A Theory of ICOs:

Diversification, Agency, and Information Asymmetry

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Abstract

We develop a theory of financing of entrepreneurial ventures via an initial coin offering (ICO). Pre-selling a venture's output by issuing tokens allows the entrepreneur to transfer part of the venture risk to diversified investors without diluting her control rights. This, however, leads to an agency conflict between the entrepreneur and investors that manifests itself in underinvestment. We show that an ICO can dominate traditional venture capital (VC) financing when VC investors are under-diversified, when the idiosyncratic component of venture risk is large enough, when the payoff distribution is sufficiently right-skewed, and when the degree of information asymmetry between the entrepreneur and ICO investors is not too large. Overall, our model suggests that an ICO can be a viable financing alternative for some but not all entrepreneurial ventures. An implication is that while regulating ICOs to reduce the information asymmetry associated with them is desirable, banning them outright is not.

Keywords: ICO, crypto tokens, venture capital, agency, information asymmetry.

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1. Introduction

As a result of developments in the blockchain technology, a new form of financing – initial coin Offering (iCO) – has emerged, whereby an entrepreneurial venture obtains funds from investors in exchange for crypto tokens that are the sole means of payment for the venture's future products or services.[1] For example, in the largest ICO of 2017, Filecoin raised about $250 million by selling around 8% of the total supply of 2 billion FIL tokens, which are to be used for payments on its decentralized data storage network. So far, over 1,000 entrepreneurial ventures have sought financing via an ICO. The size of the ICO market has grown exponentially over the last two years. Based on various estimates, ICOs generated $2.3-3.9 billion in 2017 and $1.9-6.1 billion in the first three months of 2018, compared with about $100 million in 2016.[2] The majority of ICOs up to this day raised between $10 and $50 million each (e.g., Zetsche et al. (2018) and Lyandres, Palazzo and Rabetti (2018)).

Oft-cited advantages of ICOs over traditional financing methods include low transaction costs due to technological ease of creating customized tokens using existing blockchain platforms; global investor outreach; and the ability to combine financing with building a customer base. On the flip side, a notorious feature of the ICO market is the extent of information asymmetry between entrepreneurs and investors.[3] Entrepreneurial ventures attempting to raise funds via an ICO are typically very young, and many are in the pre-R&D stage. The main source of information for potential investors is the "white paper," describing with varying levels of detail the venture's technological and financial projections, sometimes complemented by a venture's blog. In the absence of appropriate regulation, high information asymmetry could turn the ICO market into a market for lemons.

Various jurisdictions have adopted a wide range of approaches to ICO regulation, from banning them altogether, e.g., China and South Korea, to imposing relatively loose regulatory standards, e.g., Singapore and Switzerland (e.g., Kaal (2018)). In the United States, the Securities and

[1] While the popular press as well as academic literature frequently use the terms "crypto token" and "crypto coin" interchangeably, there is an important difference. The purpose of a coin is to serve as a medium of exchange (currency), whereas a token typically represents the right to a certain (future) product or service and is the sole means using which that product or service can be obtained.

[2] See www.icowatchlist.com, www.coinschedule.com, www.icodata.io, and www.coindesk.com.

[3] Other risks associated with investing in crypto tokens include potential vulnerability of smart contracts that ensure token property rights (e.g., a leakage of $50 million worth of DAO tokens in June 2016) and inadvertent participation in illegal activities, such as money laundering and tax evasion.

Exchange Commission (SEC) seems to be moving in the direction of regulating ICOs as financial security offerings, as recently stated by the SEC Chairman: "the structures of initial coin offerings that I have seen promoted involve the offer and sale of securities and directly implicate the securities registration requirements and other investor protection provisions of our federal securities laws."[4]

In this paper, we develop a theory of ICOs that identifies several determinants of optimal ICO structure and the choice between ICO and conventional equity-based financing. In doing so, we presume adequate regulation preventing the ICO market from becoming a market for lemons is in place. We take the perspective of a risk-averse entrepreneur who has an idea of an entrepreneurial venture but lacks capital to finance it. To highlight the key distinctions between the economics of ICO financing and those of traditional equity financing, such as venture capital (VC), we first consider a parsimonious "base-case" model with fully diversified investors, no systematic risk, full information, and a Gaussian payoff distribution. In this setting, we identify the following fundamental tradeoff between the two sources of financing.

On the one hand, unlike traditional VC financing, an ICO allows the entrepreneur to keep some of the capital raised in cash. This feature of an ICO is similar to the secondary component of an equity issue in that it allows a risk-averse entrepreneur to transfer part of the venture risk to diversified investors. Unlike selling equity, however, transferring risk to investors by retaining part of the ICO proceeds does not dilute the entrepreneur's control rights.

On the other hand, the entrepreneur's ability to choose the investment level after securing the financing creates an agency conflict between the entrepreneur and the ICO investors. When choosing the investment level, the entrepreneur bears the full marginal cost of investment, but internalizes only a part of the marginal investment payoff that corresponds to the proportion of tokens retained by the entrepreneur. This is similar in spirit to Myers (1977), but unlike underinvestment associated with risky debt, underinvestment due to ICO financing arises even in the absence of failure risk. A similar agency conflict does not arise under traditional VC financing, where both the entrepreneur and the VC hold equity claims to the venture's cash flows.

Under the stylized assumptions of our base-case model, the agency cost of an ICO always dominates the diversification benefit of retaining cash, and venture capital always emerges as the preferred source of financing. To examine the choice between ICO and venture capital in more

[4] Public statement on Cryptocurrencies and Initial Coin Offerings by SEC Chairman Jay Clayton, from December 11, 2017; https://www.sec.gov/news/public-statement/statement-clayton-2017-12-11

realistic settings, we generalize the base-case model by relaxing several of its assumptions.

Under-diversified VC investors. In practice, VC investors, general partners in particular, tend to have a large portion of their wealth invested in a limited number of ventures and, as a result, require a return that reflects the venture risk and their own risk aversion. In contrast, tokens are an easily divisible asset, allowing virtually perfect diversification. To recognize this distinction between tokens and venture capital, we continue to assume dispersed and fully diversified token investors, but we assume that venture capital is provided by a risk-averse and under-diversified investor. We show that in this case, ICO is the preferred source of financing when the volatility of the venture payoff is sufficiently high. High volatility favors ICO financing because it increases the value of the entrepreneur's option to retain cash, and also because it increases the return required by the under-diversified VC investor. This leads to the first prediction of our model: *ICO financing is expected to be more prevalent for ventures with highly uncertain payoffs.*

Systematic risk. As the next step, we introduce systematic risk, and examine the differential effects of systematic and idiosyncratic components of the venture risk on the relative benefits of ICO financing. We show that ICO dominates VC financing when the idiosyncratic component of venture risk is high enough. Because systematic risk has a unique equilibrium price, it affects the entrepreneur and investors in the same way, regardless of their diversification or risk aversion. This means that the entrepreneur's option to transfer risk to investors by retaining part of the ICO proceeds in cash is valuable only insofar as the risk is idiosyncratic. Similarly, an under-diversified VC investor requires a higher return than fully diversified ICO investors only to the extent that the venture risk is idiosyncratic. In other words, both aforementioned benefits of ICO financing are tied to the idiosyncratic component of the venture risk. The second prediction of our model is therefore the following: *ICO financing is expected to be more prevalent for ventures with a larger proportion of idiosyncratic risk.*

Information Asymmetry. One of the salient features of the ICO market is considerable information asymmetry between entrepreneurs and investors. Unlike VCs, who tend to perform thorough due diligence, ICO investors need to rely mainly on the contents of the white paper and the terms of the ICO when making their investment decisions. Although it is likely that the tighter regulation that is being considered will reduce the extent of information asymmetry in the ICO market, VCs are likely to be always better informed than potential ICO investors. To recognize this distinct feature of ICOs, we further extend our model by assuming that there are two types of entrepreneurial ventures, high quality and low quality. Whereas entrepreneurs and VC investors are informed about the venture type, ICO investors are not. As a result, entrepreneurs with high-quality ventures ("high types") seeking ICO financing have incentive to signal their type to investors, whereas entrepreneurs with low-quality ventures ("low types") have incentive to imitate.

We show that there exists a unique least-cost separating equilibrium, in which the low types choose the first-best ICO terms, whereas the high types retain a larger portion of tokens than they would under full information. Intuitively, the entrepreneur's willingness to retain a larger portion of tokens signals her expectation that these tokens will have a high value once the uncertainty regarding the venture's output is resolved. As the degree of information asymmetry in terms of the difference between the two types increases, so does the low types' incentive to imitate, and the high types need to incur greater signaling costs to separate. As a result, some high-quality entrepreneurs, who would prefer an ICO in a low-information-asymmetry environment, prefer VC financing when the degree of information asymmetry is high enough. This leads to the following prediction: *In the presence of high information asymmetry, ICOs are expected to be less prevalent and of lower quality on average.*

Right-skewed payoff distribution. While analytically convenient, the normal distribution is not an ideal description of uncertainty associated with entrepreneurial ventures, which tend to have highly right-skewed payoff distributions with a large upside and a considerable risk of failure. To study the effect of such asymmetry in the payoff distribution, we abandon the assumption of a Gaussian payoff and examine numerically the effect of failure risk, while keeping the mean and volatility of the payoff distribution fixed.

