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Entrepreneurs Heterogeneity and New Ventures Financing

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Abstract

In this article we study the market for entrepreneurial finance, and use entrepreneurs' heterogeneity to explain the coexistence of different financiers like Venture Capital (VC) and Angel investors. This issue has become increasingly important because i. New ventures are the major source of growth in GDP and job creation, and ii. Some estimates suggest that Angel investors are as important as VC in financing new ventures.

We capture entrepreneurial heterogeneity by assuming that entrepreneurs are motivated by several *motivational factors* beyond the motivation provided by *economic value considerations*. Motivational factors may include, for example, the entrepreneur's attitude towards risk, his need for achievement, tolerance for ambiguity and benefits of control. The difference between VC investors and angel investors is in their information acquisition capabilities, as VC investors can better learn the probability of the venture success. It seems natural that VC investors, being better informed, will dominate the market and will wipe out the less informed angel investors. This would be true if entrepreneurs only cared about economic value. With motivational factors, however, the outcome is different, as angel investors offer entrepreneurs an avenue to better capture their motivational factor.

Our model yields several empirical implications as follows: 1. Angel investors will invest smaller amount on average compared to VC investors. 2. If entrepreneurs switch financiers, they will always switch from angel financing to VC financing. 3. Angels invest in higher value ventures compared to VCs. 4. The probability of liquidating a venture is higher under VC financing than under angel financing.

1. Introduction

In this article we study the market for entrepreneurial finance, and use entrepreneurs heterogeneity to explain the coexistence of different financiers like Venture Capital (VC) and when Angel investors. This issue becomes increasingly important because i. New ventures are the major source of growth in GDP and job creation (Haltiwanger, Jarmin and Miranda 2009) and ii. Some estimates suggest that angel investors are as important as VC in financing new ventures (Goldfarb et al. 2007, Ibrahim 2008, Shane 2008, Sudek et al. 2008, Kerr et al. 2014).

We capture entrepreneurial heterogeneity by assuming that entrepreneurs are motivated by several *motivational factors* beyond the motivation provided by *economic value considerations*. *Motivational factors* are attributes of the entrepreneur's personal characteristics that interact with the venture characteristics, thereby affecting the entrepreneur's utility. For example, an entrepreneur who values a clean environment will draw utility (beyond the utility from economic value) from a venture that develops alternative energy. Another example may be of an entrepreneur with a need for achievement (McClelland 1961) that derives utility from a successful development of a treatment for a previously incurable disease. In general, motivational factors may include the entrepreneur's attitude towards risk (Shane and Venkataraman 2000), his need for achievement (McClelland 1961), tolerance for ambiguity (Schere 1982), benefit of control (Rotter 1966) and others. See Shane, Locke, and Collins 2003 for a detail description of these factors.

As per different financiers, the conventional wisdom is that angel investors provide financing at an earlier stage of the venture's life, have less funds, and are less sophisticated than VCs. As Ibrahim (2008) write: "Indeed, the conventional wisdom is that angels use simple contracts because they lack the sophistication of venture capitalists – in other words, they simply

don't know any better." In this article, we adopt an interpretation of the last convention, namely, that VC and angel investors differ in their information acquisition capabilities. Specifically, VC and angel investors have enough funds to finance any good venture and are rational in that they only finance ventures that yield positive return. They share the same information ex-ante, but, as the venture progresses, a VC will learn with more precision the probability of success. The idea here is that a VC that employs market and industry analysts is able to collect better information. In contrast, angel investors consist of wealthy individuals, who may or may not have knowledge of the product and technical issues, but have little or no information about the market for the venture's products (see Haltiwanger, Jarmin and Miranda 2009 Chaper 2). We introduce two types of angel investors: "general angel", who learn nothing, and "expert angel", who update their information in the interim.

It seems natural that VC investors, being better informed, will dominate the market and will wipe out the less informed angel investors by financing the venture with smaller dilution to the entrepreneur. This would be true if entrepreneurs only cared about economic value. With motivational factors, however, the outcome is different, as angel investors offer entrepreneurs an avenue to better capture their motivational factors. To show this, we use a dynamic two-stage financing model with learning as follows.

Suppose new venture development consists of two stages over three dates, $t = 0, 1, 2$, until the product is brought to the market. At $t = 0$ the design stage starts and requires an initial investment I_0. At $t = 1$, the design stage ends and the product development stage starts and requires additional investment I_1. The entrepreneur may raise the entire investment $I = I_0 + I_1$ at $t = 0$, or raise I_0 at the *pre-seed investment round* at $t = 0$ and I_1 at the *seed investment round* at $t = 1$. At $t = 2$ the product is brought to the market and the venture value is realized. Suppose

that the venture *exit value* at $t = 2$ can be either high or low. The stage that ends at $t = 1$ results in a probability P that the exit value at $t = 2$ will be high. With probability $1 - P$ the exit value at $t = 2$ will be low. We refer to P as the success probability. At $t = 0$ the success probability P is unknown.

After the first stage, a VC learns whether the venture will succeed, while an angel learns only part of this information, the success probability P (in the case of an expert angel), or nothing (in the case of a general angel). Value maximization calls for a VC financing, whereby the VC shuts the venture down when he learns that the venture will result in a low valuation (the option to abandon). The entrepreneur, however, maximizes his utility that consists of both the economic value and his motivational factors. When his motivational factors are high enough, the entrepreneur is willing to forgo some economic value by giving up the option to abandon, and continuing with the venture under low valuation. The VC cannot commit to continue financing when he realizes that the outcome will be low. The general angel, on the other hand, does not have such a problem, as he acquires no new information. He will price protect itself at the outset, and will not stop the venture.[1]

The implications for the structure of entrepreneurial finance markets are surprising. First, the "weak" investors - the investors with inferior information - will not only survive in the market, but will also attract some of the best ventures. This is because their "weakness" in terms of economic value is their "strength" in terms of allowing entrepreneurs to capture theirs motivational factors. Perhaps paradoxically, this strength is due to the angels' weakness in learning information, and not because of any exogenous advantage they may have. Therefore, it follows from our model that angel investors are "weak" by design. That is, they set up their

[1] An expert angel will acquires partial information and, thus, will stop financing is some cases.

business in order to avoid learning about ventures' market in order to attract entrepreneurs with high motivational factors. In this way, the structure of the market for entrepreneurial finance caters to the needs of heterogeneous entrepreneurs, as it provides entrepreneurs with their investor of choice. Entrepreneurs with relatively higher utility from economic value will choose VC financing, whereas entrepreneurs with relatively higher motivational factors will choose angel financing.

It is conceivable that the idea that the needs of different entrepreneurs are catered by different investor types may be implemented instead by financial contracts. Obviously a "weak" investor cannot mimic the contract of a "strong" investor, because he does not have the required information. Therefore, the question is whether a "strong" investor can offer contracts that mimic a "weak" investor contracts. This does not apply for the case of an expert angel because, when a VC learns the final exit value, he has no idea what is the success probability that an expert angel might have learnt. Therefore, although the VC is better informed, he cannot replicate the expert angel contract. In addition, there may be a commitment issue, as it may be impossible for a VC to commit to not using information it has already learnt. In this respect, the only way to implement financial contracts in this market is via the market structure we identify here, that includes investors with different information acquisition capabilities, each providing the specific contract it is best at.

Our model yields several empirical implications:

1. Angel investors will invest smaller amount on average compared to VCs. Note that this result is endogenous in our model, and is not due to exogenous reasons like lack of funds.

2. If entrepreneurs switch financiers, they will always switch from angel financing to VC financing. Again, this is not because of any exogenous reason.
3. Angel backed ventures have a higher value on average than VC backed ventures.
4. VC backed ventures are more likely to be liquidated than angel backed ventures.

The rest of the article is organized as follows. In section 2 we introduce the model. In Section 3 we analyze the game where VC financing is the only option. In Sections 4 and 5 we do the same for expert angel and general angel financing, respectively. In Section 6 we analyze the simultaneous choice between VC, expert angel and general angel financing. In Section 7 we provide the empirical implications of the model, and in Section 8 we conclude. All proofs are in the appendix.

2. Model

We envision a market for entrepreneurial finance consisting of many entrepreneurs and potential financiers. Entrepreneurs develop new ventures and need to raise funds. For simplicity we assume that entrepreneurs have no personal funds and need to raise the entire investment from outside investors. We assume equity financing.[2] New venture development consists of two stages over three dates, $t = 0, 1, 2$, until the product is brought to the market, as follows. At $t = 0$ the design stage starts and requires an initial investment I_0. This is the *pre-seed investment round*. At $t = 1$, the design stage ends and the product development stage starts and requires additional investment I_1 at $t = 1$. This is the *seed investment round*. The entrepreneur may raise the entire investment $I = I_0 + I_1$ at $t = 0$, or raise I_0 at the *pre-seed investment round* and I_1 at

[2] Other forms of financing like debt or preferred stocks will not change the main results of our model.

the *seed investment round* [3]. At $t = 2$ the product is brought to the market and the venture value is realized.

We assume that the venture *exit value* at $t = 2$ can be either high or low. While we formally refer to the venture value at $t = 2$ as an exit value, our model equally applies to situations where the venture continues to operate beyond $t = 2$ as an independent company, in which case the exit value represents the expected value of all future cash flows at $t = 2$. We normalize the low exit value to zero, and denote the high exit value by V. We refer to the low outcome of low exit value as "failure" and to the high outcome of high exit value as "success". The value V represents the estimate at $t = 0$ of what will be the venture exit value upon success at$t = 2$.

The design stage that ends at $t = 1$ results in a probability P that the exit value at t=2 will be V. With probability $1 - P$ the exit value at $t = 2$ will be 0. We refer to P as the success probability. A higher success probability P corresponds to a more productive design stage. At $t = 0$ the success probability P is unknown and is characterized by a distribution function $F(P)$ with an expected value $\bar{P}$.

