage. The variance of mean and the dispersion of analyst forecasts are never significantly related to leverage. Since sometimes the variance of the mean is considered a proxy for time varying probability distribution, this finding rules out the possibility that our measure is driven by a time varying distribution. The variance of variance is insignificantly related to book leverage but significantly negatively related to market leverage when our measure of ambiguity is included in the regression. The bid-ask spread has a significantly negative relation to both book and market leverage. We interpret these results as confirming that our measure of ambiguity is not simply a proxy for previously used measures. We also conclude that the previously used proxies for ambiguity, while plausibly reflecting some aspects of ambiguity, do not seem to capture the salient features of this aspect of uncertainty in the context of the leverage decision.

[Table 7]

Graham (2000) reports that firms use much less debt than predicted by a static model based upon the tradeoff of the tax benefits of debt financing against risk. The results presented here only exacerbate this low leverage puzzle, suggesting that a tax/uncertainty tradeoff does not offer a complete view of capital structure choice. Models examining alternative frictions seem more promising in offering a solution to this puzzle. Hackbarth and Sun (2017) (see also Li *et al.* (2016)) consider the interaction between investment and financing in a dynamic tradeoff theory model. The model demonstrates how the nature of the firm's investment opportunity set and the value of financial flexibility interacts with leverage choice. Their parameter estimates suggest much lower levels of optimal leverage, particularly for firms with "back-loaded" investment processes, as compared to a simple tax/risk tradeoff. Alternatively, Berk *et al.* (2010) examine the optimal labor contract for a levered firm and find that employees become entrenched under the optimal contract. As such, employees face large expected costs of bankruptcy, costs which the firm internalizes via an efficiency wage that must be offered to employees. A tradeoff between tax benefits of debt versus bankruptcy costs faced by the firm and the employees implies levels of optimal leverage that are more consistent with observed leverage.

5 Conclusion

Uncertainty plays a role in capital structure choice and many other important financial decisions. Until recently, most studies examined uncertainty only by considering risk. However, ambiguity, or Knightian uncertainty, represents a separate and distinct type of uncertainty. We contribute to a growing empirical literature by showing that ambiguity and risk both play important roles in capital structure choice.

We present a model that generates a positive relation between ambiguity and leverage, and we test this hypotheses using pooled time series cross-sectional data covering more than 56,000 firm-year observations from more than 11,500 individual Compustat firms. Theoretically, while risk should have the well known negative relation to leverage, we show that ambiguity should be positively related to leverage. As predicted, we find a positive association between ambiguity and leverage along with a negative association between equity risk (as a proxy for the risk of the cash flows) and leverage. The coefficient estimates suggest that the expectation of the future ambiguity facing the firm has an economically meaningful impact on the leverage decision. Robustness tests demonstrate that other proxies for ambiguity that have been proposed are not meaningful explanatory variables with respect to the leverage decision.

Our results are consistent with other recent papers analyzing variables affected by financial uncertainty, including the pricing of credit default swaps and the timing of executive stock option exercise. Together, these studies suggest that the role of uncertainty in financial decisions is richer and more nuanced than previously believed, and that further investigations of ambiguity have the promise of yielding additional insights that may improve our understanding of basic financial decision-making.

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A Appendix

A.1 Discrete state space

Let \mathcal{S} be a finite state space and $X : \mathcal{S} \rightarrow \mathbb{R}$ be a consumption. The outcomes of X can then be written $X = (x_1, \dots, x_n)$, where x_s stands for the outcome at state s and x_1, \dots, x_n are listed in a non-decreasing order. The investor, who doesn't know the precise probabilities of these outcomes, holds a set \mathcal{P} of possible probability measures P on a σ -algebra of subsets of \mathcal{S} . In addition, she holds a second-order belief ξ (a probability measure on a σ -algebra of subsets of \mathcal{P}).

The expected utility of the investor can then be represented by

$$\begin{aligned} V(X) = & \sum_{0 < s \leq k} \Upsilon^{-1} \left(\sum_{P \in \mathcal{P}} \Upsilon(P_X(x_s)) \xi(\{P_X\}) \right) [U(x_s) - U(x_{s+1})] + \\ & \sum_{k < s \leq n} \Upsilon^{-1} \left(\sum_{P \in \mathcal{P}} \Upsilon(P_X(x_s)) \xi(\{P_X\}) \right) [U(x_s) - U(x_{s-1})], \end{aligned} \quad (25)$$

where U is a (von Neumann-Morgenstern) utility function; and Υ is an outlook function, capturing attitude toward ambiguity. As usual, a concave U characterizes risk-averse behavior, and a convex U characterizes risk-loving behavior. Similarly, a concave Υ characterizes ambiguity-averse behavior, and a convex Υ characterizes ambiguity-loving behavior. The utility function is normalized such that $U(x_k) = 0$, for some reference point $0 \leq k$. That is, any outcome indexed by $s \leq k$ is considered unfavorable (a loss) and any outcome indexed $k < s$ is considered favorable (a gain).