One advantage of an ICO over venture capital is that in the case of partial retention of the ICO proceeds, the entrepreneur is guaranteed a positive payoff even if the venture ultimately fails. Thus, we would expect an ICO to dominate the VC alternative for ventures with right-skewed payoff distributions and a considerable risk of failure. We show that this is indeed the case. Interestingly, under right-skewed payoff distributions, an ICO can dominate VC financing even if VC investors are fully diversified, which is not the case under a normally distributed venture payoff. This analysis leads to the last prediction of our model: *ICO financing is expected to be more prevalent for ventures* *with high risk of failure and right-skewed payoff distributions.*

Our paper contributes to the emerging theoretical literature studying the economics of ICOs. Sockin and Xiong (2018) demonstrate the benefit of crypto currencies in facilitating decentralized trading among participants of a platform. Li and Mann (2018) show that an ICO can prevent undesirable prisoner-dilemma-type equilibria arising due to strategic complementarities among platform users and associated network externalities. They also highlight the ICO's ability to aggregate information of heterogeneous investors. Cong, Li and Wang (2018) study valuation of crypto tokens in the presence of both static and inter-temporal network externalities.

Sockin and Xiong (2018), Li and Mann (2018), and Cong, Li and Wang (2018) focus on platform-type ventures and explicitly model the dual role of ICO investors as users of the platform. While the majority of ICO-financed ventures have been platforms, ICOs are also being increasingly used to finance non-network-based projects.[5] Our theory is distinct from the aforementioned papers in that it does not rely on any network effects, nor does it require ICO investors to be users of the venture's product or service. Instead, we focus on contrasting ICO and traditional equity-based financing, and highlight the effect of diversification, agency, and information asymmetry considerations on equilibrium ICO terms and the optimal choice of financing method by an entrepreneur. In comparing ICO and VC financing, our paper is most closely related to Catalini and Gans (2018). Unlike us, they focus on the ability of an ICO to elicit demand information through generating buyer competition for the tokens.

In light of the high degree of information asymmetry in the ICO market, there is a growing literature discussing the optimal regulation of ICOs (e.g., Chohan (2017), Robinson (2017), Kaal (2018), Zetsche et al. (2018)). We contribute to this literature by suggesting that an objective of such regulation should be to reduce information asymmetry in the ICO market to a moderate level, at which high-quality ventures are able to separate from the low-quality ones by choosing appropriate ICO terms.

Our work is also related to the literature on reward-based crowdfunding, in which funds are also raised in exchange for the firm's future product or service.[6] Strausz (2017) studies optimal reward-

[5] Examples include Bananacoin, backed by environmentally friendly bananas grown in Laos, an upcoming ICO of Intex Resources ASA, a Norwegian mining company, and a planned ICO of the Plaza Hotel in New York.

[6] A fundamental difference between ICO financing and reward-based crowdfunding is that crypto tokens can be traded in the secondary market, so ICO investors are not necessarily the eventual users of the product. Therefore, whereas participation in a crowdfunding campaign is essentially a consumption decision, investing in an ICO is akin to purchasing a security with an uncertain monetary reward.

based crowdfunding design under entrepreneurial moral hazard. Babich, Marinesi and Tsoukalas (2018) focus on complementarity between crowdfunding and venture capital. In both these papers, participants in the crowdfunding campaign are consumers, and the benefit of crowdfunding is elicitation of demand information, similar to Catalini and Gans (2018). In our paper, investors are not necessarily consumers and the benefit of ICO financing stems from the entrepreneur's ability to transfer risk to diversified investors.

Our paper is a part of the broader literature on economic effects of the blockchain technology (see Lee (2016) for a discussion of the potential of blockchain to upend the stock market; Yermak (2017) for an analysis of effects of blockchains on corporate governance; Biais et al. (2018) for a game-theoretic model of a blockchain protocol; Chod et al. (2018) for a study of the benefits of blockchain technology in overcoming information asymmetry frictions in supply chain finance; and Cong and He (2018) for an examination of various competitive effects of blockchains).

Finally, as the advantage of ICO financing that we focus on is the entrepreneur's ability to control the venture's cash retention and investment policy, our paper is related to the literature on control implications of ownership structure (e.g., Zingales (1995), Burkart, Gromb and Panunzi (1997), and Pagano and Roell (1998)), and to the literature on optimal contracting in the presence of incomplete contracts (e.g., Aghion and Bolton (1992) and Aghion and Tirole (1994)).

2. Base-Case Model

We consider a risk-averse entrepreneur who has an idea of a risky venture, but does not have the wealth to finance it. The focus of our analysis is on the benefits and costs of financing the venture via an initial coin offering, i.e., by issuing tokens that represent claims to the venture's future product or service, compared with conventional financing. While there are various conventional ways in which an early-stage venture can raise external funds, we use venture capital as a benchmark.[7]

[7] Debt financing is typically not a viable option for entrepreneurial ventures lacking collateral and stable cash flows (e.g., Berger and Udell (1998)). There are other types of equity financing besides venture capital, e.g., angel investment. However, in terms of size, ICOs are most similar to VC financing rounds. Among 429 ICOs that occurred between March, 2014 and January, 2018 with information available on either www.icobench.com or www.icodrops.com or www.cryptocompare.com, the mean (median) ICO proceeds amounted to \$20.3 million (12.7 million). The mean (median) size of a VC round is \$13.3 million (6 million) in the overall sample of US VC investments between 1990 and 2005, and \$17.9 million (8.6 million) within a subsample of fintech firms (e.g., Cumming and Schweinbacher (2016)). In contrast, a typical angel investment tends to be below \$100,000 (e.g., Shane (2012)).

2.1. Model assumptions

Assumption 1 *The entrepreneur is risk averse and has CARA utility*

$$u(\omega) = \gamma^{-1} - \gamma^{-1} \exp(-\gamma\omega), \tag{1}$$

where ω is her terminal wealth and γ is her coefficient of absolute risk-aversion.

Assumption 2 *The venture involves producing Q units of output at a quadratic cost Q^2. The output can be sold at an uncertain unit price ξ. Therefore, a capital investment of Q^2 results in an uncertain payoff $\Pi = \xi Q$.*

Assumption 3 *The entrepreneur does not have investable funds of her own, and the venture can only be financed using one of two options: an ICO or venture capital that takes the form of equity.*

In practice, it is common for VCs to receive convertible preferred equity in the ventures they fund.[8] We assume that the VC receives equity to highlight the tradeoffs between ICO and traditional financing in the most parsimonious way. These tradeoffs remain qualitatively intact under VC financing with convertible securities.

Assumption 4 *In the case of VC financing, the venture capitalist has full control rights over the use of the funds contributed to the venture. In particular, these funds are invested directly in production and no part of them is kept as cash.*

The assumption of the VC having control over the venture's investment strategy is consistent with the large empirical literature documenting VC control rights (e.g., Gompers (1995), Lerner (1995), and Kaplan and Stromberg (2003)) and the theoretical literature on optimal contracting between VCs and entrepreneurs (e.g., Hellmann (1998) and Dessein (2005)). The assumption that the funds provided by the VC are fully invested is consistent with staged financing common in VC contracts, whereby gradual staging of capital infusions and low cash reserves aim to keep the entrepreneur "on a tight leash" (e.g., Sahlman (1990) and Gompers (1995)).

Assumption 5 *In the case of ICO financing, the entrepreneur has full control rights over the use of the funds contributed by the ICO investors. In particular, she is allowed to invest in production only part of the ICO proceeds and keep the rest as cash.*

This assumption reflects the fact that in the majority of ICOs, token holders are not given any

[8] See Gompers (1997) and Kaplan and Stromberg (2003) for evidence on VC financing using convertible securities, and Schmidt (2003) and Hellmann (2006) for models showing optimality of such financing.

meaningful control rights (e.g., Catalini and Gans (2018) and Kaal (2018)). Adhami, Giudici and Martinazzi (2018) report that in a sample of 253 ICOs between 2014 and 2017, ICO investors could participate in governance decisions, i.e., take part in decision polls, in only 25% of cases, and they were given contribution rights, i.e., the opportunity to influence product/service-related decisions, in only 16% of cases.[9]

Assumption 6 *Tokens are perfectly divisible, allowing ICO investors to fully diversify venture-specific risk.*

Assumption 7 *The risk-free rate is normalized to zero.*

To highlight the fundamental tradeoff between ICO and VC financing in a most parsimonious and analytically convenient framework, we initially make the following additional assumptions. Later, we will relax each of these assumptions to examine the choice between ICO and VC financing in more realistic settings.[10]

Assumption 8 *VC investors are dispersed and able to fully diversify venture-specific risk.*

Assumption 9 *The uncertainty associated with the output price ξ is entirely idiosyncratic.*

Assumption 10 *All parties have full information regarding the distribution of ξ.*

Assumption 11 *The output price is normally distributed, $\xi \sim \mathcal{N}(\mu, \sigma)$, and $\sigma < \frac{2}{\sqrt{\gamma}}$.*

When ξ is normally distributed, so is the entrepreneur's payoff, and maximizing the expected CARA utility, $\mathbb{E}u(\omega)$, is equivalent to maximizing the mean-variance criterion, $\mathbb{E}(\omega) - \frac{\gamma}{2}\mathrm{Var}(\omega)$. Assuming a normally distributed price is reasonable only if the probability of negative realizations is small. The condition $\sigma < \frac{2}{\sqrt{\gamma}}$ prevents the pathological scenario in which $\Pr(\xi < 0)$ is so large relative to the entrepreneur's relative risk aversion, that her expected utility under VC financing decreases in her share of the venture; see the proof of Lemma 1 for details.

Next, we characterize the equilibrium VC contract, which we then use as a "traditional financing benchmark" to assess the benefits and costs of ICO financing.

[9] Many recent white papers include explicit statements such as "the founders of Telegram will be responsible for the efficient use of funds resulting from any sale of tokens"; "MEST token holders have no voting rights or shares in the Monaco Estate company"; or "VENDI Coin confers no other rights in any form, included but not limited to any ownership, distribution (including but not limited to profit), redemption, liquidation, proprietory ... or other financial or legal rights, other than those specifically described in the White paper."

[10] Because each of the following assumptions is ultimately relaxed, we indicate in each lemma and proposition which of these assumptions it relies on.