We assume that entrepreneurs are motivated by both *economic value considerations* and *motivational factors*. Value considerations imply that the entrepreneur's utility increases with the venture's value. In addition, the entrepreneur draws utility from other aspects of the venture that interact with his personal characteristics. We refer to these interactions as motivational factors. For example, a venture that develops alternative energy increases the utility of an entrepreneur who values a clean environment. Another example may be of an entrepreneur with a need for achievement (McClelland 1961) that derives utility from a successful development of a treatment

[3] The investment I_1 may be positive or negative. A negative I_1 represents the case where there are intermediate positive cash flows before the exit event at $t = 2$.

for a previously incurable disease. In general, motivational factors may include the entrepreneur's attitude towards risk (Shane and Venkataraman 2000), his need for achievement (McClelland 1961), tolerance for ambiguity (Schere 1982), benefit of control (Rotter 1966) and others. See Shane, Locke, and Collins (2003) for a detail description of these factors. For simplicity of the analysis, we capture the value of the entrepreneur's motivational factors by a single number, $MF \in (-\infty, \infty)$ representing the utility to the entrepreneurs from the motivational factors from developing the venture. We envision an economy with a large pool of entrepreneurs with different motivational factors MF and many ventures with different economic characteristics. We assume that the cumulative distribution function of MF across potential entrepreneurs is $Q(MF)$ and that this cumulative distribution function is independent of the distribution of ventures' economic characteristics.

The entrepreneur's utility consists of his share of the venture value and the level of motivational factors he derives from the venture. For simplicity we assume that the entrepreneur's utility function is additively separable and linear in the venture value and the motivational factors as follows[4]

$$U = \alpha\gamma V + MF \tag{1}$$

where $\alpha\gamma$ is the entrepreneur's equity stake in the venture following the venture funding at $t = 0$ and $t = 1$, as explained below. The *pre-seed round investor* of $t = 0$ gets a stake $1 - \alpha$ of the

[4] As described above, in general the entrepreneur's utility depends on the venture value V and a vector of motivational factors, $(MF_1, MF_2, MF_3 \ldots\ldots MF_n)$ as follows: $U = U(MF_1, MF_2, MF_3 \ldots\ldots MF_n, V)$. In this respect, $\alpha\gamma V$ of Equation (1) represents the utility the entrepreneur derive from economic value considerations and MF represents the level of utility he derives from the underlying vector of motivational factors $(MF_1, MF_2, MF_3 \ldots\ldots MF_n)$ associated with the venture.

venture at the time of its investment. The *seed round investor* of $t = 1$ gets a stake $1 - \gamma$ of the venture at the time of its investment.[5] The entrepreneur gets the remaining equity stake, $\alpha\gamma$[6].

The game evolves as follows. At $t = 0$ the structure of the game, $V, I_0, I_1, F(P)$ and MF are common knowledge to the entrepreneur and all potential investors[7]. The entrepreneur can raise funds from three types of investors that differ in their information acquisition capabilities at $t = 1$. The most informed investors learn at $t = 1$ if the venture will succeed or fail at $t = 2$[8]. The least informed investors don't learn new information at $t = 1$, and remains with the prior beliefs $F(P)$. The third type of investors learns at $t = 1$ the success probability P, but does not know whether the venture will ultimately succeed or fail at $t = 2$. We also assume that the entrepreneur learns the success probability P at $t = 1$, thus he has the same information as the third investor. In addition, once a pre-seed investment is made, the identity of the pre-seed investor becomes public information.[9]

We refer to the most informed type as a "Venture Capital" (VC) investor, and to the other two types as "angel" investors. Further, we refer to an angel that learns P as an "Expert Angel" (EA), and to an angel that does not learn new information as a "General Angel" (GA). The identification of the most informed type as a VC investor is based on the fact that VC funds employ technological, product and market specialists that understand the product and its markets.

[5] While the pre-seed investor gets $1 - \alpha$ at $t = 0$, it will be diluted at the seed round of $t = 1$ by $1 - \gamma$, resulting in a final stake $(1 - \alpha)\gamma$.

[6] Consequently, the claims on the final venture are $\alpha\gamma$, $(1 - \alpha)\gamma$, and $1 - \gamma$ for the entrepreneur, the pre-seed round investor and the seed round investor, respectively.

[7] The analysis is identical if the entrepreneur motivational factors MF are private information of the entrepreneur.

[8] A more realistic assumption would be that the most informed investor obtains an informative signal about the probability of success that improves his information about the prospects of success or failure at $t = 2$, relative to the success probability P obtained by the entrepreneur. Our simplifying assumption that the most informed investor gets a perfect signal and knows the actual outcome that will prevail at $t = 2$ is without loss of generality and was assumed for the ease of exposition.

[9] In practice, the identities of all pre-seed investors are summarized in a "cap table" and are available for current and future investors.

An expert angel is a financier with some understanding of the product and its market, but his knowledge is inferior to that of a VC, perhaps because it was involved in companies from the same or closely related industry. A general angel is a financier with little or no knowledge of the product and its market.[10] We assume a competitive financing market with many GAs, EAs and VCs.

In what follows we solve the financing game by first analyzing the case where only VC financing is available in the economy (Section 3), where only EA financing is available (Section 4), and where only *GA* financing is available (Section 5). In Section 6 we analyze the case where all type of financiers are available.

3. Venture Capital Financing

In this section, we consider the case where only VC financing is available for the two investments, I_0 and I_1. We first solve the seed financing problem at $t = 1$, taking the solution of the pre-seed financing problem at $t = 0$, α, as given. Recall that at $t = 1$ a VC investor knows whether the venture will succeed or fail at $t = 2$. Since ventures with $I_1 > V$ are not fundable, the analysis that follows is constrained to ventures with $V \geq I_1$. Consequently, a VC investor invests if the venture will succeed and does not invest otherwise. When the venture will succeed, a VC can recover its investment I_1, and thus it is willing to invest. When the venture will fail, a VC cannot recover its investment and thus it does not invest. Consequently, the entrepreneur's expected utility at $t = 1$ after learning P but before approaching a VC investor is

[10] To focus on the implications of the structure of the entrepreneurial financing market on the matching between entrepreneurs and financiers at the pre-seed and seed stages, we abstract away from the advising and networking roles of start-up investors by implicitly assuming that all investor types have the same contribution, so the exit value is independent of the financier type. This assumption is without loss of generality and is made for ease exposition.

$$U_{vc}^{1} = P \times (\alpha\gamma V + MF). \quad (2)$$

In Equation (2), the term $P\alpha\gamma V$ is the entrepreneur's expected share of the venture value. It consists of the product of the entrepreneur's share of the exit value at $t = 1$, $\alpha\gamma V$, and the success probability P. Similarly, $P \times MF$ is the expected value of the entrepreneur's motivational factors. The assumption that financial markets are competitive implies that the VC share of the venture equals its investment I_1. Consequently, γ is the solution to

$$(1 - \gamma)V = I_1. \quad (3)$$

Rearranging Equation (3) yields

$$\gamma = \frac{V - I_1}{V}. \quad (4)$$

Substituting Equation (4) into Equation (2) yields that the entrepreneur's expected utility at $t = 1$ is

$$U_{vc}^{1} = P(\alpha(V - I_1) + MF). \quad (5)$$

Equation (5) implies that the entrepreneur expects to obtain his share of the venture value before the seed investment of $t = 1$, $\alpha(V - I_1)$, plus the value of his motivational factors, MF, all multiplied by the success probability P.

Likewise, the pre-seed VC's expected payoff at $t = 1$, before it learns whether the venture will succeed, is

$$\pi_{vc}^{1} = P(1 - \alpha)(V - I_1). \quad (6)$$

Equation (6) implies that the VC's expected payoff at $t = 1$, before it learns whether the venture will succeed, equals its share of the venture, $(1 - \alpha)$, times the venture value before the seed investment at $t = 1$, $V - I_1$, times the success probability P.

The pre-seed VC breaks even at $t = 0$ if its expected payoff from the venture equals its investment,

$$\int_0^1 \pi_{vc}^1 \, dF(P) = (1-\alpha)(V-I_1)\int_0^1 P \, dF(P) = (1-\alpha)\bar{P} \times (V-I_1) \; = I_0. \tag{7}$$

Equation (7) is the expected payoff to the VC investor at $t = 0$ where the expectation is taken over the success probability P at $t = 0$. Rearranging Equation (7) yields

$$\alpha = \frac{\bar{P}\times(V-I_1)- I_0}{\bar{P}\times(V-I_1)}. \tag{8}$$

Equation (8) reveals that the venture is fundable by a VC at the pre-seed round whenever

$$\bar{P} \times (V-I_1) \geq I_0. \tag{9}$$

Substituting α from Equation (8) into Equation (5) and taking expectation with respect to P reveals that the entrepreneur's expected utility at $t = 0$, provided that the venture is fundable, is

$$U_{vc}^0 = \bar{P} \times (V-I_1) - I_0 + \bar{P} \times MF. \tag{10}$$

Equation (10) implies that the entrepreneur obtains the expected NPV of the venture plus his expected value of his motivational factors.

4. Expert Angel Financing

We now consider the case where only EA financing is available for the two investments, I_0 and I_1. As before, we first solve the financing problem at the seed round, taking the solution of the financing problem at the pre-seed round as given. Recall that at $t = 1$ an EA knows the realized value of P but does not know whether the venture will succeed. Consequently, an EA invests at $t = 1$ according to the venture expected value $P \times V$. The EA invests if and only if $P \times V \geq I_1$, that is, when he recovers his investment I_1. We define by P^c the critical value of P above which an EA will agree to fund the venture at $t = 1$. An EA breaks even if $P \times V = I_1$, implying that $P^c = \frac{I_1}{V}$. For any $P < P^c$ an EA does not finance the venture at $t = 1$, and the

venture is liquidated. For $P \geq P^c$ an EA finances the venture. In what follows we analyze this case. The entrepreneur's expected utility at $t = 1$ after learning P, provided that $P \geq P^c$, is

$$U_{EA}^1 = P\alpha\gamma V + MF. \quad (11)$$

The term $P\alpha\gamma V$ is the entrepreneur's expected share of the venture value. Similarly, MF is the value of the entrepreneur's motivational factors. The assumption that financial markets are competitive implies that the EA share of the venture value at $t = 1$ equals his investment I_1. Consequently, γ is the solution to

$$(1 - \gamma)P \times V = I_1. \quad (12)$$

Solving for γ yields

$$\gamma = \frac{P \times V - I_1}{P \times V}. \quad (13)$$

Substituting Equation (13) into Equation (11) yields that the entrepreneur expected utility at $t = 1$ is

$$U_{EA}^1 = \alpha(P \times V - I_1) + MF. \quad (14)$$

Equation (14) states that at $t = 1$, the entrepreneur obtains his share of the expected venture value net of the required investment I_1, plus the value of his motivational factors.