By Izhakian (Theorem 2, 2016), the dual representation of the expected utility in Equation (25), can be approximated to obtain

$$\begin{aligned} W(X) \approx & \sum_{s \leq k} U(x_s) E[\varphi_X(x_s)] \left(1 - \frac{\Upsilon''(E[P_X(x_s)])}{\Upsilon'(E[P_X(x_s)])} \text{Var}\varphi_X(x_s) \right) + \\ & \sum_{s > k} U(x_s) E[\varphi_X(x_s)] \left(1 + \frac{\Upsilon''(E[P_X(x_s)])}{\Upsilon'(E[P_X(x_s)])} \text{Var}\varphi_X(x_s) \right), \end{aligned} \quad (26)$$

where $\varphi_X(x)$ is the probability mass function of X being *greater* than x ; $P_X(x)$ is a cumulative probability $P \in \mathcal{P}$ of X ; and $E[\cdot]$ and $\text{Var}[\cdot]$ are respectively the expectation and the variance of probabilities taken using the second order probability ξ . To simplify notations, the subscript X , designating the random variable, is omitted. Using this functional form, the state prices of state $s \in \mathcal{S}$ can be defined by

$$q_s = E[\varphi(s)] \Lambda(s) \frac{\partial_s U}{\partial_0 U}, \quad (27)$$

where

$$\Lambda(s) = \begin{cases} 1 + \eta(\mathbb{E}[P(s)]) \text{Var}[\varphi(s)], & s \leq k \\ 1 - \eta(\mathbb{E}[P(s)]) \text{Var}[\varphi(s)], & s > k \end{cases}, \quad (28)$$

Accordingly, the risk neutral probability of state s is

$$\pi_s^* = \frac{q_s}{\sum_s q_s} = \frac{\mathbb{E}[\varphi(s)] \Lambda(s) \frac{\partial_s U}{\partial_0 U}}{\sum_s \mathbb{E}[\varphi(s)] \Lambda(s) \frac{\partial_s U}{\partial_0 U}}, \quad (29)$$

and the risk-free rate of return is

$$r_f = \frac{1}{\sum_s q_s} - 1 = \frac{1}{\sum_s \mathbb{E}[\varphi(s)] \Lambda(s) \frac{\partial_s U}{\partial_0 U}} - 1. \quad (30)$$

Let F be the face value of a zero-coupon debt and d is index. The objective function can then be written

$$\begin{aligned} \max_F \quad & S_0(F) + D_0(F) \\ \text{s.t.} \quad & S_0 = \frac{1}{1+r_f} \sum_{d < s} \pi_s^* (1-\tau)(x_s - F) \\ & D_0 = \frac{1}{1+r_f} \left[\sum_{s \leq k} \pi_s^* (1-\alpha)x + \sum_{k < s \leq d} \pi_s^* (1-\alpha)x + \sum_{d < s} \pi_s^* F \right]. \end{aligned}$$

A.2 Proofs

Proof of Theorem 1.

Assume that $x \leq k$, i.e., an unfavorable outcome. Substituting the budget constraints into the objective function in Equation (9) and solving the maximization problem, differentiation with respect to θ conditional on a given x (state of nature), provides

$$q(x) \partial_0 U = \mathbb{E}[\varphi(x)] (1 + \eta(\mathbb{E}[P(x)]) \text{Var}[\varphi(x)]) \partial_x U.$$

Organizing terms completes the proof. The proof for $k < x$, i.e., a favorable outcome, is similar. ■

Proof of Theorem 2.

Obtained from the same arguments of the Modigliani-Miller's (1958) Proposition 1. ■

Proof of Theorem 3.