2.2. VC financing

Suppose that the entrepreneur approaches a VC firm with a take-it-or-leave-it offer of a contract that stipulates (i) the VC's capital investment Q^2 and (ii) the VC's share of the venture payoff, or equity stake, α. The VC accepts the offer if, and only if, it can expect to at least break even, i.e., $\alpha\mathbb{E}\Pi \geq Q^2$, where $\Pi = \xi Q$. The entrepreneur chooses α and Q so as to maximize her expected utility

$$Q^{vc}, \alpha^{vc} = \underset{\alpha\in(0,1), Q\geq 0}{\arg\max}\ \mathbb{E}u\left((1-\alpha)\Pi\right) \text{ s.t. } \alpha\mathbb{E}\Pi \geq Q^2. \tag{2}$$

Next, we characterize the equilibrium VC contract and the mean-variance form of the entrepreneur's expected utility given this contract, $U^{vc} = \mathbb{E}\left((1-\alpha^{vc})\Pi\left(Q^{vc}\right)\right) - \frac{\gamma}{2}\text{Var}\left((1-\alpha^{vc})\Pi\left(Q^{vc}\right)\right)$.

Lemma 1 *Under $\xi \sim \mathcal{N}(\mu, \sigma)$, fully diversified VC investors, no systematic risk, and full information, the equilibrium VC contract is such that $Q^{vc} = \frac{\mu}{2}$, $\alpha^{vc} = \frac{1}{2}$, and $U^{vc} = \frac{\mu^2}{4}\left(1 - \frac{\gamma\sigma^2}{8}\right)$.*

Because for any given output level Q, the entrepreneur is better off holding a larger share of the venture, she always chooses the smallest α that is acceptable to the VC. In other words, the equilibrium α is such that the VC breaks even, i.e., $Q^2 = \alpha\mathbb{E}\Pi$. Interestingly, the equilibrium output level satisfies

$$Q^{vc} = \arg\max_{Q\geq 0}\left[\mathbb{E}\Pi - Q^2\right], \tag{3}$$

that is, VC financing results in the investment/output level that maximizes expected profit. Because the entrepreneur does not contribute her own funds to the venture, the investment level is not distorted by her risk aversion. Furthermore, because both the entrepreneur and the VC hold equity claims, the investment level is not distorted by any agency conflict between the two parties.

2.3. ICO financing

Suppose that the entrepreneur legally commits to issue a certain number of tokens, and to accept these tokens as the sole means of payment for the venture's output.[11] She then sells a proportion α of the tokens to investors via an ICO, while retaining the rest.[12] We normalize the

[11] In our model as well as in the vast majority of ICOs to date, the entrepreneur commits to a pre-specified number of tokens in circulation. See Catalini and Gans (2018) for a discussion of such commitment.

[12] The average share of tokens retained by founders and/or the venture within 429 ICOs with available information that occurred between March, 2014 and January, 2018 is 47%.

total number of tokens issued to one, so the terms "number of tokens" and "proportion of tokens" are interchangeable.

Let p be the equilibrium price of a token at the initial offering. Thus, by selling α tokens, the entrepreneur raises the total amount αp. Retaining control rights over the use of the ICO proceeds, she then decides to produce Q units of output by investing Q^2 in production, while keeping the remaining $\alpha p - Q^2 \geq 0$ in cash. Eventually, uncertainty regarding the unit output price ξ is resolved and the entrepreneur sells the remaining $(1-\alpha)$ tokens. These tokens as well as the tokens held by ICO investors are purchased by consumers who redeem them for the venture's output.

Because in equilibrium, the monetary value of total output, $\Pi = \xi Q$, has to be equal to the value of tokens outstanding, whose number is normalized to one, Π is also the future value of one token. Thus, the entrepreneur's revenue from selling her remaining tokens is $(1-\alpha)\,\Pi$. Finally, since the ICO investors are fully diversified, the venture risk is entirely idiosyncratic, and the risk-free rate is normalized to zero, the ICO price of a token must equal its expected future value in equilibrium, i.e., $p = \mathbb{E}\Pi$. The sequence of events is summarized below:

1. The entrepreneur sells α tokens to investors at equilibrium price p, raising αp.
2. The entrepreneur invests Q^2 in production and keeps the remaining $\alpha p - Q^2 \geq 0$ in cash.
3. Uncertainty regarding the monetary value of output, $\Pi = \xi Q$, is resolved.
4. The entrepreneur sells the remaining $(1-\alpha)$ tokens, earning $(1-\alpha)\Pi$; token holders exchange their tokens for the venture's output.

The entrepreneur has two decisions to make. She first decides α, the number of tokens to initially offer to investors. After selling the tokens to investors, she decides the output level Q or, equivalently, the investment level Q^2. We consider these two decisions sequentially starting with the latter.

When choosing the optimal output level, the entrepreneur's capital available is αp and she maximizes the expected utility from her terminal payoff, which is $\left(\alpha p - Q^2\right) + (1-\alpha)\,\Pi$. The first term is the amount raised through the ICO, αp, minus the production investment, Q^2. The second term is the uncertain future value of the $(1-\alpha)$ tokens retained, where $\Pi = \xi Q$. Therefore, we have

$$Q^{ico}(\alpha) = \arg\max_{Q \geq 0} \mathbb{E}u\left(\alpha p - Q^2 + (1-\alpha)\,\Pi\right) \text{ s.t. } Q^2 \leq \alpha p, \tag{4}$$

where p and α are given.

The difference between the optimal investment decision under ICO in (4) and that under VC financing in (3) is twofold. First, unlike VC financing, an ICO allows the risk-averse entrepreneur to keep some of the capital raised in cash. As the entrepreneur retains 100% equity in the firm, this is equivalent to the entrepreneur distributing the uninvested part of the ICO proceeds to herself and thereby transferring some of the venture risk to diversified investors.

Second, the entrepreneur's ability to choose the investment level *after* securing the financing creates an agency conflict between her and the ICO investors. When choosing the investment level, the entrepreneur bears the full marginal cost of investment, but internalizes only a fraction $(1 - \alpha)$ of the marginal investment payoff. This is similar in spirit to the underinvestment problem of Myers (1977), which arises in the presence of risky debt and limited liability. However, there is an important difference. In Myers (1977) underinvestment occurs because equityholders bear the full marginal cost of investment, whereas the marginal investment payoff realized in bankruptcy states accrues to debtholders. Here, a fraction α of the marginal investment payoff accrues to the ICO investors in each state of the world. Thus, unlike underinvestment associated with debt, underinvestment under ICO financing arises even in the absence of uncertainty.

When choosing the fraction of tokens to sell during the ICO, the entrepreneur maximizes the expected utility from her terminal payoff, $\alpha p - Q^2 + (1 - \alpha)\,\Pi(Q)$, anticipating the subsequent investment $Q = Q^{ico}(\alpha)$. Because investors too anticipate the entrepreneur's optimal investment and price the ICO accordingly, we have $p = \mathbb{E}\Pi\left(Q^{ico}(\alpha)\right)$. The optimal α is thus given by

$$\alpha^{ico} = \arg\max_{\alpha\in(0,1)} \mathbb{E}u\left(\alpha\mathbb{E}\Pi\left(Q^{ico}(\alpha)\right) + (1-\alpha)\,\Pi\left(Q^{ico}(\alpha)\right) - \left(Q^{ico}(\alpha)\right)^2\right), \tag{5}$$

where $Q^{ico}(\alpha)$ is given in (4). Equation (5) highlights the tradeoff in choosing the optimal α. On the one hand, larger α allows the entrepreneur to remove uncertainty from a larger portion of the investment payoff Π. On the other hand, larger α exacerbates the distortion of the anticipated investment $Q^{ico}(\alpha)$ due to the aforementioned agency problem, which in turn affects how investors price the ICO.

The next lemma characterizes the equilibrium ICO structure and the corresponding mean-variance form of the entrepreneur's expected utility, U^{ico}.

Lemma 2 *Under $\xi \sim \mathcal{N}(\mu, \sigma)$, no systematic risk, and full information, there exists a threshold*

$s < 2$ such that the equilibrium ICO structure is the following:

(a) If $\sigma \leq \frac{s}{\sqrt{\gamma}}$, then $\alpha^{ico} = \tilde{\alpha}$, $Q^{ico} = \tilde{\alpha}\mu$, and $U^{ico} = \frac{\mu^2}{2}\left(\tilde{\alpha} + \tilde{\alpha}^2\right)$, where $\tilde{\alpha}$ is the unique root of $3\alpha + \gamma\sigma^2 (1-\alpha)^2 \alpha - 1 = 0$.

(b) If $\sigma > \frac{s}{\sqrt{\gamma}}$, then $\alpha^{ico} = \frac{\sigma^2\gamma + 1 - \sqrt{2\sigma^2\gamma+1}}{\sigma^2\gamma}$, $Q^{ico} = \frac{\mu}{2}\frac{1}{\sqrt{2\sigma^2\gamma+1}}$, and $U^{ico} = \mu^2 \frac{\sqrt{2\sigma^2\gamma+1}-1}{4\sigma^2\gamma}$.

The form of the equilibrium solution depends on the entrepreneur's risk aversion γ and the venture risk captured by the output price volatility σ. When risk aversion and/or volatility are relatively low (case (a)), the entrepreneur is less concerned with offloading risk to investors, the option to retain cash is less valuable, and underinvestment becomes the dominant consideration. Thus, the entrepreneur sells a relatively small portion of the tokens to investors, and subsequently invests the entire ICO proceeds in production, i.e., $Q^2 = \alpha p$.

If risk and/or risk aversion are high (case (b)), the entrepreneur's incentive to retain cash is strong, she sells a large portion of the tokens to investors, and subsequently invests only part of the ICO proceeds in prodution, i.e., $Q^2 < \alpha p$. This is the case in which the potential benefit of ICO financing – the ability to lock in part of the uncertain future revenue – materializes, albeit at the cost of underinvestment, which rational investors price into the ICO valuation.