The pre-seed EA investor obtains its share of the expected venture value net of the required investment of $t = 1$,[11]

$$\pi_{EA}^1 = (1 - \alpha)(P \times V - I_1). \quad (15)$$

The *EA* breaks even at $t = 0$ if his expected payoff equals its investment,

$$\int_{P^c}^1 \pi_{EA}^1 \, dF(P) = (1 - \alpha) \int_{P^c}^1 (P \times V - I_1) \, dF(P) = I_0. \quad (16)$$

[11] Note that since the EA does not know whether the venture will succeed, there are two possible inefficiencies at $t = 1$. First, when $P < P^c$, the venture is not financed, resulting in value loss of the exit value V when the venture would have succeeded. Second, when $P \geq P^c$, the venture is financed, leading to a loss of I_1 when the venture will fail.

The LHS of Equation (16) represents the expected payoff to the pre-seed EA investor at $t = 0$ where expectation is taken over the success probability P. For any $P < P^c$ the venture is liquidated and the payoff is zero. Solving for α yields

$$\alpha = \frac{\int_{Pc}^{1}(P\times V - I_1)dF(P) - I_0}{\int_{Pc}^{1}(P\times V - I_1)dF(P)}. \quad (17)$$

It follows from Equation (17) that the venture is fundable at the pre-seed round if

$$\int_{Pc}^{1}(P \times V - I_1)\, dF(P) - I_0 \geq 0. \quad (18)$$

Substituting α from Equation (17) into Equation (14) and taking expectation with respect to P, implies that the entrepreneur expected utility at $t = 0$, provided that the venture is fundable at $t = 0$, is

$$U_{EA}^{0} = \int_{Pc}^{1}(P \times V - I_1)\, dF(P) - I_0 + MF(1 - F(P^c)). \quad (19)$$

Equation (19) implies that the entrepreneur's expected utility consists of the venture expected NPV at $t = 0$ plus the expected value of the entrepreneur's motivational factors.

5. General Angel Financing

We now consider the case where only GA financing is available for the two investments, I_0 and I_1. Recall that a GA does not learn any new information beyond what he knows at $t = 0$. Therefore, at $t = 0$ a GA has to decide whether to fund the entire investment $I = I_0 + I_1$ or refuse funding it. A GA funds the venture at the pre-seed round if the expected venture value at $t = 0$, $\int_0^1 P \times V\, dF(P) = \bar{P} \times V$ is higher than the total investment:

$$\overline{\mathrm{P}} \times \mathrm{V} \geq I_0 + I_1 \quad (20)$$

The resulting expected utility at $t = 0$ for the entrepreneur is

$$U_{GA}^{0} = \overline{\mathrm{P}} \times \mathrm{V} - I_0 - I_1 + MF. \quad (21)$$

Equation (21) implies that the entrepreneur obtains the expected NPV of the venture plus his motivational factors. In this scenario, the entrepreneur obtains his motivational factors for sure once he secures financing.

6. The Entrepreneur's Choice of Investor: Venture Capital, Expert Angel, General Angel

So far we considered VC financing, EA financing, or GA financing in isolation. We now allow the entrepreneur to choose the type of investor at each period. The entrepreneur will choose each period the investor that provides him the highest expected utility. We first consider the case where the entrepreneur chooses VC financing at $t = 0$. Since the pre-seed VC becomes an insider of the firm, he learns at $t = 1$ whether the venture will succeed. This fact is common knowledge, implying that if the entrepreneur approaches an angel investor at $t = 1$, the angel will infer that the *VC* knows that the venture will fail and will not provide financing. Therefore, the only remaining alternative for the entrepreneur is to seek VC financing at $t = 1$. Consequently, the analysis of this case is identical to that presented in Section 3 above. The entrepreneur obtains VC financing at $t = 1$ only when the venture will succeed, resulting in expected utility of U_{vc}^{0} (see Equation (10)) to the entrepreneur at $t = 0$.

We now consider the case where the entrepreneur chooses EA financing at $t = 0$. The analysis of this case is different from that of Section 4 above, as now the entrepreneur can also seek GA or VC financing at $t = 1$. GA financing is irrelevant at $t = 1$, as a GA will not finance a venture that a better informed EA refuses to finance. Thus, we only need to consider the entrepreneur's choice between VC and EA financing at $t = 1$. If the entrepreneur chooses VC financing at $t = 1$, his expected utility is given by U_{vc}^{1} of Equation (5)[12]. In contrast, the entrepreneur's expected utility from EA financing at $t = 1$ is given by U_{EA}^{1} of Equation (14),

[12] Note that the term of financing α is determined according to EA finanicng

provided that $P \geq P^c$. Otherwise, for $P < P^c$ the EA does not provide financing and the entrepreneur's only option is to attempt a VC financing. In this case, his expected utility is given by Equation (5).

Finally, the solution of the case where the entrepreneur chooses GA financing at $t = 0$ is equivalent to the analysis of Section 5, as the GA provides all funds necessary for periods $t = 0$ and $t = 1$, implying that the entrepreneur does not need to seek financing at $t = 1$[13].

Lemma 1 characterizes the entrepreneur's choice at $t = 1$:

Lemma 1

1. Suppose the entrepreneur obtains VC financing at $t = 0$. Then, at $t = 1$ the entrepreneur seeks VC financing and obtains it iff the VC learns that the venture will be successful.
2. Suppose the entrepreneur obtains EA financing at $t = 0$. Then, at $t = 1$, if $P \geq P^c$ and $MF - \alpha I_1 \geq 0$, the entrepreneur obtains EA financing. Otherwise, if $P < P^c$ or $MF - \alpha I_1 < 0$ the entrepreneur seeks VC financing and obtains it iff the VC learns that the venture will be successful.

Lemma 1.1 states that an entrepreneur that start with VC financing will continue with VC financing at $t = 1$ as explained above. Lemma 1.2 states that an entrepreneur that starts with EA financing will continue with EA financing under two conditions: First, the EA is willing to finance the venture ($P \geq P^c$). Second, the entrepreneur prefers EA financing over VC financing. The benefit for the entrepreneur from VC financing is from exercising the option to abandon at $t = 1$ when the VC learns that the venture will fail, saving I_1, out of which the entrepreneur's part is αI_1. In contrast, EA financing enables the entrepreneur to realize the motivational factors MF.

[13] It is also possible that a GA will provide financing at t=0 for only the design stage, and the entrepreneur will ask for VC financing at t=1. This possibility is similar to only VC financing, and thus, it is ignored.

When the motivational factors are larger than the value to abandon, $MF - \alpha I_1 \geq 0$, the entrepreneur prefers angel financing.

We now consider the financing game at $t = 0$. There are two cases to consider, according to the entrepreneur's preferences at $t = 1$:

Case 1: $MF - \alpha I_1 \geq 0$. In this case, the entrepreneur prefers EA financing. When the EA is willing to finance the venture, $P \geq P^c$, the entrepreneur obtains financing from an EA. Otherwise, if $P < P^c$ the entrepreneur is forced to seek VC financing. Consequently, the entrepreneur's expected utility at $t = 0$ from EA financing is given by

$$U_{EA}^0 = \int_0^{P^c} P(V - I_1 + MF)\, dF(P) + \int_{P^c}^1 (P \times V - I_1 + MF)\, dF(P) - I_0. \quad (22)$$

The first integral in Equation (22) is the expected utility to the entrepreneur from VC financing at $t = 1$. The second integral represents his expected utility from EA financing at $t = 1$.

Note that the venture is fundable by an EA at $t = 0$ if

$$\int_0^{P^c} P(V - I_1)\, dF(P) + \int_{P^c}^1 (P \times V - I_1)\, dF(P) - I_0 \geq 0 \quad (23)$$

Equation (23) represents the NPV of the venture under EA financing. The first integral is the expected value under VC financing at $t = 1$, while the second integral represents the expected value under EA financing.

Case 2: $MF - \alpha I_1 < 0$. In this case, the entrepreneur prefers VC financing at $t = 1$. The analysis of this case is equivalent to that of Section 3 where only VC financing is available. The entrepreneur's expected utility is given by Equation (10) and the venture is fundable if Inequality (9) holds.

The Following proposition characterizes the equilibrium of the game:

Proposition 1

1. Suppose Inequality (20) holds. Then,
    a. If $MF > I_1$, the entrepreneur obtains $I_0 + I_1$ from a GA at $t = 0$.
    b. If $I_1 > MF > I_1 - V + \frac{I_0}{\bar{P}}$, the entrepreneur obtains VC financing at $t = 0$. At $t = 1$ the entrepreneur seeks financing from a VC and obtains it if the VC learns that the venture will succeed.
    c. $I_1 - V + \frac{I_0}{\bar{P}} > MF$, the entrepreneur does not initiate the venture
2. Suppose Inequality (20) does not hold but inequality (23) holds. Then,
    a. If $MF > I_1$, the entrepreneur obtains EA financing at $t = 0$. At $t = 1$ the entrepreneur obtains EA financing if $P \geq P^c$, and VC financing if $P < P^c$ and the VC learns that the venture will succeed.
    b. If $I_1 > MF > I_1 - V + \frac{I_0}{\bar{P}}$, the entrepreneur obtains VC financing at $t = 0$. At $t = 1$ the entrepreneur seeks financing from a VC and obtains it if the VC learns that the venture will succeed.
    c. $I_1 - V + \frac{I_0}{\bar{P}} > MF$, the entrepreneur does not initiate the venture.
3. Suppose Inequality (23) does not hold but Inequality (9) holds. Then,
    a. If $MF \geq I_1 - V + \frac{I_0}{\bar{P}}$, the entrepreneur obtains VC financing at $t = 0$. At $t = 1$ the entrepreneur seeks financing from the VC and obtains it if the VC learns that the venture will succeed.
    b. $I_1 - V + \frac{I_0}{\bar{P}} > MF$, the entrepreneur does not initiate the venture.
4. Suppose Inequality (9) is reversed. Then, the venture is unfundable.

Proof – See Appendix.