The first order condition, obtained by differentiating the firm value in Equation (15) with respect to its leverage, is

$$\frac{\partial V(\cdot)}{\partial F} = \frac{1}{1+r_f} \left(\begin{aligned} & - \int_F^\infty \pi^*(x) (1-\tau) dx \\ & + \pi^*(F) F (1-\alpha) + \int_F^\infty \pi^*(x) dx - \pi^*(F) F \end{aligned} \right) = 0.$$

Rearranging terms provides

$$F = \frac{\tau}{\pi^*(F)\alpha} \int_F^\infty \pi^*(x) dx.$$

■

Proof of Proposition 1. Consider a payoff $F \leq x$. Suppose that risk increases such that instead of x the payoff is $x + \Delta$ or $x - \Delta$, with equal probabilities, i.e., $x \pm \Delta$ is mean-preserving spread of x . If $\Delta \leq x - F$ this will not affect to value of the equity. If $\Delta > x - F$, then $\frac{1}{2}(x - F + \Delta) > x - F$. This holds for any $F \leq x$. Thus, by Equation (14), the value of the equity increases in the risk. ■

Proof of Proposition 2. Since $F > k$, every $x \geq F$ is considered favorable. By Equation (7), for favorable outcomes $\pi^*(x)$ decreases in ambiguity. This holds for any $F \leq x$. Thus, by Equation (14), S_0 decreases in ambiguity. ■

Proof of Proposition 3.

Substitute the explicit expression of $\pi^*(\cdot)$ into the optimal leverage in Equation (16) to obtain

$$F = \frac{\tau}{\alpha q(F)} \int_F^\infty q(x) dx. \quad (31)$$

The first order condition (FOC) can be defined to be the implicit function³⁶

$$G(F, \eta, \mathcal{U}^2, \gamma, \mathcal{R}) = \tau \int_F^\infty q(x) dx - \alpha q(F) F. \quad (32)$$

By the second-order condition, at a maximum

$$\frac{\partial G}{\partial F} < 0.$$

Suppose that the ambiguity increases such that the variance $\text{Var}[\varphi(x)] \times t$ of the probability of each $x \in X$ increases, i.e., $t \geq 1$. Differentiating the implicit function in Equation (32) with respect to t , provides

$$\frac{\partial G}{\partial t} = -\tau \int_F^\infty \text{E}[\varphi(x)] \eta(\text{E}[P(x)]) \text{Var}[\varphi(x)] \frac{\partial_x \mathcal{U}}{\partial_0 \mathcal{U}} dx + \alpha \text{E}[\varphi(F)] \eta(\text{E}[P(F)]) \text{Var}[\varphi(F)] \frac{\partial_F \mathcal{U}}{\partial_0 \mathcal{U}} F$$

By Equation (32),

$$\frac{\partial G}{\partial t} = \alpha q(F) F - \tau \int_F^\infty \text{E}[\varphi(x)] \frac{\partial_x \mathcal{U}}{\partial_0 \mathcal{U}} dx - \alpha q(F) F + \alpha \text{E}[\varphi(F)] \frac{\partial_F \mathcal{U}}{\partial_0 \mathcal{U}} F.$$

Thus, $\frac{\partial G}{\partial t} \geq 0$ when

$$F \geq \frac{\tau}{\alpha \text{E}[\varphi(F)] \frac{\partial_F \mathcal{U}}{\partial_0 \mathcal{U}}} \int_F^\infty \text{E}[\varphi(x)] \frac{\partial_x \mathcal{U}}{\partial_0 \mathcal{U}} dx. \quad (33)$$

Since the right hand side of this inequality is the optimal F assuming there is no ambiguity (alternatively, no aversion to ambiguity), the inequality holds for the optimal F when ambiguity is present.

³⁶In the notation $G(F, \eta, \mathcal{U}^2, \gamma, \mathcal{R})$, γ stands for the risk aversion coefficient and η for the ambiguity aversion coefficient.

By the implicit function theorem,

$$\frac{\partial G}{\partial F} dF + \frac{\partial G}{\partial t} dt = 0.$$

Therefore,

$$\frac{dF}{dt} = -\frac{\frac{\partial G}{\partial t}}{\frac{\partial G}{\partial F}} \geq 0.$$

■

Proof of Proposition 4.

Suppose that for any event with an outcome x the outcome is instead xt , where $0 < t$. The greater t the greater is the risk and so is the expected outcome. Differentiating the implicit function in Equation (32) with respect to t , provides

$$\frac{\partial G}{\partial t} = \tau \int_F^\infty \mathbb{E}[\varphi(x)] (1 - \eta(\mathbb{E}[P(x)])) \text{Var}[\varphi(x)] \frac{\partial_{xx} U}{\partial_0 U} x dx.$$

Since, by risk aversion, $\partial_{xx} U < 0$, $\frac{\partial G}{\partial t} \leq 0$. Because, by the second-order condition, $\frac{\partial G}{\partial F} < 0$,

$$\frac{dF}{dt} = -\frac{\frac{\partial G}{\partial t}}{\frac{\partial G}{\partial F}} \leq 0.$$