The equilibrium α and Q under ICO financing are illustrated and compared with the corresponding quantities under equilibrium VC financing in Panels (a) and (b) of Figure 1, respectively. In this figure, $\mu = 1$, σ varies over the entire permissible range $\left[0, \frac{2}{\sqrt{\gamma}}\right)$, and $\gamma = 8$.[13] The blue lines correspond to the VC benchmark. Because under VC financing, the entrepreneur does not contribute her own funds to the venture and the VC investors are assumed to be fully diversified, neither the equilibrium investment nor the equity share required by the VC is affected by the venture payoff volatility.

The red curves illustrate the equilibrium under ICO financing. At low levels of volatility, the entrepreneur structures the ICO so as to commit herself to investing the entire ICO proceeds, passing on the option to retain cash. She does so by keeping a relatively large portion of the tokens, which in turn limits the amount of money that she is able to raise. Thus, even though

[13] With respect to our choice of the numerical value of γ, note that the coefficient of absolute risk aversion depends strongly on the individual's wealth w, whereas the coefficient of relative risk aversion, $R = \gamma \times w$, appears to be roughly constant (e.g., Rabin (2000)). Furthermore, most empirical estimates of relative risk aversion R vary between 1 and 3 (e.g., Chiappori and Paiella (2011) and the references therein). If we proxy for the entrepreneur's wealth by the expected value of her share of the venture under VC financing, i.e., $w = (1-\alpha^{vc})\mathbb{E}\Pi\left(Q^{vc}\right) = \frac{1}{4}$, our choice of $\gamma = 8$ corresponds to $R = 2$, consistent with empirical estimates.

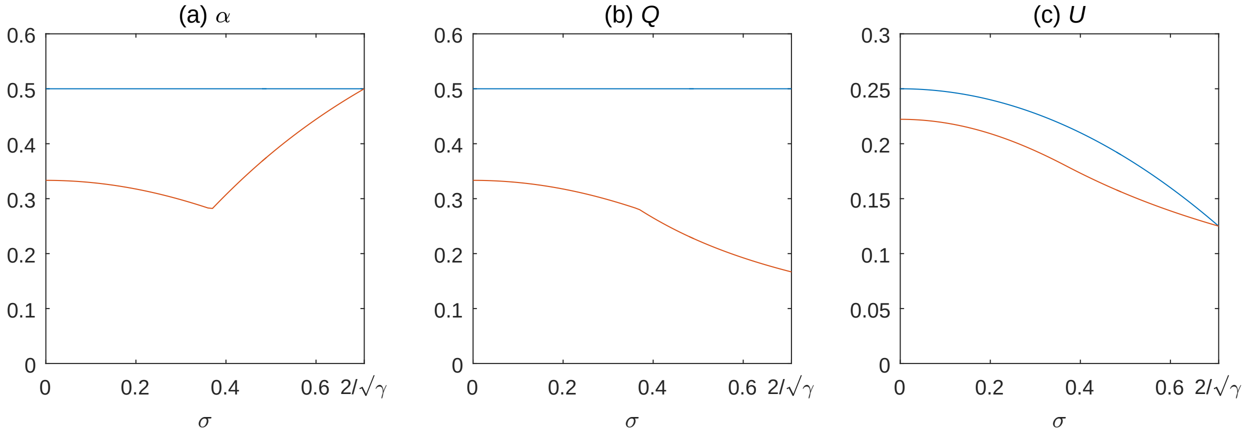

Figure 1: The equilibrium α, Q, and the resulting mean-variance utility U under ICO (red) and VC financing (blue), as a function of $\sigma \in (0, 2/\sqrt{\gamma})$ for μ=1 and γ=8.

underinvestment does not arise in the sense that the entire ICO proceeds are invested, the capital raised and the resulting investment are below those arising under VC financing. As volatility increases, the entrepreneur needs to keep a larger portion of the tokens to maintain her commitment and, as a result, raises less money, which in turn leads to a lower investment.

When volatility exceeds a certain threshold, $\frac{s}{\sqrt{\gamma}} \approx 0.37$ in the example, the entrepreneur's risk considerations become pre-eminent, and the option to retain cash becomes too valuable to pass. As volatility increases, the entrepreneur sells a larger share of the tokens, which exacerbates the agency problem, and the investment level drops precipitously as a result of risk aversion and underinvestment.

2.4. Equilibrium financing choice

Comparing the entrepreneur's expected utilities under ICO and VC financing leads to the following result.

Proposition 1 *Under $\xi \sim \mathcal{N}(\mu, \sigma)$, fully diversified VC investors, no systematic risk, and full information, ICO is always dominated by VC financing, i.e., $U^{vc} \geq U^{ico}$.*

Proposition 1 shows that in our base-case model, the underinvestment problem always trumps the diversification benefit of retaining part of the ICO proceeds, and venture capital is the optimal source of financing. This is also illustrated in Panel (c) of Figure 1, in which the entrepreneur's expected utility under VC financing exceeds that under ICO financing for any admissible volatility $\sigma \in \left[0, \frac{2}{\sqrt{\gamma}}\right)$.

While the base-case model is useful for illustrating the fundamental tradeoff between the two financing methods, it relies on several restrictive assumptions. In the next section, we relax some of these assumptions and examine the choice between ICO and VC financing in more realistic settings. We first examine the case of under-diversified VC investors. Second, we introduce systematic risk. Third, we consider information asymmetry between entrepreneurs and ICO investors. Fourth and finally, we examine the effect of asymmetric, right-skewed payoff distributions, which tend to characterize entrepreneurial ventures seeking early-stage financing.

3. When is ICO financing optimal?

3.1. Under-diversified VC investor

In our base-case model, we assumed risk-neutral valuation of the venture by both VC and ICO investors. This is a rather innocuous assumption in the case of ICO investors because tokens are an easily divisible asset, allowing virtually perfect diversification. In contrast, VC investors, and general partners of VC funds in particular, tend to be under-diversified. Since general partners have to invest considerable time and effort in managing each venture, VC funds tend to invest in a limited number of projects.[14] As a result, VC investors are likely to require a higher expected return, reflecting the venture risk and their own risk aversion. To recognize this distinction between ICO and VC investors, we replace Assumption 8 of dispersed and fully diversified VC investors with the following alternative:

Assumption 8a *The VC firm is wholly owned by an investor with CARA utility and the coefficient of absolute risk aversion $\delta \leq \gamma$, who does not have any other holdings.*

The assumption that the VC investor is less risk-averse than the entrepreneur captures two empirical regularities. First, the coefficient of absolute risk aversion tends to decrease in wealth (e.g., Arrow (1971) and Kimball (1993)). Second, VC investors tend to be wealthier than entrepreneurs.[15] The assumption that the VC investor does not have other holdings is equivalent to assuming that

[14] See Fulghieri and Sevilir (2009) for a model of optimal VC portfolio size, Ewens, Jones and Rhodes-Kropf (2013) for a model of interactions between a risk-averse, under-diversified VC general partner and entrepreneurs, and Cumming (2006) for empirical evidence on VC portfolios sizes.

[15] Bitler, Moskowitz and Vissing-Jorgensen (2005) report the mean (median) wealth of entrepreneurs to be around \$1 million (300,000). Although we are not aware of any evidence on the wealth distribution for VC general partners, Bottazzi, Da Rin and Hellmann (2008) report that a mean (median) VC fund manages \$330 million (60 million) and has three general partners. A back-of-the-envelope calculation using a typical 2% annual management fee implies that the lower bound on the annual income of a mean (median) VC general partner is \$2 million (400,000), suggesting substantially larger levels of wealth than those of entrepreneurs.

the payoff of such holdings is normally distributed and independent of ξ, and is made for ease of exposition.

When approaching an under-diversified VC investor, the entrepreneur faces the same problem as in (2) except that the VC's participation constraint $Q^2 \leq \alpha \mathbb{E}\Pi$ becomes

$$Q^2 \leq \alpha \mathbb{E}\Pi - \frac{\delta}{2} \mathrm{Var}\left(\alpha \Pi\right), \tag{6}$$

reflecting the VC's concern with payoff volatility. The next lemma characterizes the equilibrium solution.

Lemma 3 *Under $\xi \sim \mathcal{N}(\mu, \sigma)$, under-diversified VC investor, no systematic risk, and full information, the equilibrium VC contract is such that $Q^{vc} = \frac{\mu}{2}\sqrt{\frac{2}{2+\sigma^2\delta}}$, $\alpha^{vc} = \frac{\sqrt{2\sigma^2\delta+4}-2}{\sigma^2\delta}$, and $U^{vc} = \frac{\mu^2}{2}\frac{\sqrt{2\sigma^2\delta+4}-2}{\sigma^2\delta}\left(1 - \frac{\gamma}{\delta}\frac{\sqrt{2\sigma^2\delta+4}-2}{4}\right)$.*

Comparing the entrepreneur's expected utility when financed by an under-diversified VC with her expected utility under ICO financing leads to the following result.

Proposition 2 *Under $\xi \sim \mathcal{N}(\mu, \sigma)$, under-diversified VC investor, no systematic risk, and full information, ICO dominates VC financing if, and only if, the volatility of the venture's output price is sufficiently high, i.e., there exists $\bar{\sigma}$ such that*

$$U^{ico} > U^{vc} \Longleftrightarrow \sigma > \bar{\sigma}.$$

Importantly, the threshold $\bar{\sigma}$, which we characterize in the proof of the proposition, satisfies $\bar{\sigma} < \frac{2}{\sqrt{\gamma}}$. This means that there are levels of volatility within the admissible range $\left[0, \frac{2}{\sqrt{\gamma}}\right)$ for which ICO dominates VC financing. Furthermore, $\bar{\sigma} > \frac{s}{\sqrt{\gamma}}$, which means that an ICO can be the preferred financing strategy only when the entrepreneur's option to retain part of the ICO proceeds is "in the money."