In Proposition 1.1 all types of investors are willing to finance the venture at $t = 0$ and the entrepreneur selects at $t = 0$ the type of investor that maximizes his expected utility. The benefit for the entrepreneur from VC financing at $t = 0$ is the value of the option to abandon the venture at $t = 1$. This option saves the investment I_1 when the VC learns that the venture will fail. The cost to the entrepreneur from the option to abandon is the loss of the motivational factors MF when the VC learns that the venture will fail. Therefore, when the motivational factors MF exceed the investment I_1, the entrepreneur prefers angel financing. Since GA financing enables the entrepreneur to capture his motivational factors with certainty, and EA financing has a positive probability of losing the motivational factors, the entrepreneur selects GA over EA financing at $t = 0$. In contrast, when the investment I_1 exceeds the motivational factors MF, the entrepreneur opts for VC financing at $t = 0$, as long as his expected utility is positive. This happens when $P(MF + V - I_1) - I_0 \geq 0$ or equivalently when $MF \geq I_1 - V + \frac{I_0}{\bar{P}}$. Otherwise, when $MF < I_1 - V + \frac{I_0}{\bar{P}}$ the entrepreneur does not initiate the venture[14].

The difference between Proposition 1.1 and Proposition 1.2 is that under the conditions of Proposition 1.2, a GA investor is unwilling to fund the venture. In this case, when the motivational factors MF exceeds the investment I_1, the entrepreneur opts for EA financing at $t = 0$.

Under the conditions of Proposition 1.3 only VC investors are willing to finance the venture at $t = 0$, and under the conditions of Proposition 1.4 no investor is willing to finance the venture at $t = 0$.

[14] Note that when $MF \geq 0$ the entrepreneur always initiate the venture, provided that he can secure financing, which happens under the conditions of Proposition 1.1 – 1.3. Only when MF is negative enough he does not enter the entrepreneurship game.

Having characterized the equilibrium of the game, we now derive the empirical implications of the model.

Our theory connects ventures financing with ventures' economic value and entrepreneurs' motivational factors. In this environment, Proposition 1 has immediate implications for the structure of entrepreneurial finance markets. First, it shows that the "weak" investors - the investors with inferior information - will not only survive in the market, but will also attract some of the best ventures. This is because their "weakness" in terms of economic value is their "strength" in terms motivational factors, as it enables entrepreneurs to capture their motivational factors with a higher certainty. Perhaps paradoxically, this strength is due to the angels' weakness in learning the information, and not due to exogenous advantages they may have.

It follows from Proposition 1 that the the market for entrepreneurial finance structures itself to cater to the needs of heterogeneous entrepreneurs so that entrepreneurs with a relatively high utility from economic value will choose VC financing, whereas entrepreneurs with relatively high motivational factors will choose angel financing whenever possible.

It is conceivable that the idea that the needs of different entrepreneurs are catered by different investor types may be implemented instead by financial contracts. Obviously a "weak" investor cannot mimic the contract of a "strong" investor, as he does not have the required information. Therefore, the question is whether a "strong" investor (a VC) can offer contracts that mimic the weak investor contracts. The answer is negative for the case of an EA investor. The VC learns the final exit value and has no idea what is the success probability that an EA might have learnt. Therefore, although the VC is better informed, he cannot replicate the EA contract. In addition, there may be a commitment issue, as it may be impossible for a VC to commit to not using information it has already learnt. In this respect, the only way to implement

financial contracts in this market is via the market structure we identify here, that includes investors with different information, each providing the specific contract it is best at.

7. Empirical implications

The analysis of the equilibrium described in Proposition 1 reveals several empirical implications that are summarized in Propositions 2 and 3 below.

Proposition 2

There are three possible sequences of financing a venture

1. Starting with an EA financing at $t = 0$ and either staying with EA financing or switching to VC financing at $t = 1$.
2. Starting with VC financing at $t = 0$ and continuing with VC financing at $t = 1$.
3. Financing the entire investment I at $t = 0$ by a GA investor.

Proposition 2 states that, whenever both VC and angel investors finance early stage ventures, the angel investor finances the pre-seed investment and the VC finances the follow-up seed investment. This sequence of financing is consistent with the common wisdom that angel investors tend to invest before VC investors because, for example, angel investors don't have enough resources to support both stages or that the pre-seed investment is "too small" for VC funds. In contrast, we obtain this result as an equilibrium outcome of the model where we don't put any constraint on the resources available to angel investors and we don't have "too small" investments for VC investors.

Proposition 3 describes the implication of our theory to the likelihood that a venture will be shut down at the seed round, depending on the identity of the pre-seed round financier.

Proposition 3

Ventures that are backed by GA investors at $t = 0$ are less likely to be shut down at $t = 1$ than EA backed ventures, who in turn are less likely to be shut down than VC backed ventures.

GA backed ventures are never shut down in our model at $t = 1$ because GA investors provide the entire investment at$t = 0$. In the other extreme, VC backed ventures are shut down whenever the VC learns that the venture will fail. EA backed ventures may be financed at $t = 1$ even when the venture will ultimately fail whenever $P \geq P^c$.

We now consider the relation between new venture financing and the required level of investment. We first consider a cross section distribution of the initial investment I_0 while keeping other characteristics unchanged. Proposition 4 describes the empirical implications for different values of the pre-seed investment.

Proposition 4

Suppose the only source of variation among ventures is the pre-seed round investment, I_0. Then:

1. GA investors finance ventures with a lower average initial investment I_0 and a lower average total investment I compared to EA and VC investors.
2. Angel investors, defined by the union of GA and EA investors, finance ventures with a lower average initial investment I_0 and a lower average total investment I compared to VC investors.

We now consider a cross section distribution of the seed round investment I_1 while keeping the other monetary characteristics unchanged. Proposition 5 describes the empirical implications for this case:

Proposition 5

Suppose the only source of variation among ventures is the seed round investment I_1 at $t = 1$. Then:

1. GA investors finance ventures with a lower average initial investment I_1 and a lower average total investment I compared to EA and VC investors.
2. Angel investors, defined by the union of GA and EA investors, finance ventures with a lower average initial investment I_1 and a lower average total investment I compared to VC investors.

We now turn to the relations between new ventures financing and the exit value by considering a cross section distribution of the exit value V while keeping the other monetary characteristics unchanged. Proposition 6 describes the empirical implications for this case:

Proposition 6

Suppose the only source of variation among ventures is the exit value V. Then:

1. GA investors finance ventures with a higher average exit value V compared to EA and VC investors.
2. Angel investors, defined by the union of GA and EA investors, finance ventures with a higher average exit value V compared to VC investors.

Propositions 4-6 are the result of a simple feature of our model, namely, that the least informed investor demands a higher stake in the firm (that is, he price protects itself against inferior information). Therefore, the combined results of these propositions are as follows:

1. Consider two industries, 1 and 2, where industry 1 is more profitable than industry 2 (either because it requires less investment or has a higher exit value). Then, industry 1 will have a higher fraction of Angel financing.

2. Consider two periods, 1 and 2, where period 1 is "better" in terms of value. Then, period 1 will have higher fraction of Angel financing.
3. Consider two geographic locations, 1 and 2, where location 1 has better terms for ventures. Then location 1 will have a higher fraction of Angel financing.

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Appendix – Proofs

Proof of Lemma 1

1. Suppose the entrepreneur obtains VC financing at $t = 0$. Then, from the discussion preceding Lemma 1, an angel investor will not finance the venture at $t = 1$. Consequently, the entrepreneur only option is to seek VC financing, which he obtains if the VC learns that the venture will succeed.
2. Suppose the entrepreneur obtains EA financing at $t = 0$ and $P \geq P^c$. Then, by the definition of P^c, there is $\gamma \in [0,1]$ such that Equation (12) holds, and the EA is willing to finance the required investment I_1 at $t = 1$. Therefore, at this juncture, the entrepreneur has to decide between EA and VC financing. Comparing U_{vc}^1 of Equation (5) to U_{EA}^1 of Equation (14), reveals that the entrepreneur prefers EA financing if $MF - \alpha I_1 \geq 0$. When $MF - \alpha I_1 < 0$, the entrepreneur prefers VC financing. Since the VC learns if the venture will succeed before he finances the venture, the entrepreneur is only willing to provide funding when he learns that the venture will succeed.

   Suppose now that $P < P^c$. Then, by the definition of P^c, the EA does not finance the venture. The entrepreneur's only option is to seek VC financing, which he obtains if the entrepreneur learns that the venture will succeed.

Q.E.D.

Proof of Proposition 1

1. Suppose Inequality (20) holds. It follows that Inequalities (23) and (9) also hold. Thus, all investors are willing to finance the venture. The entrepreneur selects the investor that maximizes his utility at $t = 0$. Consequently, it remains to compare U_{vc}^0 of Equation (10) to U_{GA}^0 of Equation (21) and U_{EA}^0 of Equation (22).

A comparison of U_{GA}^0 to U_{EA}^0 reveals that $U_{GA}^0 > U_{EA}^0$ if

$$\overline{\mathrm{P}}\mathrm{V} - I_0 - I_1 + MF$$

$$> \int_0^{P^c} P(V - I_1 + MF)\, dF(P) + \int_{P^c}^1 (P \times V - I_1 + MF)\, dF(P) - I_0$$

Rearranging terms yields that the entrepreneur prefers GA financing at the pre-seed stage, $t = 0$, if $MF > I_1$ and EA financing if $I_1 > MF$.

A comparison of U_{EA}^0 to U_{vc}^0 reveals that $U_{GA}^0 > U_{vc}^0$ if

$$\int_0^{P^c} P(V - I_1 + MF)\, dF(P) + \int_{P^c}^1 (P \times V - I_1 + MF)\, dF(P) - I_0$$

$$> \bar{P} \times (V - I_1) - I_0 + \bar{P} \times MF$$

Rearranging terms yields that the entrepreneur prefers EA financing at the seed stage, $t = 0$, if $MF > I_1$ and VC financing if $I_1 > MF$.

The two comparisons above yield that

$$U_{GA}^0 > U_{EA}^0 > U_{vc}^0 \qquad \text{if } MF > I_1 \qquad \text{(A-1)}$$

This proves part (1.a) of the proposition.

Likewise, the two comparisons yield that

$$U_{vc}^0 > U_{EA}^0 > U_{GA}^0 \qquad \text{if } I_1 > MF \qquad \text{(A-2)}$$

There are two possibilities in this case.