Note that this relation holds when the expected outcome (compensating for risk) increases in t . Thus, it also holds when the expected outcome remains unchanged. ■

A.3 Tables and Figures

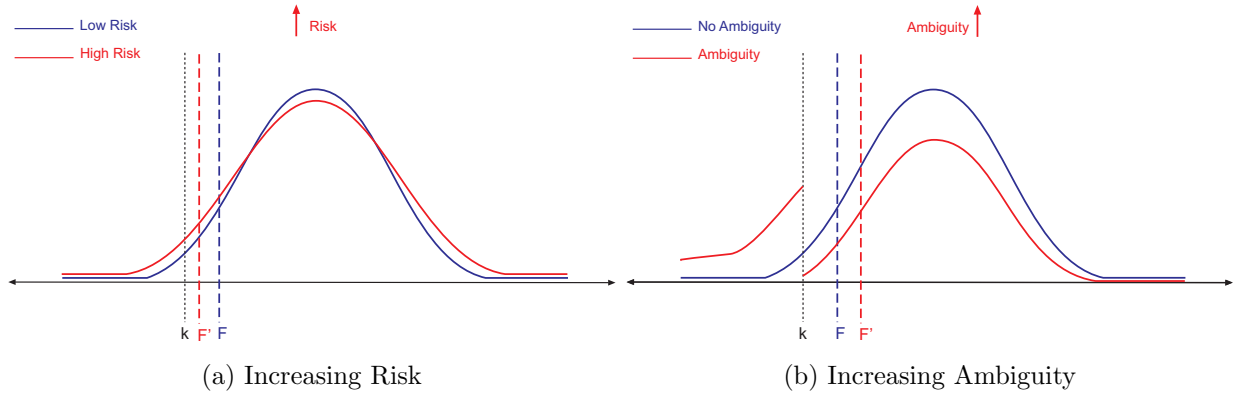


Figure 1: **Optimal leverage**

This figure depicts the effect of an increase in risk (left panel) and an increase in ambiguity (right panel) upon optimal leverage. Each panel represents a stylized perceived probability distribution in which the x -axis illustrates the range of the firm's possible future values. The point k on the x -axis is a reference point, below which any outcome is considered unfavorable (a loss) and above which any outcome is considered favorable (a gain). The point F is the optimal choice for the firm's leverage, at which the expected marginal benefit of interest tax shields just equals the expected marginal cost of bankruptcy. In the left panel, the blue series shows a lower-risk perceived probability distribution, which is assumed to shift to the red series, representing a higher-risk perceived probability distribution. The optimal choice of leverage then moves to the left on the x -axis. In the right panel, an increase in ambiguity is assumed to occur, resulting in higher perceived probabilities of expected outcomes below k and lower perceived probabilities of outcomes above k , as shown by the change from the blue to the red probability distribution. In this case the optimal choice of leverage moves to the right.

Table 1: **Descriptive statistics**

The full sample includes 56,325 annual observations associated with 11,527 individual firms between 1993 and 2016, using records from the Compustat database. Expected volatility, based on CRSP daily stock price records, and expected ambiguity, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Book leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total book assets. Market leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus “debt in current liabilities” plus “long-term debt”. Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Profitability is the operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. Median leverage is the median of the annual leverage in the industry.

Variable	Mean	Std Dev	Minimum	Median	Maximum	N
Book Leverage	0.1943	0.1994	0.0000	0.1483	0.9980	56325
Market Leverage	0.1713	0.2048	0.0000	0.0949	0.9999	56325
Ambiguity	0.0300	0.0189	0.0028	0.0251	0.1161	56325
Risk	0.0335	0.0466	0.0000	0.0187	1.3803	56325
Expected Ambiguity	0.0328	0.0188	0.0035	0.0296	0.1073	40680
Expected Risk	0.0210	0.0263	0.0003	0.0126	0.7600	40680
Median Book Lev.	0.1622	0.1479	0.0000	0.1236	0.9906	56325
Median Market Lev.	0.1444	0.1589	0.0000	0.0880	1.0000	56325
Profitability	0.0474	0.2447	-2.5559	0.1093	0.5709	56325
Tangibility	0.2416	0.2190	0.0000	0.1685	0.9369	56325
Market to Book	1.9796	1.9431	0.0000	1.3759	36.4716	56325
R&D	0.4464	3.8677	0.0000	0.0075	222.2798	56325
Sales	5.7925	2.2146	0.0000	5.9293	11.9069	56325
Tax Rate	0.1476	0.1522	0.0000	0.0452	0.3948	56325
Firm Age	15.2657	12.7442	1.0000	11.0000	58.0000	56325

Table 2: **Correlation matrix**

The full sample includes 56,325 annual observations associated with 11,527 individual firms between 1993 and 2016, using records from the Compustat database. Expected volatility, based on CRSP daily stock price records, and expected ambiguity, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Book leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total book assets. Market leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus “debt in current liabilities” plus “long-term debt”. Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Profitability is the operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. *p*-values appear in parentheses.