Volatility of the venture payoff affects the choice between ICO and VC financing in two ways: (i) Unlike VC financing, an ICO allows the entrepreneur to mitigate her risk exposure by converting part of the uncertain future revenue into cash at fair value. This diversification benefit of ICO financing is greatest when volatility is high. However, as shown in Proposition 1, it is never sufficient to dominate the underinvestment problem when the benchmark is financing by a fully

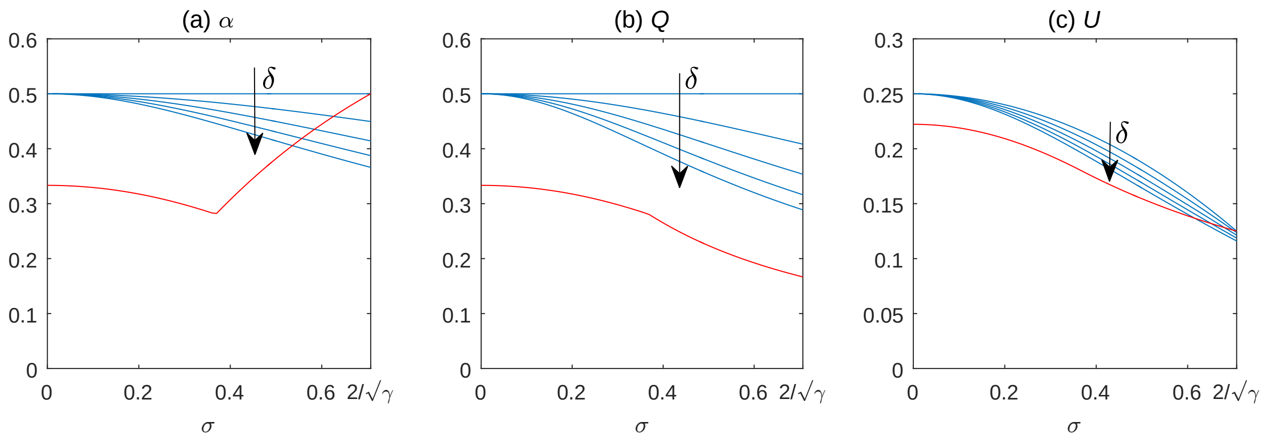

Figure 2: The equilibrium α, Q, and the resulting mean-variance utility U under ICO (red) and VC financing (blue), as a function of $\sigma \in (0, 2/\sqrt{\gamma})$ for $\mu = 1$, $\gamma = 8$, and $\delta \in (0, \gamma)$.

diversified VC. (ii) When the VC investor himself is under-diversified, venture capital becomes less attractive as the VC requires a higher expected return than ICO investors, and more so when payoff volatility is high. Taken together, these two effects result in preference for ICO financing for ventures with highly volatile payoffs.

The effect of the VC's under-diversification is illustrated in Figure 2, which is similar to Figure 1, except that it plots the equilibrium α^{vc}, Q^{vc}, and U^{vc} under different levels of the VC's risk aversion δ. As δ increases, the VC invests less (Panel (b)), receives a smaller share of the venture in return (Panel (a)), and the entrepreneur is worse off (Panel (c)). Importantly, the higher the VC's risk aversion δ, the lower the volatility threshold above which ICO dominates VC financing, as evident from Panel (c).

3.2. Systematic risk

In this section, in addition to allowing the VC investor to be under-diversified, we relax our assumption that the output price uncertainty is entirely idiosyncratic. In particular, we replace Assumption 9 with the following alternative:

Assumption 9a *The output price ξ is correlated with the return on the market portfolio, $r_m \sim \mathcal{N}(\bar{r}_m, \sigma_m)$. The standard assumptions of the CAPM hold. Namely, the VC investor as well as the entrepreneur can borrow and lend at the risk-free rate and allocate their wealth outside the venture optimally between cash and the market portfolio.*

The only feature of our model that deviates from the standard CAPM is that the entrepreneur and the VC investor hold, in addition to the optimal combination of cash and the market portfolio, their entire stakes in the venture.

Although we allow the entrepreneur to borrow at the risk-free rate and to invest in the market portfolio so as to make the concept of systematic risk meaningful, we retain our initial Assumption 3 that the venture is entirely financed either by venture capital or from ICO proceeds. In other words, we do not allow the entrepreneur to finance the venture by using her personal wealth or by borrowing.

Let ρ be the correlation coefficient between ξ and r_m, and let

$$\hat{\mu} = \mathbb{E}\xi - \bar{r}_m \frac{\operatorname{cov}(\xi, r_m)}{\sigma_m^2} = \mu - \bar{r}_m \rho\sigma/\sigma_m \text{ and} \tag{7}$$

$$\hat{\sigma}^2 = \left(1-\rho^2\right)\operatorname{Var}(\xi) = \left(1-\rho^2\right)\sigma^2. \tag{8}$$

Whereas μ is the expected value of the unit revenue ξ, $\hat{\mu}$ denotes its CAPM valuation. Parameter $\hat{\sigma}$ measures the idiosyncratic volatility of ξ. Namely, if we decompose $\xi = \xi_{idio} + \xi_{sys}$, where ξ_{idio} is independent of r_m, whereas ξ_{sys} is perfectly correlated with r_m, we can write $\operatorname{Var}(\xi_{idio}) = \hat{\sigma}^2$. Finally, note that ρ^2 is the proportion of total risk that is systematic.

VC financing

Let y denote the VC's investment in the market portfolio. The VC will accept the entrepreneur's offer to invest in the venture if, and only if,

$$\max_y \left[\mathbb{E}\left(r_m y\right) - \frac{\delta}{2}\operatorname{Var}\left(r_m y\right)\right] \leq \max_y \left[\mathbb{E}\left(r_m y + \alpha\Pi - Q^2\right) - \frac{\delta}{2}\operatorname{Var}\left(r_m y + \alpha\Pi\right)\right]. \tag{9}$$

The left-hand side of (9) is the VC's mean-variance utility from the optimal investment in the market portfolio, provided it is his only investment. The right-hand side of (9) is the VC's mean-variance utility if he accepts the entrepreneur's offer to invest Q^2 in the venture in exchange for a share α of the venture payoff Π, in addition to optimally investing in the market portfolio. The optimal solutions to the left-hand and right-hand sides of (9) are $y_1 = \frac{\bar{r}_m}{\delta\sigma_m^2}$ and $y_2 = \frac{\bar{r}_m}{\delta\sigma_m^2} - \alpha Q\rho\sigma/\sigma_m$, respectively. Plugging these quantities back to the respective sides of (9), the VC's participation constraint simplifies into

$$Q^2 \leq \alpha Q\hat{\mu} - \frac{\delta}{2}\alpha^2 Q^2 \hat{\sigma}^2. \tag{10}$$

Let x denote the entrepreneur's investment in the market portfolio. Her decision problem is

$$x^{vc}, Q^{vc}, \alpha^{vc} = \arg\max_{x,Q,\alpha} \mathbb{E}u\left((1-\alpha)\,\Pi + r_m x\right), \tag{11}$$

subject to the VC's participation constraint (10). The next lemma characterizes the equilibrium under VC financing.

Lemma 4 *Under $\xi \sim \mathcal{N}(\mu, \sigma)$, under-diversified VC investor, systematic risk, and full information, the equilibrium VC contract is such that $Q^{vc} = \frac{\hat{\mu}}{2}\sqrt{\frac{2}{2+\hat{\sigma}^2\delta}}$, $\alpha^{vc} = \frac{\sqrt{2\hat{\sigma}^2\delta+4}-2}{\hat{\sigma}^2\delta}$, and $U^{vc} = \frac{\hat{\mu}^2}{2}\frac{\sqrt{2\hat{\sigma}^2\delta+4}-2}{\hat{\sigma}^2\delta}\left(1-\frac{\gamma}{\delta}\frac{\sqrt{2\hat{\sigma}^2\delta+4}-2}{4}\right)+\frac{1}{2}\frac{\bar{r}_m^2}{\gamma\sigma_m^2}$.*

The effect of systematic risk and the opportunity to trade the market portfolio can be seen by comparing the expressions for α^{vc}, Q^{vc}, and U^{vc} in Lemma 4 with those in Lemma 3. The difference is threefold. (i) The entrepreneur's expected utility is increased by $\frac{1}{2}\frac{\bar{r}_m^2}{\gamma\sigma_m^2}$, which is the gain from trading the market portfolio. (ii) Output price volatility σ is replaced by idiosyncratic volatility $\hat{\sigma}$, which continues to affect the risk-averse entrepreneur and VC investor in the same way, regardless of their ability to trade the market portfolio. (iii) The expected unit revenue μ is replaced by $\hat{\mu}$. Because systematic risk can be hedged by trading the market portfolio, its effect on both the entrepreneur and the VC depends on how it is priced by the market. In the CAPM equilibrium, it alters the value of uncertain cash flow ξ from μ to $\hat{\mu}$.

ICO financing

According to the CAPM, the relation between the token price at the initial offering, p, and its uncertain future value, Π, is given by

$$p = \mathbb{E}\Pi - \bar{r}_m \frac{\operatorname{cov}(\Pi, r_m)}{\sigma_m^2} \Longleftrightarrow p = Q\hat{\mu}. \tag{12}$$

The entrepreneur's problem post-ICO is to choose the output level Q^{ico} and the investment in the market portfolio x^{ico} that maximize her expected utility, i.e.,

$$x^{ico}(\alpha), Q^{ico}(\alpha) = \arg\max_{x,Q} \mathbb{E}u\left(\alpha p - Q^2 + (1-\alpha)\Pi + x r_m\right) \text{ s.t. } Q^2 \leq \alpha p, \tag{13}$$

where p and α are given. Without loss of generality, we focus on the scenario, in which the constraint, $Q^2 \leq \alpha p$, is not binding, i.e., the entrepreneur does not invest the entire ICO proceeds in production.[16]

When choosing the optimal α, the entrepreneur maximizes her expected utility in (13), but now she anticipates the equilibrium token price p to be determined according to (12) and the subsequent choice of x and Q to be determined by the optimal solution to (13). The next lemma characterizes the equilibrium under ICO financing.