First, $MF > I_1 - V + \frac{I_0}{\bar{P}}$. In this case the entrepreneur's utility is positive and he obtains VC financing. This proves the first statement of part (1.b) of the proposition. The second statement of part (1.b) of the proposition is proven in Lemma 1.1.

Second, $I_1 - V + \frac{I_0}{\bar{P}} > MF$. In this case the entrepreneur's utility is negative and he does not seek financing.

2. Suppose Inequality (20) does not hold and Inequality (21) holds. The fact that Inequality (21) holds implies that Inequality (9) also holds. Thus, only EA and VC investors are willing to finance the venture. The entrepreneur will select the investor that will maximize his utility at $t = 0$. Consequently, as in part 1 of the proposition, it remains to compare U_{vc}^{0} to U_{EA}^{0}.

   Equation (A-1) states that $U_{EA}^{0} > U_{vc}^{0}$ when $MF > I_1$. This proves the first statement of part (2.a) of the proposition. The second statement of part (2.a) of the proposition is proven in Lemma 1.2.

   Equation (A-2) states that $U_{vc}^{0} > U_{EA}^{0}$ when $I_1 > MF$. As before, there are two cases to consider.

   First, $MF > I_1 - V + \frac{I_0}{\bar{P}}$. In this case the entrepreneur's utility is positive and he obtains VC financing. This proves the first statement of part (2.b) of the proposition. The second statement of part (2.b) of the proposition is proven in Lemma 1.1.

   Second, $I_1 - V + \frac{I_0}{\bar{P}} > MF$. In this case the entrepreneur's utility is negative and he does not seek financing.

   Suppose Inequality (22) does not hold and Inequality (9) holds. Then, at $t = 0$ the venture is only fundable by a VC. As before, there are two cases to consider.

   First, $MF > I_1 - V + \frac{I_0}{\bar{P}}$. In this case the entrepreneur obtains VC financing at $t = 0$. Lemma 1.1 proves the second part of the statement.

   Second, $I_1 - V + \frac{I_0}{\bar{P}} > MF$. In this case the entrepreneur's utility is negative and he does not seek financing.

3. Suppose Inequality (9) does not hold. Then, the venture is not fundable.

Q.E.D.

Proof of Proposition 2

The Proposition follows directly from Proposition 1.

Q.E.D.

Proof of Proposition 3

The GA does not get any new information and, thus, he never shuts down a venture. The VC shuts down whenever he learns that the venture will fail. This happens with probability $1 - \overline{\mathrm{P}}$. An EA backed venture is shut down whenever a VC backed venture is shut down, except for the case where $P \geq P^c$, in which case the EA finances the venture even when it will fail. Thus, the probability that an EA investor shuts down a venture is lower than that of a VC.

Q.E.D.

Proof of Proposition 4

Let Z_{I_0} be the cumulative distribution function of I_0 with a positive support over [0, ∞]. From Equation (20), GA financing at $t = 0$ is feasible for any $I_0 \leq I_0^*$, where

$$I_0^* \equiv \overline{\mathrm{P}} \times \mathrm{V} - I_1 \tag{A-3}$$

Rearranging Equation (23) yields that EA financing is feasible for any $I_0 \leq I_0^{**}$, where

$$I_0^{**} \equiv \overline{\mathrm{P}}(\mathrm{V} - I_1) - \int_{P^c}^{1} \big((1 - P) I_1\big)\, dF(P) \tag{A-4}$$

From Equation (9), VC financing is feasible for any $I_0 \leq I_0^{***}$, where

$$I_0^{***} \equiv \overline{\mathrm{P}}(\mathrm{V} - I_1) \tag{A-5}$$

Equations (A-3) through (A-5) imply that $I_0^{***} > I_0^{**} > I_0^*$.

Consider all I_0 satisfying $I_0 \leq I_0^*$. In this case the venture is fundable by GA, EA and VC investors. Inequality (A-1) implies that $U_{GA}^0 > U_{EA}^0$ if $MF - I_1 > 0$ and inequality (A-2) implies that $U_{vc}^0 > U_{EA}^0$ if $MF - I_1 < 0$. Therefore, the entrepreneur will seek financing

from either a VC or a GA. Since the distribution function Q over MF is independent of the distribution function Z_{I_0} over I_0, it follows that the probability of VC investment in this region is $Q(I_1)$ and of GA investment is $1 - Q(I_1)$.

Consider all I_0 satisfying $I_0^{**} \geq I_0 > I_0^*$. In this case the venture is only fundable by EA and VC investors. Proposition 1.2 implies that the entrepreneur obtains investment from EA if $MF - I_1 > 0$ and from VC if $MF - I_1 < 0$. Again, the independence of the distribution function Q over MF and the distribution function Z_{I_0} over I_0 implies that the probability of VC investment in this region is $Q(I_1)$ and of EA investment is $1 - Q(I_1)$.

Finally, for all I_0 satisfying $I_0^{***} \geq I_0 > I_0^{**}$ the venture is only fundable by a VC, so the entrepreneur obtains investment from a VC for sure.

Since GA financing is possible for all I_0 satisfying $I_0^* \geq I_0$, the average investment for GA is given by

$$\bar{I}_{GA} = \frac{\int_0^{I_0^*} I_0 dZ_{I_0}(I_0)}{Z_{I_0}(I_0^*)} \tag{A-6}$$

Since EA financing is possible for all I_0 satisfying $I_0^{**} \geq I_0 > I_0^*$, the average investment for EA is given by

$$\bar{I}_{EA} = \frac{\int_{I_0^*}^{I_0^{**}} I_0 dZ_{I_0}(I_0)}{Z_{I_0}(I_0^{**}) - Z_{I_0}(I_0^*)} \tag{A-7}$$

Since VC financing is possible for all I_0 satisfying for $I_0^{***} \geq I_0$, the average investment for VC is given by

$$\bar{I}_{VC} = \frac{\int_0^{I_0^{**}} I_0 dZ_{I_0}(I_0) \times Q(I_1) + \int_{I_0^{**}}^{I_0^{***}} I_0 dZ_{I_0}(I_0)}{Z_{I_0}(I_0^{**}) \times Q(I_1) + (Z_{I_0}(I_0^{***}) - Z_{I_0}(I_0^{**}))} \tag{A-8}$$

1. We first prove that $\bar{I}_{EA} > \bar{I}_{GA}$. Given that for GA, $I_0 \in [0,\ I_0^*]$, it follows that $\bar{I}_{GA} < I_0^*$. Similarly, given that for EA, $I_0 \in (I_0^*,\ I_0^{**}]$, it follows that $\bar{I}_{EA} > I_0^*$. Consequently, $\bar{I}_{EA} > I_0^* > \bar{I}_{GA}$.

   Next we show that $\bar{I}_{VC} > \bar{I}_{GA}$. Equation (A-8) can be written as a weighted average, as follows:

$$\bar{I}_{VC} = \omega_1 \times \bar{I}_{GA} + \omega_2 \times \bar{I}_{EA} + (1 - \omega_1 - \omega_2) \frac{\int_{I_0^{**}}^{I_0^{***}} I_0 dZ_{I_0}(I_0)}{(Z_{I_0}(I_0^{***}) - Z_{I_0}(I_0^{**}))} \tag{A-9}$$

   where $\omega_1 = \frac{Z_{I_0}(I_0^*) \times Q(I_1)}{Z_{I_0}(I_0^{**}) \times Q(I_1) + (Z_{I_0}(I_0^{***}) - Z_{I_0}(I_0^{**}))}$

   and $\omega_2 = \frac{(Z_{I_0}(I_0^{**}) - Z_{I_0}(I_0^*)) \times Q(I_1)}{Z_{I_0}(I_0^{**}) \times Q(I_1) + (Z_{I_0}(I_0^{***}) - Z_{I_0}(I_0^{**}))}$.

   Since $\bar{I}_{GA} < I_0^*$, and $\bar{I}_{EA} > I_0^*$ and since $I_0 > I_0^*$ for all $I_0 \in [I_0^{**},\ I_0^{***}]$, it follows that the weighted average on the right-hand-side of Equation (A-9) exceeds its lowest argument, $\bar{I}_{GA}$. This proves the results regarding the initial investment, I_0. The result about total investment I follows since I_1 is the same for all ventures.

2. Since angel financing is possible for all $I_0 \in [0,\ I_0^{**}]$ the average investment for angel investors is

$$\bar{I}_A = \frac{\int_0^{I_0^{**}} I_0 dZ_{I_0}(I_0)}{Z_{I_0}(I_0^{**})}. \tag{A-10}$$

   As before, Equation (A-8) can be written as a weighted average, as follows:

$$\bar{I}_{VC} = \omega_3 \times \bar{I}_A + (1 - \omega_3) \frac{\int_{I_0^{**}}^{I_0^{***}} I_0 dZ_{I_0}(I_0)}{(Z_{I_0}(I_0^{***}) - Z_{I_0}(I_0^{**}))} \tag{A-11}$$

   where $\omega_3 = \frac{Z_{I_0}(I_0^{**}) \times Q(I_1)}{Z_{I_0}(I_0^{**}) \times Q(I_1) + (Z_{I_0}(I_0^{***}) - Z_{I_0}(I_0^{**}))}$.

Again, the weighted average on the right-hand-side of Equation (A-11) exceeds its lowest argument, $\bar{I}_A$. This proves the results regarding the initial investment, I_0. The result about total investment Ifollows since I_1 is the same for all ventures.

Q.E.D.

Proof of Proposition 5

Let Z_{I_1}be the cumulative distribution function of I_1 with a positive support over [0, ∞].

From Equation (20), GA financing at $t = 0$ is feasible for any $I_1 \leq I_1^*$, where

$$I_1^* \equiv \bar{\mathrm{P}} \times \mathrm{V} - I_0 \tag{A-12}$$

Rearranging Equation (23) yields that EA financing is feasible for any $I_1 \leq I_1^{**}$, where

$$I_1^{**} \equiv \frac{\bar{\mathrm{P}} \times V - I_0}{\bar{\mathrm{P}} + \int_{P^C}^{1} (1-P) dF(P)} \tag{A-13}$$

From Equation (9), VC financing is feasible for any $I_1 \leq I_1^{***}$, where

$$I_1^{***} \equiv \frac{\bar{\mathrm{P}} \times V - I_0}{\bar{\mathrm{P}}} \tag{A-14}$$

Equations (A-12) through (A-14) imply that $I_1^{***} > I_1^{**} > I_1^*$.