	Book Leverage	Market Leverage	Ambiguity	Risk	Expected Ambiguity	Expected Risk	Profitability	Tangibility	Market to Book	R&D	Sales	Tax Rate	Firm Age
Book Leverage	1												
Market Leverage	0.8248 (<.0001)	1											
Ambiguity	0.1498 (<.0001)	0.0555 (<.0001)	1										
Risk	-0.0733 (<.0001)	-0.0285 (<.0001)	-0.4042 (<.0001)	1									
Expected Ambiguity	0.1509 (<.0001)	0.0287 (<.0001)	0.9836 (<.0001)	-0.5400 (<.0001)	1								
Expected Risk	-0.0785 (<.0001)	-0.0227 (<.0001)	-0.5119 (<.0001)	0.8400 (<.0001)	-0.5227 (<.0001)	1							
Profitability	0.1011 (<.0001)	0.1055 (<.0001)	0.2973 (<.0001)	-0.3495 (<.0001)	0.3577 (<.0001)	-0.3677 (<.0001)	1						
Tangibility	0.3340 (<.0001)	0.3414 (<.0001)	0.0859 (<.0001)	-0.1135 (<.0001)	0.0676 (<.0001)	-0.0996 (<.0001)	0.1982 (<.0001)	1					
Market to Book	-0.2112 (<.0001)	-0.3759 (<.0001)	-0.0980 (<.0001)	0.1217 (<.0001)	-0.0554 (<.0001)	0.0940 (<.0001)	-0.1720 (<.0001)	-0.1793 (<.0001)	1				
R&D	-0.0486 (<.0001)	-0.0667 (<.0001)	-0.0648 (<.0001)	0.0453 (<.0001)	-0.0783 (<.0001)	0.0581 (<.0001)	-0.2639 (<.0001)	-0.0720 (<.0001)	0.0874 (<.0001)	1			
Sales	0.2986 (<.0001)	0.3325 (<.0001)	0.6185 (<.0001)	-0.4024 (<.0001)	0.6289 (<.0001)	-0.3976 (<.0001)	0.5898 (<.0001)	0.2344 (<.0001)	-0.2854 (<.0001)	-0.2264 (<.0001)	1		
Tax Rate	-0.0070 (.0949)	-0.0186 (<.0001)	0.1247 (<.0001)	-0.1979 (<.0001)	0.1600 (<.0001)	-0.2032 (<.0001)	0.3825 (<.0001)	0.1327 (<.0001)	-0.0431 (<.0001)	-0.0955 (<.0001)	0.2957 (<.0001)	1	
Firm Age	0.0974 (<.0001)	0.1121 (<.0001)	0.4756 (<.0001)	-0.2815 (<.0001)	0.4565 (<.0001)	-0.3356 (<.0001)	0.2167 (<.0001)	0.1181 (<.0001)	-0.1695 (<.0001)	-0.0556 (<.0001)	0.4588 (<.0001)	0.1375 (<.0001)	1

Table 3: Level regression estimates of capital structure

Linear model regression estimates of the level of annual leverage ratio explained by one-year lagged variables. The full sample includes 56,325 annual observations associated with 11,527 individual firms between 1993 and 2016, using records from the Compustat database. Expected volatility, based on CRSP daily stock price records, and expected ambiguity, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Book leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total book assets. Market leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus “debt in current liabilities” plus “long-term debt”. Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Profitability is the operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. *t*-statistics clustered by firm and year appear in parentheses below each coefficient estimate. All errors are double clustered by firm and year.