Lemma 5 *Under $\xi \sim \mathcal{N}(\mu, \sigma)$, systematic risk, and full information, the equilibrium ICO structure is such that $\alpha^{ico} = \frac{\hat{\sigma}^2\gamma+1-\sqrt{2\hat{\sigma}^2\gamma+1}}{\hat{\sigma}^2\gamma}$, $Q^{ico} = \frac{\hat{\mu}}{2}\frac{1}{\sqrt{2\hat{\sigma}^2\gamma+1}}$, and $U^{ico} = \hat{\mu}^2\frac{\sqrt{2\hat{\sigma}^2\gamma+1}-1}{4\hat{\sigma}^2\gamma} + \frac{1}{2}\frac{\bar{r}_m^2}{\gamma\sigma_m^2}$.*

The effects of introducing systematic risk are the same as in the case of VC financing in the sense that the expressions in Lemma 5 are identical to those in Lemma 2(b) except that μ and σ are replaced by $\hat{\mu}$ and $\hat{\sigma}$, respectively, and the entrepreneur's expected utility is increased by the gain from trading the market portfolio, $\frac{1}{2}\frac{\bar{r}_m^2}{\gamma\sigma_m^2}$.

Comparing the entrepreneur's expected utilities under VC and ICO financing given in Lemmata 4 and 5, respectively, leads to the following result.

Proposition 3 *Under $\xi \sim \mathcal{N}(\mu, \sigma)$, under-diversified VC investor, systematic risk, and full information, ICO dominates VC financing if, and only if, the proportion of systematic risk is sufficiently low, i.e., there exists $\bar{\rho}$ such that*

$$U^{ico} > U^{vc} \iff \rho^2 < \bar{\rho}^2.$$

When all parties have the opportunity to trade the market portfolio, systematic nature of the venture risk favors VC financing for the following reason. Because systematic risk has a given price determined by the CAPM equilibrium, it affects the entrepreneur and investors the same way, regardless of their diversification or risk aversion. This has two implications. First, the entrepreneur's option to off-load some of the venture risk to investors by retaining part of the ICO proceeds is valuable only to the extent that this risk is idiosyncratic. Second, an under-diversified VC requires a higher return than fully diversified ICO investors only to the extent that the venture

[16] This is the case when $\hat{\sigma} > \frac{s}{\sqrt{\gamma}}$. When $\hat{\sigma} \leq \frac{s}{\sqrt{\gamma}}$, we obtain a boundary solution, which is analogous to the one characterized in part (a) of Lemma 2. However, because this boundary solution is always dominated by VC financing, assuming $\hat{\sigma} > \frac{s}{\sqrt{\gamma}}$ is without any loss of generality when it comes to identifying the parameter set for which an ICO is the preferred financing strategy.

risk is idiosyncratic. Therefore, the larger the systematic component of the venture risk, the less valuable the option to retain part of the ICO proceeds, and the smaller the valuation premium of ICO investors over the VC.

Finally, recall Proposition 2 according to which venture capital dominates ICO financing when volatility is low enough, even if it is entirely idiosyncratic. In other words, for low enough volatility, the threshold $\bar{\rho}$ equals zero.

3.3. Information asymmetry

In this section, we continue to consider an under-diversified VC investor and systematic risk, but add more realism to the informational environment of ICO financing. One of the paramount features of the ICO market is the notorious information asymmetry between entrepreneurs and investors. Therefore, we replace Assumption 10 regarding full information of all parties with the following alternative:

Assumption 10a *There are two types of entrepreneurial ventures: high-quality and low-quality, denoted by indices H and L, respectively. The two types differ only in the expected value of the output price, in particular, $\xi_i \sim \mathcal{N}\left(\mu_i, \sigma^2\right)$ for type i, with $\mu_H > \mu_L$. Each entrepreneur and VC is informed about the venture quality, whereas ICO investors are not.*

The assumption that the VC is fully informed about the venture quality reflects in a most parsimonious form the informational advantage of VCs over ICO investors, stemming from a rigorous due diligence process typical of VC financing (e.g., Chan (1983) and Ueda (2004)).

With a fully informed VC, the equilibrium under VC financing is identical to that characterized in the previous section, with the expected output price μ replaced by μ_H and μ_L for high-quality and low-quality ventures, respectively. In the remainder of this section, we characterize the equilibrium ICO, and compare it with VC financing. In what follows we use the terms "high/low-quality entrepreneur" and "high/low-quality venture" interchangeably.

ICO financing

Because the ICO valuation depends on investors' beliefs regarding the venture's type, high-quality entrepreneurs have an incentive to signal their type, whereas low-quality entrepreneurs have an incentive to imitate. Because investors form their beliefs regarding an ICO's quality upon observing the fraction α of tokens that the entrepreneur offers through the ICO, this fraction is a potential signaling device. Let $\beta(\alpha)$ be the investors' belief regarding the type of an entrepreneur who offers fraction α of tokens at the ICO. Following Spence (1973), we assume that the belief has a threshold structure, i.e.,

$$\beta(\alpha) = \begin{cases} H & \text{if } \alpha \leq t \\ L & \text{otherwise} \end{cases} \tag{14}$$

where the threshold t has to be consistent with entrepreneurs' actions and is determined endogenously in equilibrium. In other words, if the proportion of tokens that an entrepreneur offers through the ICO is lower than or equal to t, investors believe that her venture is of high quality. Otherwise they believe that the venture is of low quality. Intuitively, the entrepreneur's willingness to retain a larger portion of the tokens is a signal of her expectation that these tokens will have a high value once uncertainty regarding the output price is resolved. Note that in our model, the entrepreneur can sell the tokens through the ICO and then after the resolution of uncertainty, but not in-between these two events. Not allowing the entrepreneur to trade on her insider information post ICO is critical for the signal conveyed through α to be credible.[17]

Let $U_{ij}(\alpha)$ be the expected utility of an entrepreneur of type i who is perceived as type j by investors, and post-IPO invests optimally in production and the market portfolio, consistent with her true type. A separating equilibrium (SE) is characterized by the low type's and the high type's optimal actions, α_L and α_H, respectively, and a consistent belief structure given by (14) that satisfies the following necessary and sufficient conditions:

$$\max_{\alpha>t} U_{HL}(\alpha) \leq \max_{\alpha\leq t} U_{HH}(\alpha), \text{ and} \tag{15}$$

$$\max_{\alpha>t} U_{LL}(\alpha) \geq \max_{\alpha\leq t} U_{LH}(\alpha). \tag{16}$$

Condition (15) ensures that the high type chooses α at or below the threshold t. Condition (16) ensures that the low type chooses α above this threshold. Because these conditions may lead to multiple separating equilibria, we adopt the intuitive criterion refinement by Cho and Kreps (1987), which eliminates any Pareto-dominated equilibria. We refer to any equilibria that survive as least-cost separating equilibria (LCSE).

[17] This assumption is consistent with the common practice, whereby entrepreneurs' tokens gradually vest and cannot be sold for some time following the ICO. In addition, a large portion of tokens retained is typically earmarked for future payments to contributors such as developers.

We continue to focus on the more interesting parameter range, in which the entrepreneur of either type does not invest the entire ICO proceeds in production and in which an ICO can potentially dominate VC financing.[18] Note that under full information, it is optimal for both types to choose α^{ico} that is characterized in Lemma 5, and to which we refer as the "first-best."

Lemma 6 *Under $\xi \sim \mathcal{N}(\mu, \sigma)$, systematic risk, and asymmetric information, there exists a unique least-cost separating equilibrium, in which the two types choose $\alpha_L = \alpha^{ico}$ and $\alpha_H = t < \alpha^{ico}$, and the investors' belief threshold t is given by*

$$t = 1 - \frac{2\hat{\sigma}^2\gamma\theta + 2\sqrt{\hat{\sigma}^2\gamma\,(\theta - 1)\left(\hat{\sigma}^2\gamma - 2\sqrt{2\hat{\sigma}^2\gamma + 1} + \hat{\sigma}^2\gamma\theta + 2\right)}}{\hat{\sigma}^2\gamma\left(4\theta + \sqrt{2\hat{\sigma}^2\gamma + 1} - 3\right)}, \tag{17}$$

where $\theta = \hat{\mu}_H^2/\hat{\mu}_L^2$ and $\hat{\mu}_i = \mu_i - \bar{r}_m\rho\sigma/\sigma_m$.

In the least-cost separating equilibrium, low-quality entrepreneurs choose the first-best ICO terms, are perceived as low types, and their ventures are valued accordingly. In contrast, high-quality entrepreneurs retain a larger portion of tokens than they would under full information to signal their quality to investors. Obtaining the correct valuation thus comes at a cost to them. Parameter θ measures the difference between the two types and thereby the degree of information asymmetry.[19] As this difference increases, so does the low type's incentive to imitate. As a consequence, the high type needs to retain a larger share of tokens to separate, resulting in greater signaling costs.

The effect of the degree of information asymmetry θ on high-quality ICOs is illustrated in Figure 3 (in red) together with the VC benchmark (in blue). Because we are examining the effect on high-quality ventures, we fix μ_H and vary μ_L so that θ ranges between 1 and 1.5. The figure is based on a relatively high volatility ($\sigma = 0.7$), so that in the absence of information asymmetry ($\theta = 1$), ICO dominates VC financing. As the degree of information asymmetry θ increases, high-quality entrepreneurs reduce the portion of tokens they sell at the ICO to sustain separation (Panel (a)). The larger stake in the venture retained by the entrepreneur moderates the agency problem, while increasing the entrepreneur's risk exposure, the net result being a moderately positive relation

[18] Recall that under full information, this is the case when $\hat{\sigma} > \frac{s}{\sqrt{\gamma}}$. By continuity, it must also be the case when $\hat{\sigma} > \frac{s}{\sqrt{\gamma}}$ and the difference between μ_H and μ_L is not too large.