Consider all I_1 satisfying $I_1 \leq I_1^*$. In this case the venture is fundable by GA, EA and VC investors. Proposition 1.1 implies that the entrepreneur obtains GA financing if $MF - I_1 \geq 0$ and that he obtains VC financing if $MF - I_1 < 0$.

Consider all I_1 satisfying $I_1^{**} \geq I_1 > I_1^*$. In this case the venture is only fundable by EA and VC investors. Proposition 1.2 implies that the entrepreneur obtains EA financing if $MF - I_1 \geq 0$ and VC financing if $MF - I_1 < 0$.

Finally, for all I_1 satisfying $I_1^{***} \geq I_1 > I_1^{**}$ the venture is only fundable by a VC, so the entrepreneur obtains investment from a VC for sure.

We denote by $\bar{I}_{1GA}, \bar{I}_{1EA}, \bar{I}_{1A}$ and $\bar{I}_{1VC}$ the average expected seed round investment of a GA, EA, all angels and VC at the pre-seed round.

1. We first prove that $\bar{I}_{1EA} > \bar{I}_{1GA}$. Given that for GA, $I_1 \in [0, I_1^*]$, it follows that $I_1^* > \bar{I}_{1GA}$. Similarly, given that for EA, $I_1 \in (I_1^*, I_1^{**}]$, it follows that $\bar{I}_{1EA} > I_1^*$. Consequently, $\bar{I}_{1EA} > I_1^* > \bar{I}_{1GA}$.

   Next we show that $\bar{I}_{1VC} > \bar{I}_{1GA}$. For all $I_1 \in [0, I_1^*]$ the expected investment of a VC exceeds that of a GA. This is so because as I_1 increases within the range, the likelihood of VC financing increases and the likelihood of GA financing decreases. Since VC also invests in the range $I_1 \in (I_1^*, I_1^{***}]$, it follows that the unconditional average investment of a VC, $\bar{I}_{1VC}$, exceeds $\bar{I}_{1GA}$.

2. Next we show that $\bar{I}_{1VC} > \bar{I}_{1A}$. As in the proof of part 1 of the proposition, the expected investment of a VC exceeds that of angel investors for all $I_1 \in [0, I_1^{**}]$. Since VC also invests in the range $I_1 \in (I_1^{**}, I_1^{***}]$, it follows that the unconditional average investment of a VC, $\bar{I}_{1VC}$, exceeds that of angel investors, $\bar{I}_{1A}$.

Q.E.D.

Proof of Proposition 6

Let Z_V be the cumulative distribution function of V with a positive support over [0, ∞].

Equation (9) implies that VC financing at the seed stage, $t = 0$, is feasible for any $V \geq V^*$, where

$$V^* \equiv \frac{I_0 + \bar{P} I_1}{\bar{P}} \tag{A-15}$$

From Equation (23), EA financing is feasible for any $V \geq V^{**}$, where

$$V^{**} \equiv \frac{I_0 + \bar{P} I_1 + \int_{Pc(V^{**})}^{1} (1-P) I_1 dF(P)}{\bar{P}} \tag{A-16}$$

From Equation (20), GA financing is feasible for any $V \geq V^{***}$, where

$$V^{***} \equiv \frac{I_0+I_1}{\bar{P}} \tag{A-17}$$

Equations (A-15) through (A-17) that $V^{***} > V^{**} > V^{*}$.

Consider all V satisfying $V \geq V^{***}$. In this case the venture is fundable by GA, EA and VC investors. Inequality (A-1) implies that $U_{GA}^0 > U_{EA}^0$ if $MF - I_1 > 0$ and inequality (A-2) implies that $U_{vc}^0 > U_{EA}^0$ if $MF - I_1 < 0$. Therefore, the entrepreneur will seek financing from either a VC or a GA. Since the distribution function Q over MF is independent of the distribution function Z_V over V, it follows that the probability of VC investment in this region is $Q(I_1)$ and of GA investment is $1 - Q(I_1)$.

Consider all V satisfying $V^{***} > V \geq V^{**}$. In this case the venture is only fundable by EA and VC investors. Proposition 1.2 implies that the entrepreneur obtains investment from EA if $MF - I_1 > 0$ and from VC if $MF - I_1 < 0$. Again, the independence of the distribution function Q over MF and the distribution function Z_V over V implies that the probability of VC investment in this region is $Q(I_1)$ and of EA investment is $1 - Q(I_1)$.

Finally, for all V satisfying $V^{**} > V \geq V^{*}$ the venture is only fundable by a VC, so the entrepreneur obtains investment from a VC for sure.

Since GA financing is possible for all V satisfying $V \geq V^{***}$, the average exit value for GA is

$$\overline{V_{GA}} = \frac{\int_{V^{***}}^{\infty} V dZ_V(V)}{1-Z_V(V^{***})}. \tag{A-18}$$

Since EA financing is possible for all V satisfying $V^{***} > V \geq V^{**}$, the average exit value for EA is

$$\overline{V_{EA}} = \frac{\int_{V^{**}}^{V^{***}} V dZ_V(V)}{Z_V(V^{***}) - Z_V(V^{**})}. \tag{A-19}$$

Since VC financing is possible for all V satisfying $V \geq V^*$, the average exit for VC is

$$\overline{V_{VC}} = \frac{\int_{V^{**}}^{\infty} V dZ_V(V) \times Q(I_1) + \int_{V^*}^{V^{**}} V dZ_V(V)}{(1-Z_V(V^{**})) \times Q(I_1) + (Z_V(V^{**}) - Z_V(V^*))} \quad \text{(A-20)}$$

1. We first prove that $\overline{V_{GA}} > \overline{V_{EA}}$. Given that for GA, $V \geq V^{***}$, it follows that $\overline{V_{GA}} > V^{***}$. Similarly, given that for EA, $V \in [V^{**}, V^{***})$, it follows that $V^{***} > \overline{V_{EA}}$. Consequently, $\overline{V_{GA}} > V^{***} > \overline{V_{EA}}$.

   We now show that $\overline{V_{GA}} > \overline{V_{VC}}$. Equation (A-20) can be written as a weighted average, as follows:

$$\overline{V_{VC}} = \omega_1 \times \overline{V_{GA}} + \omega_2 \times \overline{V_{EA}} + (1 - \omega_1 - \omega_2) \frac{\int_{V^*}^{V^{**}} V dZ_V(V)}{(Z_V(V^{**}) - Z_V(V^*))} \quad \text{(A-21)}$$

   where $\omega_1 = \frac{(1-Z_V(V^{***})) \times Q(I_1)}{(1-Z_V(V^{**})) \times Q(I_1) + (Z_V(V^{**}) - Z_V(V^*))}$

   and $\omega_2 = \frac{(Z_V(V^{***}) - Z_V(V^{**})) \times Q(I_1)}{(1-Z_V(V^{**})) \times Q(I_1) + (Z_V(V^{**}) - Z_V(V^*))}$.

   Since $\overline{V_{GA}} > \overline{V_{EA}}$, and $\overline{V_{EA}} > V^{**}$ and since $V^{**} \geq V$ for all $V \in [V^*, V^{**}]$, it follows that the weighted average on the right-hand-side of Equation (A-21) is below its highest argument, $\overline{V_{GA}}$. This proves that $\overline{V_{GA}} > \overline{V_{VC}}$.

2. Since angel financing is possible for all $V \geq V^{**}$ the average exit value for angel investors is

$$\overline{V_A} = \frac{\int_{V^{**}}^{\infty} V dZ_V(V)}{1-Z_V(V^{**})}. \quad \text{(A-22)}$$

   As before, Equation (A-20) can be written as a weighted average, as follows:

$$\overline{V_{VC}} = \omega_3 \times \overline{V_A} + (1 - \omega_3) \frac{\int_{V^*}^{V^{**}} V dZ_V(V)}{(Z_V(V^{**}) - Z_V(V^*))} \quad \text{(A-23)}$$

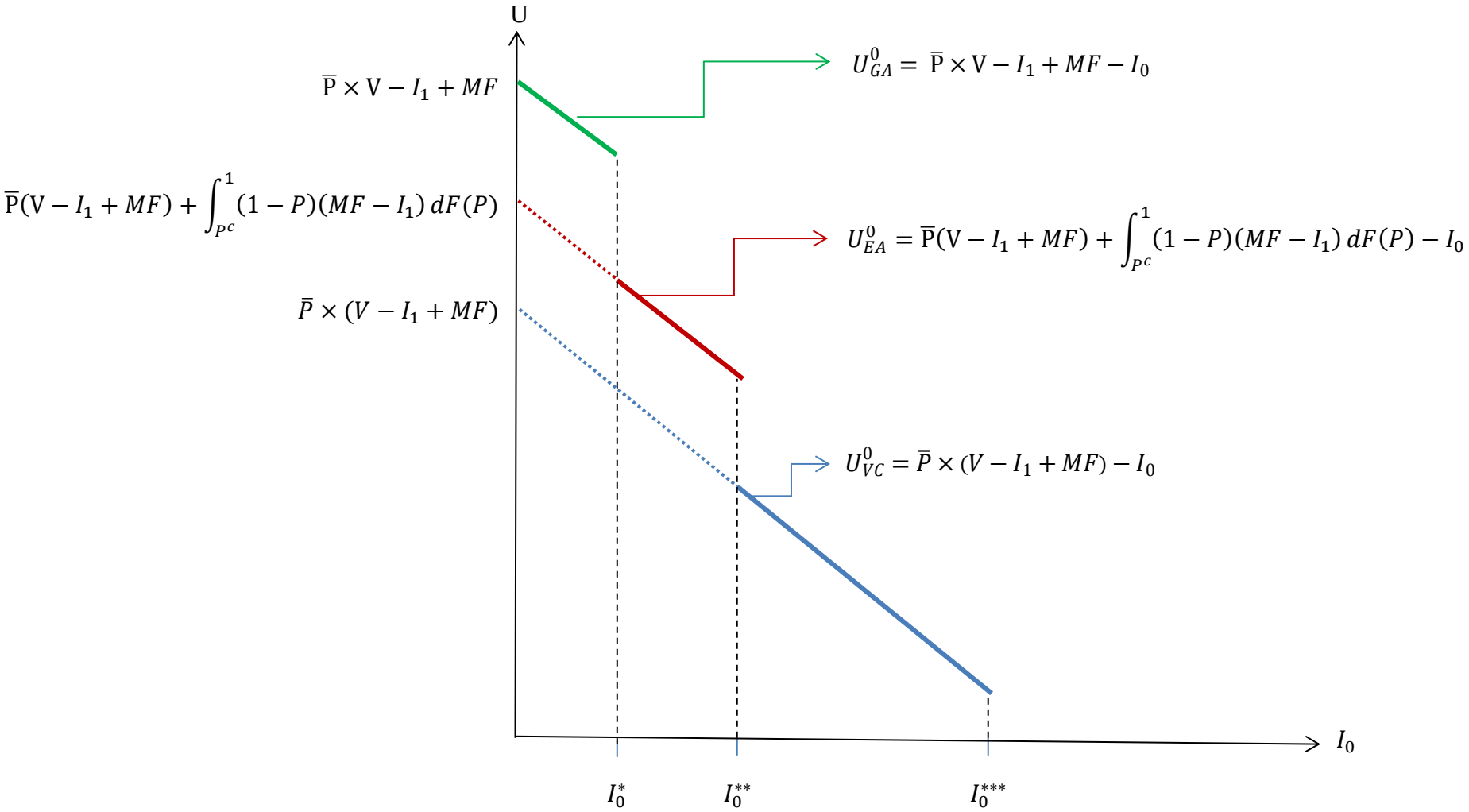
   where $\omega_3 = \frac{(1-Z_V(V^{**})) \times Q(I_1)}{(1-Z_V(V^{**})) \times Q(I_1) + (Z_V(V^{**}) - Z_V(V^*))}$.