Panel A: Book leverage								
Intercept	0.144 (87.18)	0.211 (191.84)	0.155 (74.55)	0.014 (4.44)	0.139 (65.49)	0.216 (160.34)	0.141 (47.65)	-0.004 (-1.01)
Ambiguity	1.717 (39.70)		1.543 (32.49)	0.271 (4.92)				
Risk		-0.453 (-20.91)	-0.177 (-8.45)	0.071 (3.43)				
Expected Ambiguity					1.873 (36.06)		1.833 (29.92)	0.527 (7.60)
Expected Risk						-0.751 (-16.26)	-0.051 (-1.02)	0.182 (3.76)
Median Leverage				0.615 (92.39)				0.618 (79.68)
Profitability				-0.066 (-13.97)				-0.080 (-13.13)
Tangibility				0.106 (25.32)				0.093 (19.40)
Market to Book				-0.005 (-12.53)				-0.004 (-6.50)
R&D				0.001 (2.59)				0.001 (2.49)
Sales				0.016 (28.59)				0.017 (24.45)
Tax Rate				-0.100 (-19.75)				-0.105 (-18.07)
Firm Age				-0.001 (-16.80)				-0.001 (-12.99)
N	49082	49082	49082	49082	35439	35439	35439	35439
R-Square	0.0266	0.0102	0.0279	0.3349	0.0312	0.0096	0.03116	0.3368

Panel B: Market leverage

Intercept	0.155 (89.31)	0.188 (164.37)	0.167 (74.37)	0.030 (9.28)	0.154 (69.70)	0.190 (136.90)	0.166 (52.67)	0.018 (4.40)
Ambiguity	0.786 (18.32)		0.586 (12.20)	-0.859 (-15.43)				
Risk		-0.308 (-13.63)	-0.203 (-8.34)	0.145 (6.18)				
Expected Ambiguity					0.784 (15.09)		0.588 (9.45)	-0.697 (-10.00)
Expected Risk						-0.490 (-10.37)	-0.265 (-4.84)	0.127 (2.58)
Median Leverage				0.569 (81.88)				0.568 (70.64)
Profitability				-0.073 (-18.60)				-0.087 (-17.26)
Tangibility				0.111 (25.01)				0.105 (20.25)
Market to Book				-0.017 (-37.57)				-0.018 (-30.42)
R&D				0.000 (1.72)				0.001 (2.97)
Sales				0.022 (36.97)				0.023 (31.48)
Tax Rate				-0.102 (-19.21)				-0.104 (-17.25)
Firm Age				-0.001 (-14.54)				-0.001 (-11.08)
N	49082	49082	49082	49082	35439	35439	35439	35439
R-Square	0.005011	0.004224	0.006523	0.3987	0.004965	0.003702	0.005701	0.3971

Table 4: **Changes regression estimates of capital structure**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables. The full sample includes 56,325 annual observations associated with 11,527 individual firms between 1993 and 2016, using records from the Compustat database. Expected volatility, based on CRSP daily stock price records, and expected ambiguity, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Book leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total book assets. Market leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus “debt in current liabilities” plus “long-term debt”. Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Profitability is the operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. *t*-statistics clustered by firm and year appear in parentheses below each coefficient estimate. All errors are double clustered by firm and year.

Panel A: Book leverage								
Intercept	0.007 (14.34)	0.007 (15.51)	0.007 (14.30)	0.006 (11.13)	0.005 (10.23)	0.006 (12.04)	0.005 (10.17)	0.005 (9.06)
Ambiguity	0.348 (6.42)		0.306 (5.44)	0.308 (5.43)				
Risk		-0.068 (-4.03)	-0.052 (-2.99)	-0.057 (-3.29)				
Expected Ambiguity					0.671 (10.70)		0.638 (9.78)	0.648 (9.86)
Expected Risk						-0.129 (-3.72)	-0.046 (-2.25)	-0.051 (-2.41)
Median Leverage				-0.020 (-2.60)				-0.014 (-1.65)
Profitability				-0.008 (-1.30)				0.000 (-0.05)
Tangibility				0.066 (5.03)				0.063 (4.06)
Market to Book				-0.001 (-2.31)				-0.001 (-2.42)
R&D				0.000 (-0.39)				0.000 (0.57)
Sales				0.011 (5.70)				0.007 (2.83)
Tax Rate				0.000 (-0.09)				0.000 (-0.08)
N	39906	39906	39906	39906	25961	25961	25961	25961
R-Square	0.0011	0.0007	0.0015	0.0059	0.0035	0.0007	0.0036	0.0065