[19] To be precise, θ is equal to the ratio of the expected utilities that the high type and the low type derive from the venture under full information.

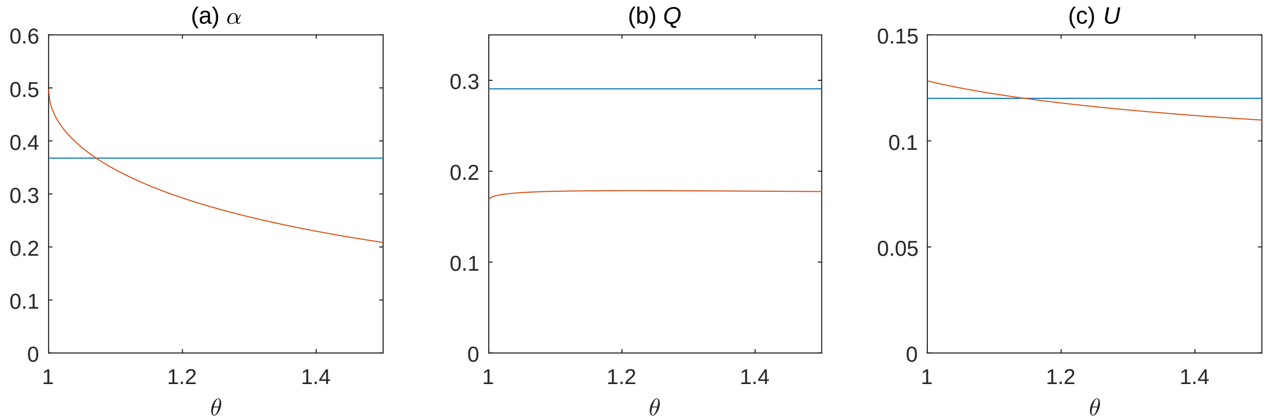

Figure 3: The equilibrium α, Q, and the resulting mean-variance utility U of the high-quality entrepreneur under VC financing (blue) and the LCSE ICO (red), as a function of the degree of information asymmetry $\theta \in (1, 1.5)$ for $\mu_H = 1$, $\sigma = 0.7$, $\rho = 0$, $\bar{r}_m = 0.05$, $\sigma_m = 0.25$, and $\gamma = \delta = 8$.

between the degree of information asymmetry and investment in production (Panel (b)). Most important, as the proportion of tokens sold at the ICO deviates further from the first best level, the entrepreneur's expected utility decreases, and eventually falls below the VC benchmark (Panel (c)). We formalize this result in the following proposition.

Proposition 4 *Under $\xi \sim \mathcal{N}(\mu, \sigma)$, under-diversified VC investor, systematic risk, and asymmetric information, ICO dominates VC financing for high-quality ventures if, and only if, the degree of information asymmetry is sufficiently low, i.e., there exists $\bar{\theta}$ such that*

$$U^{ico} > U^{vc} \iff \theta < \bar{\theta}.$$

Recall that under VC financing, information asymmetry does not affect ventures of either type by assumption. According to Lemma 6, low-quality ventures are not affected by information asymmetry even under ICO financing in equilibrium. Information asymmetry thus impacts only high-quality entrepreneurs seeking ICO financing. According to Proposition 4, some high-quality entrepreneurs, who would prefer an ICO in a low-information-asymmetry environment, prefer VC financing if the degree of information asymmetry is high enough. Finally, recall that venture capital is in some cases (well-diversified VC or low idiosyncratic volatility) the preferred source of financing even under full information, i.e., the threshold $\bar{\theta}$ can equal one.

3.4. Right-skewed payoff distribution

Our analysis thus far has relied on a normally distributed output price and resulting venture payoff. While analytically convenient, the normal distribution is not an ideal description of payoff uncertainty for ventures considering ICO financing. Payoff distribution of such ventures is likely to be highly right-skewed with a large probability of failure and a small probability of a very high payoff.[20] In this section, we examine numerically the effect of asymmetry in payoff distribution that captures both high failure risk and right-skewness. To make the analysis as transparent as possible, we revert to the parsimony of our base case model with fully diversified VC investors, fully idiosyncratic volatility, and full information. At the same time, we replace Assumption 11 of a normally distributed output price with the following alternative:

Assumption 11a *The output price ξ follows the following mixture distribution:*

$$\xi = \begin{cases} 0 & \textit{with probability } \phi \\ X & \textit{with probability } 1-\phi \end{cases} \tag{18}$$

where X is log-normally distributed.

Under this assumption, the venture either completely fails, or it is a "success" with a log-normally distributed payoff. We examine the effect of failure risk ϕ, while keeping the mean and volatility of ξ fixed.[21] This effect is shown in Figure 4, which plots the expected utilities U^{ico} and U^{vc} as a function of $\phi \in (0.1, 0.5)$ for $\mathbb{E}\xi = 1$, $\mathrm{Var}(\xi) = 1$, and varying levels of the entrepreneur's absolute risk aversion γ.[22]

An advantage of ICO over VC financing is that the former enables a risk-averse entrepreneur to obtain a positive payoff by retaining part of the ICO proceeds even if the venture ultimately fails. Thus, we would expect ICO financing to dominate the VC alternative for ventures with right-skewed payoff distributions that have a considerable probability mass at zero, especially if the entrepreneur is strongly risk averse. As shown in Figure 4, a higher failure risk indeed favors

[20] While it is too early to measure the distribution of ICO returns, the evidence on entrepreneurial ventures backed by venture capital suggests a failure rate (probability of a write-off) ranging between 25% and 50% (e.g., Cumming and MacIntosh (2003) and Laine and Torstilla (2005)).

[21] In particular, as we vary ϕ, we simultaneously vary $\mathbb{E}X$ and $\mathrm{Var}(X)$ to keep $\mathbb{E}\xi = (1-\phi)\,\mathbb{E}X$ and $\mathrm{Var}(\xi) = (1-\phi)\,\phi\,(\mathbb{E}X)^2 + (1-\phi)\mathrm{Var}(X)$ constant.

[22] Under non-Gaussian ξ, maximizing the expected utility is not equivalent to maximizing the mean-variance criterion. Therefore, unlike all previous figures, Figure 4 plots the actual expected utility rather than the mean-variance criterion.

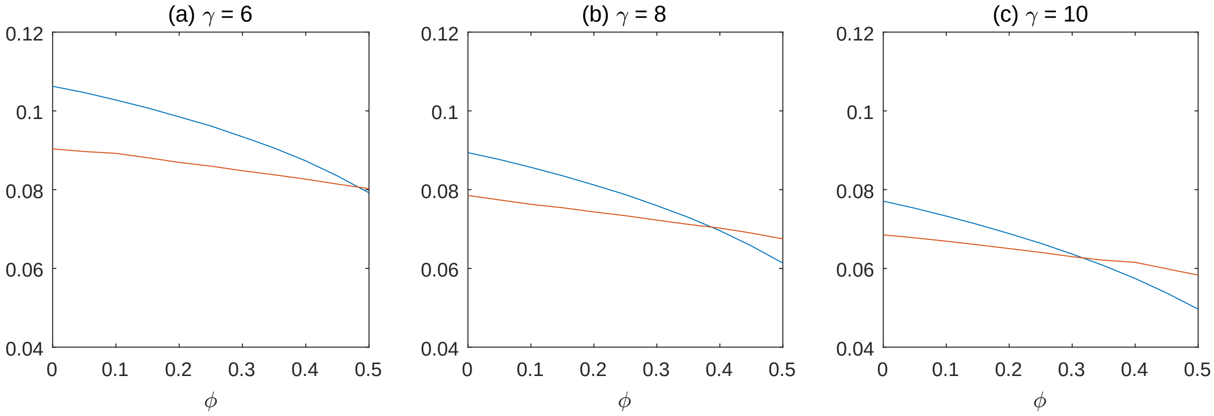

Figure 4: The entrepreneur's expected utility under equilibrium VC financing (blue) and ICO (red) as a function of ϕ for $\mathbb{E}\xi = 1$ and $\mathrm{Var}(\xi) = 1$.

ICO financing. Namely, ICO dominates VC financing when the failure risk $\phi \gtrsim 32\%$ (38%, 48%) for $\gamma = 10$ $(8, 6)$. According to Proposition 1, under a normally distributed payoff, ICO cannot dominate financing by a fully diversified VC. As shown in Figure 4, under a more realistic, non-Gaussian payoff distribution, it can.

4. Empirical Predictions

Prediction 1 *ICO financing is expected to be more prevalent for ventures with highly uncertain payoffs.*

This prediction, which follows directly from Proposition 2, is consistent with anecdotal evidence and with the fact that in the relatively short period of their existence, token returns have been substantially more volatile than equity returns, even those of small and risky technology firms.[23] Designing an empirical test of the self selection of ventures with more uncertain projects into ICO financing is not trivial for two reasons. First, token returns can be more volatile than equity returns for reasons unrelated to underlying project risk. Second, the choice of financing mode may by itself influence return volatility through its effect on the venture's investment strategy. An alternative interpretation of the prediction, which would be more amenable to empirical testing, is as a relation between return or cash flow volatility in a given industry and the prevalence of ICOs in this industry.

Prediction 2 *ICO financing is expected to be more prevalent for ventures with a larger proportion*

[23] See e.g., Earnst and Young research report on ICOs from December 2017, http://www.ey.com/Publication/vwLUAssets/ey-research-initial-coin-offerings-icos/File/ey-research-initial-coin-offerings-icos.pdf

of idiosyncratic risk.