Again, the weighted average on the right-hand-side of Equation (A-23) is below its highest argument, $\overline{V_A}$.

Q.E.D.

Figure 1

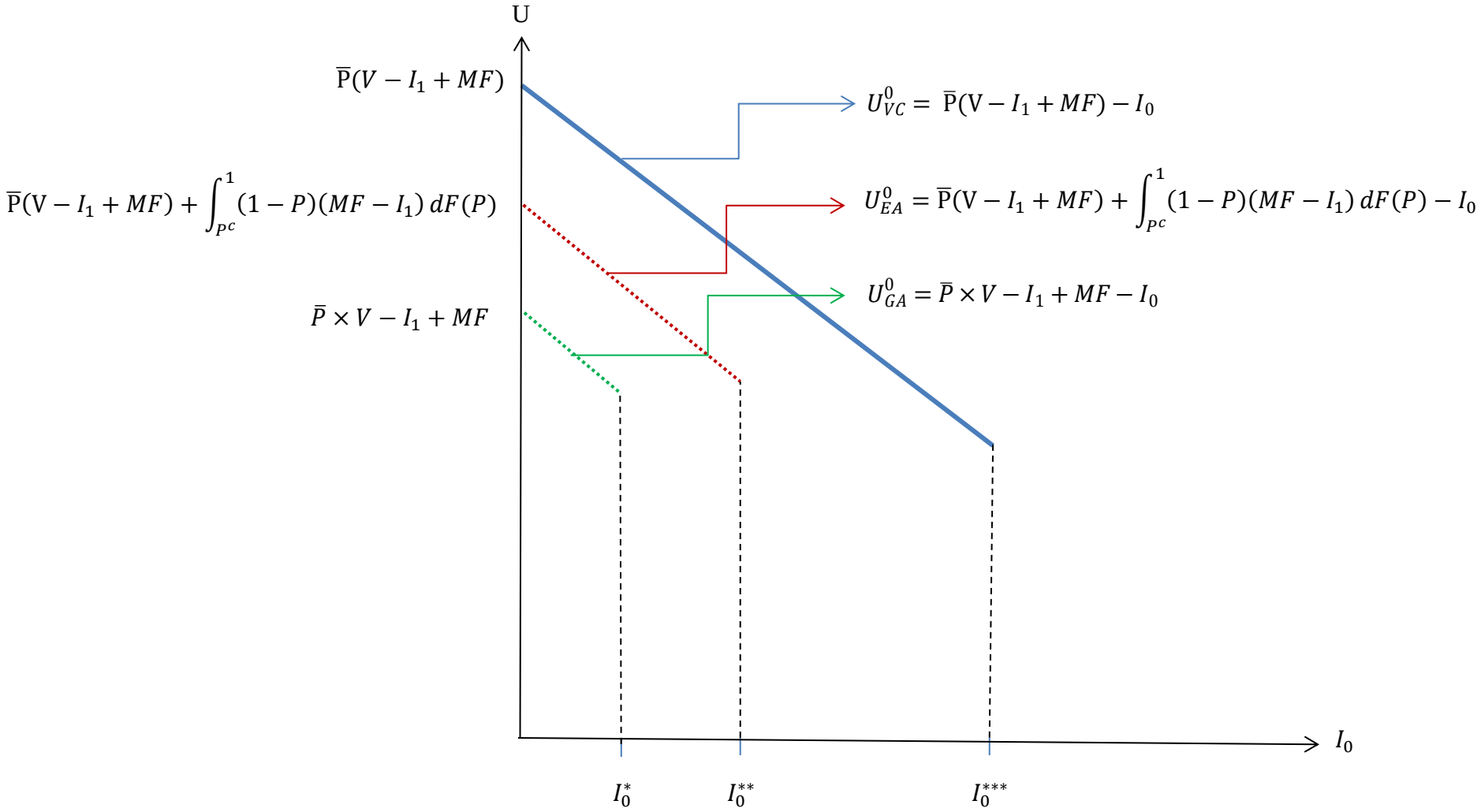
The entrepreneur's financing choice for different initial investment levels I_0 when $MF - I_1 > 0$



I_0^*, I_0^{**}, and I_0^{***} are the highest investment that *GA*, *EA* and *VC* are willing to invest, respectively, and are given by equations (A-3), (A-4), and (A-5) respectively. The solid lines describe the entrepreneur choice of financing. For $I_0 \leq I_0^*$ all investors are willing to finance the venture at t=0. The entrepreneur prefers *GA* financing. For $I_0^{**} \geq I_0 \geq I_0^*$ only *EA* and *VC* are willing to finance the venture. The entrepreneur prefers *EA* financing. For $I_0^{***} \geq I_0 \geq I_0^{**}$ only *VC* is willing to finance and the entrepreneur raises financing from the VC. For $I_0 \geq I_0^{***}$ the venture is not fundable.

Figure 2

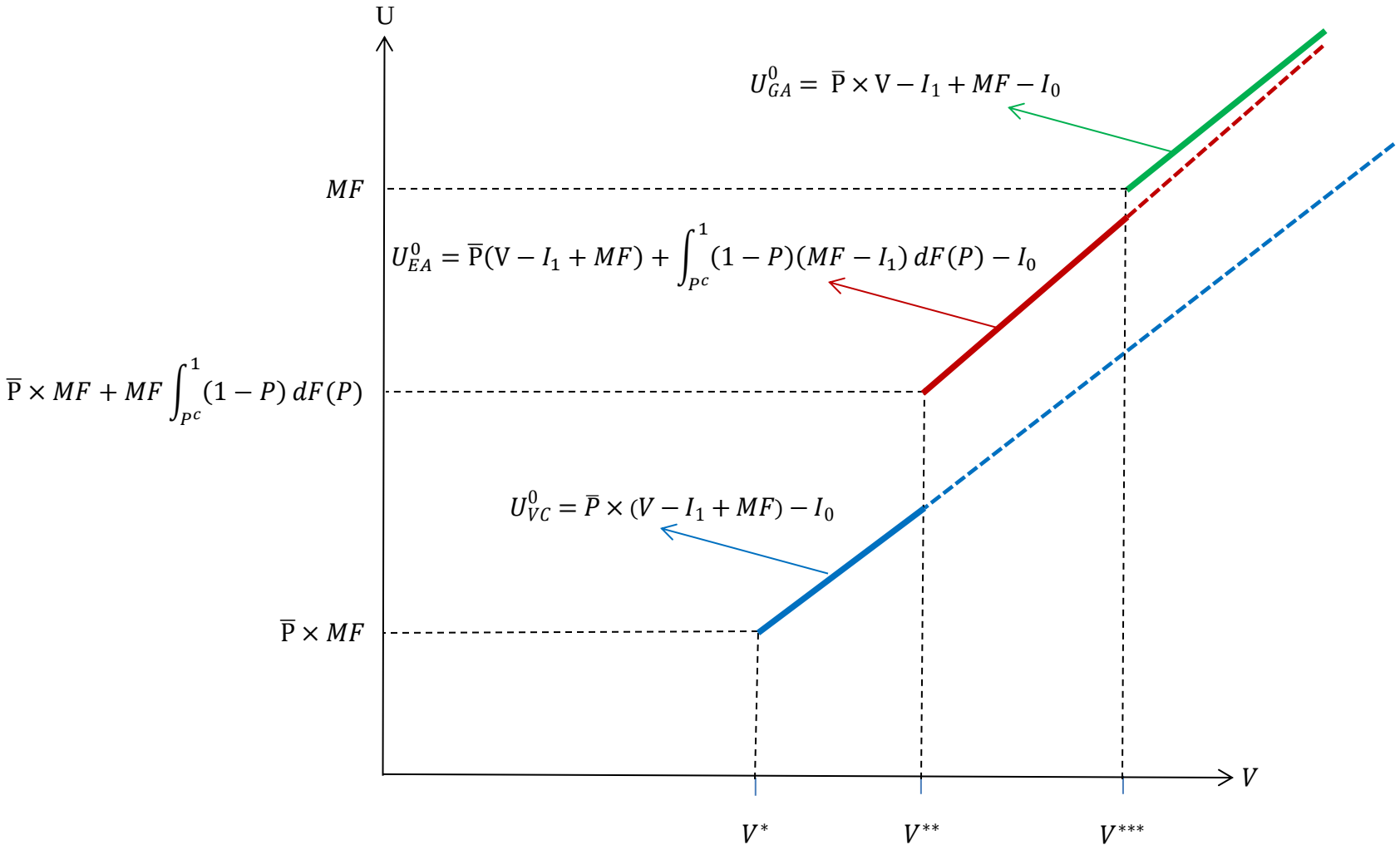
The entrepreneur's financing choice for different initial investment levels I_0 when $MF - I_1 < 0$



I_0^*, I_0^{**}, and I_0^{***} are the highest investment that *GA*, *EA* and *VC* are willing to invest, respectively, and are given by equations (A-3), (A-4), and (A-5) respectively. The solid line describes the entrepreneur choice of financing. For $I_0 \leq I_0^{***}$ the entrepreneur prefers *VC* financing and the V*C* is willing to finance the venture, resulting in *VC* financing. For $I_0 \geq I_0^{***}$ the venture is not fundable.