Panel B: Market leverage

Intercept	0.009 (15.46)	0.010 (18.11)	0.009 (15.37)	0.007 (11.09)	0.005 (7.75)	0.007 (11.69)	0.005 (7.62)	0.004 (6.28)
Ambiguity	0.945 (14.48)		0.790 (12.01)	0.743 (11.28)				
Risk		-0.235 (-10.40)	-0.193 (-8.80)	-0.196 (-9.11)				
Expected Ambiguity					1.788 (21.16)		1.658 (19.31)	1.588 (18.60)
Expected Risk						-0.395 (-8.63)	-0.182 (-3.91)	-0.185 (-4.03)
Median Leverage				-0.018 (-2.18)				-0.015 (-1.65)
Profitability				-0.018 (-3.88)				-0.012 (-2.14)
Tangibility				0.059 (4.21)				0.043 (2.47)
Market to Book				0.000 (1.30)				0.000 (0.60)
R&D				0.000 (0.48)				0.000 (0.23)
Sales				0.025 (11.68)				0.017 (6.75)
Tax Rate				0.000 (-0.09)				0.001 (0.12)
N	39906	39906	39906	39906	25961	25961	25961	25961
R-Square	0.0063	0.0059	0.0101	0.0183	0.0177	0.0047	0.0186	0.0219

Table 5: **Changes regression estimates of capital structure in subgroups by median**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables. The full sample includes 56,325 annual observations associated with 11,527 individual firms between 1993 and 2016, using records from the Compustat database. Expected volatility, based on CRSP daily stock price records, and expected ambiguity, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Book leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total book assets. Market leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus “debt in current liabilities” plus “long-term debt”. Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Profitability is the operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. *t*-statistics clustered by firm and year appear in parentheses below each coefficient estimate. All errors are double clustered by firm and year.

	Book leverage				Market leverage			
	Below Median		Above Median		Below Median		Above Median	
Intercept	-0.012 (-16.96)	-0.011 (-14.86)	0.031 (29.60)	0.027 (24.31)	-0.014 (-20.55)	-0.014 (-18.16)	0.033 (26.13)	0.028 (20.00)
Ambiguity	0.351 (4.56)		0.372 (3.58)		0.515 (6.92)		1.068 (8.20)	
Risk	-0.080 (-3.30)		-0.086 (-2.05)		-0.113 (-4.01)		-0.487 (-9.63)	
Expected Ambiguity		0.672 (7.83)		0.703 (5.88)		1.185 (11.24)		2.168 (13.52)
Expected Risk		-0.025 (-0.48)		-0.209 (-2.62)		-0.027 (-0.35)		-0.495 (-5.20)
Median Leverage	-0.035 (-3.01)	-0.034 (-2.68)	-0.050 (-4.12)	-0.029 (-2.23)	-0.068 (-5.57)	-0.068 (-4.73)	-0.022 (-1.78)	-0.011 (-0.77)
Profitability	0.001 (0.08)	0.002 (0.23)	-0.027 (-1.51)	-0.001 (-0.07)	-0.004 (-0.63)	-0.005 (-0.61)	-0.055 (-3.35)	-0.038 (-1.93)
Tangibility	0.082 (5.05)	0.116 (6.67)	0.079 (3.34)	0.052 (1.89)	0.010 (0.76)	0.028 (1.64)	0.130 (4.84)	0.083 (2.48)
Market to Book	-0.002 (-3.47)	-0.002 (-3.10)	-0.003 (-1.53)	-0.002 (-1.11)	0.001 (4.78)	0.001 (3.68)	-0.005 (-2.56)	-0.004 (-1.69)
R&D	-0.001 (-1.61)	0.000 (-1.16)	0.001 (0.90)	0.002 (1.89)	0.000 (-0.43)	0.000 (-0.56)	0.001 (0.80)	0.002 (1.27)
Sales	0.004 (1.40)	0.002 (0.64)	0.020 (4.70)	0.013 (2.23)	0.015 (6.04)	0.015 (4.91)	0.050 (8.53)	0.035 (4.73)
Tax Rate	0.007 (1.27)	0.001 (0.20)	0.000 (-0.04)	0.004 (0.47)	0.013 (2.38)	0.012 (1.77)	-0.006 (-0.60)	-0.003 (-0.29)
N	16202	11717	16202	11717	16202	11869	16202	11869
R-Square	0.0104	0.0138	0.0014	0.0090	0.0072	0.0308	0.0101	0.0279

Table 6: **Economic significance of the estimates of capital structure**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables. The full sample includes 56,325 annual observations associated with 11,527 individual firms between 1993 and 2016, using records from the Compustat database. Expected volatility, based on CRSP daily stock price records, and expected ambiguity, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Book leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total book assets. Market leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus “debt in current liabilities” plus “long-term debt”. Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Profitability is the operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales.