Figure 3

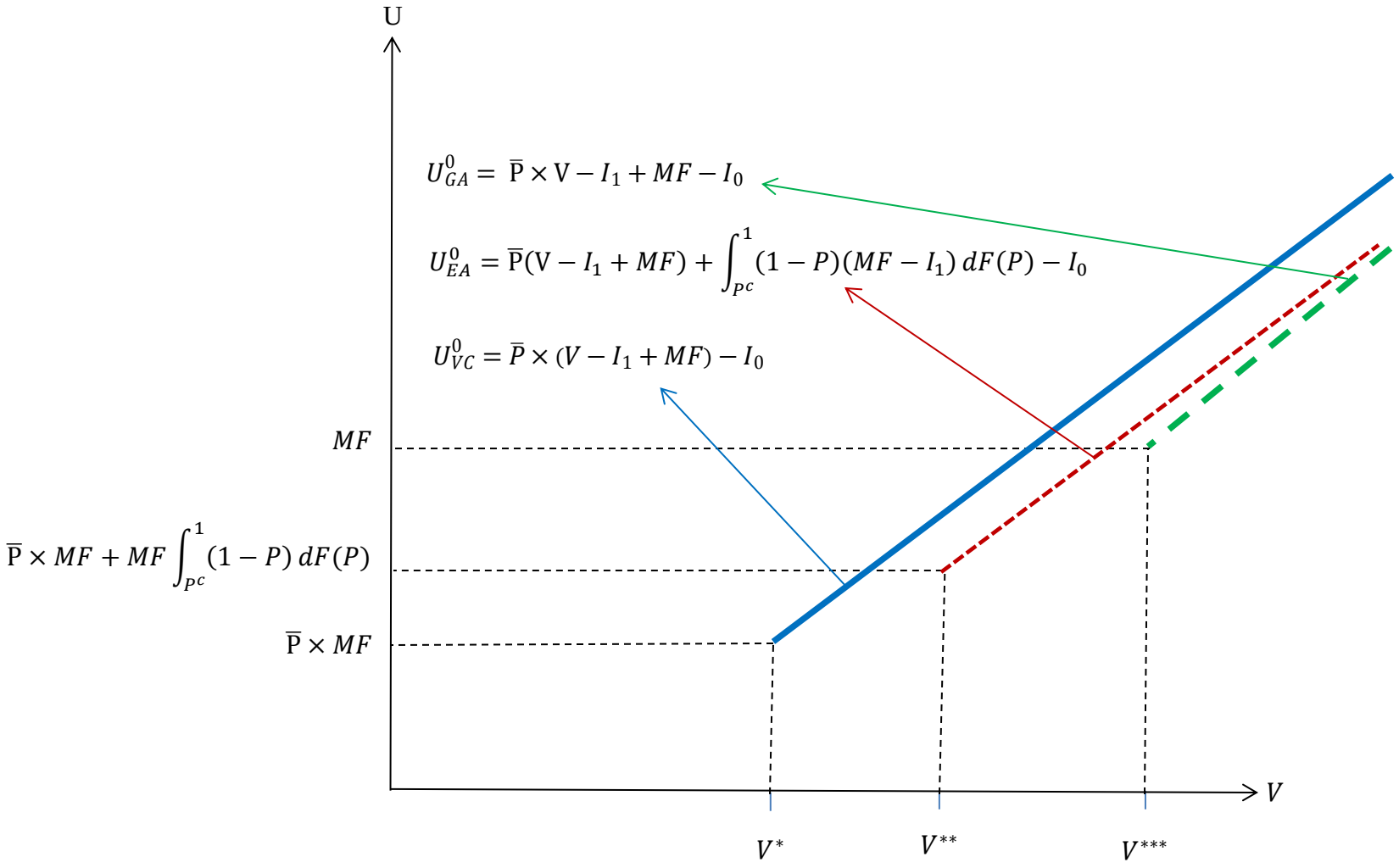
The entrepreneur's financing choice for different exit values V when $MF - I_1 > 0$



V^*, V^{**}, and V^{***} are the lowest exit value required by *VC*, *EA* and *GA* to offer financing for the venture, respectively, and are given by equations (A-13), (A-14), and (A-15) respectively. The solid lines describe the entrepreneur choice of financing. For $V \geq V^{***}$ all investors are willing to finance the venture at $t = 0$. The entrepreneur prefers *GA* financing. For $V^{***} > V \geq V^{**}$ only *EA* and *VC* are willing to finance the venture. The entrepreneur prefers *EA* financing. For $V^{**} > V \geq V^{*}$ only *VC* is willing to finance and the entrepreneur raises financing from the VC. For $V^{*} > V$ the venture is not fundable.

Figure 4

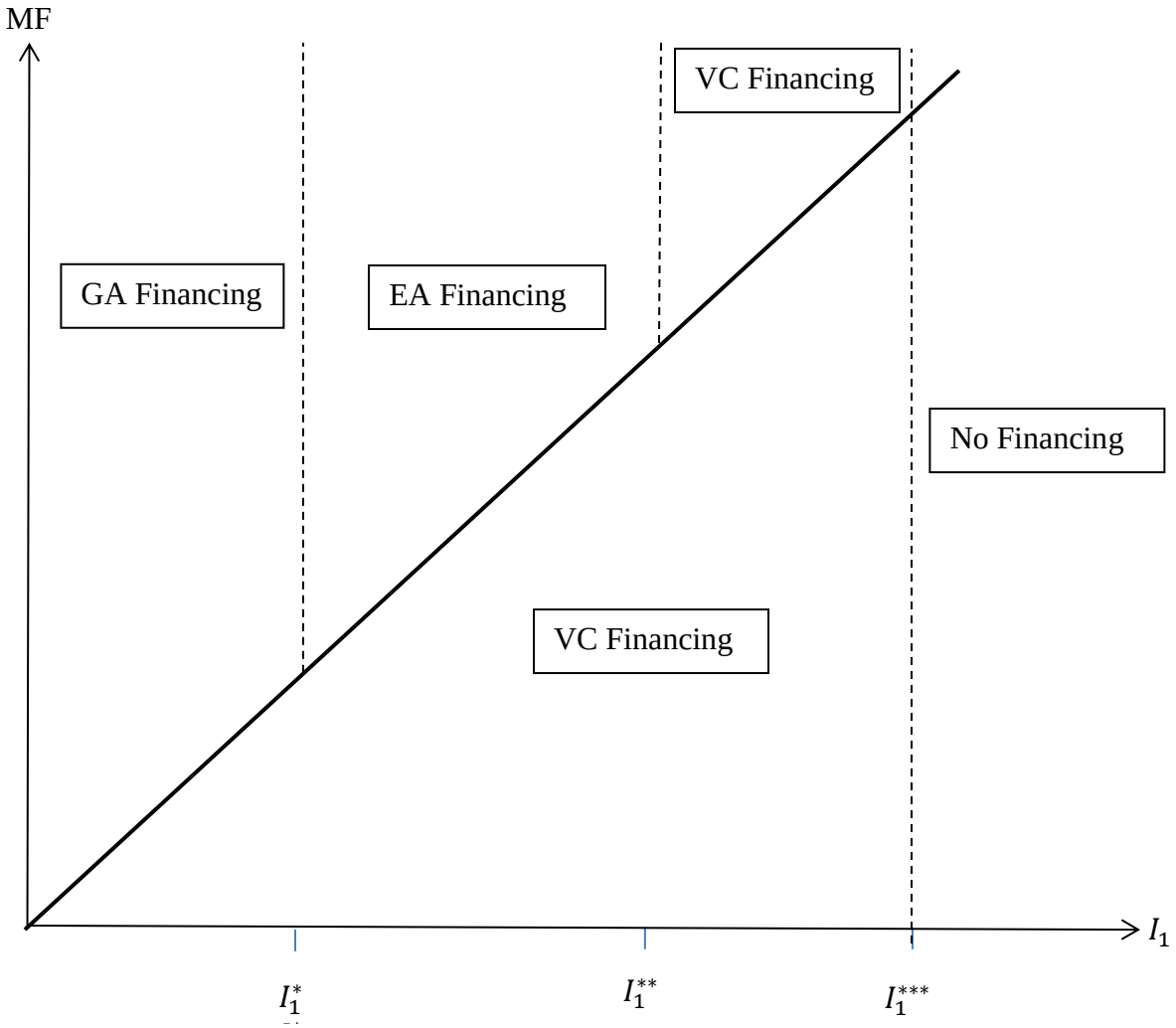
The entrepreneur's financing choice at $t = 0$ for different exit values V when $MF - I_1 < 0$



V^*, V^{**}, and V^{***} are the lowest exit value required by *VC*, *EA* and *GA* to offer financing for the venture, respectively, and are given by equations (A-13), (A-14), and (A-15) respectively. The solid lines describe the entrepreneur choice of financing. For $V \geq V^*$ the entrepreneur prefers *VC* financing and the V*C* is willing to finance the venture, resulting in *VC* financing. For $V^* > V$ the venture is not fundable .

Figure 5

The entrepreneur's financing choice at $t = 0$ for different exit values I_1



I_1^*, I_1^{**},, and I_1^{***}, are the highest seed round investment that *VC*, *EA* and *GA* are willing to finance and are given by equations (A-22), (A-23), and (A-24) respectively. The figure describes the entrepreneur choice of financing. For $I_1^* < I_1$, the entrepreneur prefers *VC* financing and the V*C* is willing to finance the venture, resulting in *VC* financing. For $V^* > V$ the venture is not fundable .