Panel A: Book leverage changes					
	Std Dev	Coefficient	Significance	Ambiguity Relative to	Ex. Ambiguity Relative to
Ambiguity	0.0092	0.3084	0.0028	1.0000	1.7992
Risk	0.0364	-0.0567	-0.0021	1.3690	2.4631
Expected Ambiguity	0.0079	0.6476	0.0051	0.5558	1.0000
Expected Risk	0.0183	-0.0514	-0.0009	3.0123	5.4199
Median leverage	0.0711	-0.0205	-0.0015	1.9431	3.4962
Profitability	0.1466	-0.0077	-0.0011	2.5119	4.5195
Tangibility	0.0652	0.0657	0.0043	0.6601	1.1876
Market to Book	1.5905	-0.0011	-0.0018	1.5719	2.8282
R&D	2.7515	-0.0001	-0.0003	10.2751	18.4873
Sales	0.4308	0.0113	0.0049	0.5826	1.0482
Tax Rate	0.1203	-0.0004	0.0000	67.0586	120.6542
Panel B: Market leverage changes					
	Std Dev	Coefficient	Significance	Ambiguity Relative to	Ex. Ambiguity Relative to
Ambiguity	0.0092	0.7429	0.0068	1.0000	1.8315
Risk	0.0364	-0.1955	-0.0071	0.9566	1.7521
Expected Ambiguity	0.0079	1.5880	0.0125	0.5460	1.0000
Expected Risk	0.0183	-0.1848	-0.0034	2.0162	3.6927
Median leverage	0.0961	-0.0181	-0.0017	3.9045	7.1511
Profitability	0.1466	-0.0178	-0.0026	2.6137	4.7870
Tangibility	0.0652	0.0589	0.0038	1.7711	3.2438
Market to Book	1.5905	0.0004	0.0006	11.1771	20.4711
R&D	2.7515	0.0001	0.0003	24.4828	44.8406
Sales	0.4308	0.0248	0.0107	0.6376	1.1678
Tax Rate	0.1203	-0.0005	-0.0001	120.7594	221.1726

Table 7: **Robustness tests**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables. The full sample includes 56,325 annual observations associated with 11,527 individual firms between 1993 and 2016, using records from the Compustat database. Book leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total book assets. Market leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus “debt in current liabilities” plus “long-term debt”. The variance of the mean (Var mean) is the variance of daily mean returns (computed from 5-minute return) over the month. The variance of the variance (Var var) is the variance of daily variance returns (computed from 5-minute return) over the month. Bid-ask spread is the effective bid-ask spread. Analysts dispersion is the variance among analyst forecasts of the stock price. All regressions include an intercept and the following control variables (whose coefficient estimates are unreported). Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Profitability is the operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. *t*-statistics clustered by firm and year appear in parentheses below each coefficient estimate. All errors are double clustered by firm and year.

Panel A: Book leverage									
Ambiguity	0.248 (3.77)					0.247 (3.76)	0.248 (3.77)	0.216 (3.26)	0.248 (3.77)
Risk	-0.113 (-3.73)	-0.139 (-4.87)	-0.139 (-4.89)	-0.107 (-3.79)	-0.138 (-4.87)	-0.112 (-3.72)	-0.113 (-3.73)	-0.085 (-2.84)	-0.112 (-3.71)
Var Mean		0.000 (0.87)				0.000 (0.82)			
Var Var			0.000 (1.06)				0.000 (0.42)		
Bid-Ask Spread				-0.003 (-0.31)				-0.870 (-5.54)	
Analysts Dispersion					-0.002 (-0.54)				-0.002 (-0.54)
N	21205	21205	21205	21205	21205	21205	21205	21205	21205
R-Square	0.0071	0.0066	0.0066	0.0089	0.0068	0.0068	0.0071	0.0093	0.0072
Panel B: Market leverage									
Ambiguity	0.644 (7.79)					0.644 (7.80)	0.644 (7.79)	0.560 (6.77)	0.644 (7.79)
Risk	-0.309 (-8.13)	-0.374 (-10.10)	-0.374 (-10.10)	-0.284 (-7.80)	-0.373 (-10.07)	-0.309 (-8.14)	-0.309 (-8.13)	-0.229 (-6.14)	-0.308 (-8.10)
Var Mean		0.000 (-0.70)				0.000 (-0.89)			
Var Var			0.000 (-0.34)				0.000 (-3.04)		
Bid-Ask Spread				-2.550 (-12.48)				-2.493 (-12.25)	
Analysts Dispersion					-0.003 (-1.10)				-0.003 (-1.11)
N	21205	21205	21205	21205	21205	21205	21205	21205	21205
R-Square	0.0087	0.0186	0.0186	0.0333	0.0187	0.0214	0.0214	0.0354	0.0215