This prediction follows from Proposition 3. Given the large intra-industry correlations in the proportion of idiosyncratic volatility (e.g., Herskovic et al. (2004)), this prediction could be potentially tested by examining the inter-industry variation in the prevalence of ICO financing and its relation to the inter-industry variation in the proportion of idiosyncratic risk.

Prediction 3 *In the presence of high information asymmetry, ICOs are expected to be less prevalent and of lower quality on average.*

This prediction follows from Proposition 4, which postulates that in the presence of asymmetric information, high-quality ventures may be deterred from raising funds via an ICO due to signaling costs, which are increasing with the degree of information asymmetry, i.e. with the quality differences among ventures seeking capital. The prediction is consistent with recent concerns, expressed by regulators in many countries, that in the absence of regulation mitigating information asymmetry, the ICO market could turn into a market for lemons. These concerns already led to actions against ICOs believed to be fraudulent, and are likely to lead to treatment of crypto tokens as securities, which are subject to tighter regulation.[24,25]

Prediction 4 *ICO financing is expected to be more prevalent for ventures with high risk of failure and right-skewed payoff distributions.*

Prediction 4 follows from the numerical analysis in Section 3.4. It is consistent with ventures of exploratory nature, aiming to use novel and unproven technologies, in early stages of development, opting for ICO financing. Importantly, this prediction is based on an analysis that abstracts from information asymmetry. Therefore, when designing an empirical test of this prediction, it is crucial to disentangle the effects of high failure risk from those of information asymmetry.

5. Conclusions

We propose a model of financing of an entrepreneurial venture by issuing crypto tokens in an initial coin offering. Following recent trends in the regulatory approach to ICOs, our model treats tokens as securities. Our focus is on the benefits and costs of ICOs relative to traditional equity-based financing such as venture capital. We demonstrate that an ICO can be preferable to venture

[24]A recent example of an ICO that was halted by a court order obtained by the SEC on charges of fraud is AriseBank, see https://www.sec.gov/news/press-release/2018-8

[25]SEC chairman Jay Clayton already stated "I believe every ICO I've seen is a security" during the U.S. Senate Committee on Banking, Housing and Urban Affairs hearing on February 6, 2018.

capital financing, and identify several factors that favor ICO financing, such as under-diversification of VC investors, large idiosyncratic component of venture risk, right-skewness of payoff distribution, and low degree of information asymmetry between entrepreneurs and investors. Our results suggest that an ICO can be a viable financing alternative for some, but not all entrepreneurial ventures. An implication is that while regulating the ICO market so as to reduce information asymmetry between entrepreneurs and investors is desirable, banning ICOs outright is not.

Our paper offers numerous empirical predictions regarding factors affecting an entrepreneur's decision to choose an ICO over traditional financing methods. While the empirical literature on crypto currencies has so far focused on analyzing their returns and identifying sources of their risk (e.g., Elendner et al. (2016), Bianchi (2017), and Adhami, Giudici and Martinazzi (2018)), examining empirically firms' decisions to issue tokens would be informative for designing optimal ICO regulation. Finally, our model is not limited to platform-based ventures, which have been the focus of the emerging theoretical ICO literature, but is applicable to financing of any venture via token issuance.

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Appendix

Proof of Lemma 1: Taking advantage of the mean-variance form of the expected utility, the entrepreneur's problem (2) can be written as

$$
\begin{aligned}
Q^{vc}, \alpha^{vc} &= \underset{\alpha\in(0,1),Q\geq 0}{\arg\max} \left[\mathbb{E}\left((1-\alpha)\,\Pi\right) - \frac{\gamma}{2}\mathrm{Var}\left((1-\alpha)\,\Pi\right)\right] \text{ such that } Q^2 \leq \alpha\mathbb{E}\Pi \qquad (19)\\
&= \underset{\alpha\in(0,1),Q\geq 0}{\arg\max} \left[(1-\alpha)\,\mu Q - \frac{\gamma\sigma^2}{2}(1-\alpha)^2 Q^2\right] \text{ such that } Q \leq \alpha\mu. \qquad (20)
\end{aligned}
$$

The corresponding Kuhn-Tucker conditions (KTC) are

$$
\begin{aligned}
\frac{d}{dQ}\left[(1-\alpha)\,\mu Q - \frac{\gamma\sigma^2}{2}(1-\alpha)^2 Q^2 - c\,(\alpha-1) - d\,(Q-\alpha\mu)\right] &= 0,\\
\frac{d}{d\alpha}\left[(1-\alpha)\,\mu Q - \frac{\gamma\sigma^2}{2}(1-\alpha)^2 Q^2 - c\,(\alpha-1) - d\,(Q-\alpha\mu)\right] &= 0,\\
c\,(\alpha-1) = 0,\ d\,(Q-\alpha\mu) = &\quad 0,\\
\alpha - 1 \leq 0,\ Q - \alpha\mu \leq &\quad 0,\\
\alpha, Q, c, d &\geq 0,
\end{aligned}
$$

which can be simplified into

$$
\begin{aligned}
(1-\alpha)\,\mu - \gamma\sigma^2(1-\alpha)^2 Q - d &= 0,\\
-\mu Q + \gamma\sigma^2(1-\alpha)\,Q^2 - c + \mu d &= 0,\\
c\,(\alpha-1) = 0,\ d\,(Q-\alpha\mu) = &\quad 0,\\
\alpha - 1 \leq 0,\ Q - \alpha\mu \leq &\quad 0,\\
\alpha, Q, c, d &\geq 0.
\end{aligned}
$$

It is straightforward to show that $c \neq 0$ cannot be a part of any optimal solution. Thus we have two cases.

Case 1: If $c = 0$ and $d \neq 0$, the KTC become $\alpha = \frac{1}{2}$, $Q = \frac{\mu}{2}$, and $\gamma\sigma^2 < 4$, which also directly leads to $U^{vc} = \frac{\mu^2}{4}\left(1 - \frac{\gamma\sigma^2}{8}\right)$.

Case 2: If $c = 0$ and $d = 0$, the KTC become $Q = \frac{\mu}{\gamma\sigma^2(1-\alpha)}$, $0 \leq \alpha \leq 1$, and $1 - \gamma\sigma^2(1-\alpha)\,\alpha \leq 0$, which has a solution only if $\gamma\sigma^2 \geq 4$. At this solution, the derivative of the entrepreneur's objective in (20) w.r.t. α is zero. Because this objective is concave in α, the entrepreneur would be worse off holding with a larger share of the firm.■

Proof of Lemma 2: Using the mean-variance form of the expected utility, the entrepreneur's choice of optimal Q in (4) and optimal α in (5) can be written, respectively, as

$$\begin{aligned} Q^{ico}(\alpha) &= \arg\max_{Q\geq 0}\left[\alpha p - Q^2 + (1-\alpha)\mu Q - \frac{\gamma}{2}(1-\alpha)^2 Q^2\sigma^2\right] \text{ such that } Q^2 \leq \alpha p, \quad (21) \\ \text{and } \alpha^{ico} &= \arg\max_{\alpha\in(0,1)}\left[\mu Q^{ico}(\alpha) - \left(Q^{ico}(\alpha)\right)^2 - \frac{\gamma}{2}(1-\alpha)^2\left(Q^{ico}(\alpha)\right)^2\sigma^2\right], \quad (22) \end{aligned}$$

where $p = \mu Q^{ico}(\alpha)$. The unconstrained optimum of (21) is

$$\bar{Q} = \frac{(1-\alpha)\mu}{2+\gamma(1-\alpha)^2\sigma^2}. \tag{23}$$

If $\bar{Q} \leq \sqrt{\alpha p}$, this solution is feasible and $Q^{ico} = \bar{Q}$; otherwise $Q^{ico} = \sqrt{\alpha p}$. Thus, problem (22) can be written as

$$\begin{aligned} \alpha^{ico} &= \arg\max_{\alpha\in(0,1)}\left[\mu Q - Q^2 - \frac{\gamma}{2}(1-\alpha)^2 Q^2\sigma^2\right], \text{ where} \\ Q &= \begin{cases} \frac{(1-\alpha)\mu}{2+\gamma(1-\alpha)^2\sigma^2} & \text{if } \frac{(1-\alpha)\mu}{2+\gamma(1-\alpha)^2\sigma^2} \leq \sqrt{\alpha p} \\ \sqrt{\alpha p} & \text{if } \frac{(1-\alpha)\mu}{2+\gamma(1-\alpha)^2\sigma^2} > \sqrt{\alpha p} \end{cases}, \end{aligned}$$

and where $p = \mu Q$. Because the investors' anticipation of Q must be consistent with the firm's actual investment, we have

$$Q = \begin{cases} \frac{(1-\alpha)\mu}{2+\gamma(1-\alpha)^2\sigma^2} & \text{if } \frac{(1-\alpha)\mu}{2+\gamma(1-\alpha)^2\sigma^2} \leq \sqrt{\alpha\mu\frac{(1-\alpha)\mu}{2+\gamma(1-\alpha)^2\sigma^2}} \\ \sqrt{\alpha\mu Q} & \text{if } \frac{(1-\alpha)\mu}{2+\gamma(1-\alpha)^2\sigma^2} > \sqrt{\alpha\mu Q} \end{cases}.$$

Therefore,

$$Q = \begin{cases} \frac{(1-\alpha)\mu}{2+\gamma(1-\alpha)^2\sigma^2} & \text{if } \frac{(1-\alpha)}{2+\gamma(1-\alpha)^2\sigma^2} \leq \alpha \\ \alpha\mu & \text{if } \frac{(1-\alpha)}{2+\gamma(1-\alpha)^2\sigma^2} > \alpha \end{cases}.$$

The discriminant of the cubic equation $\frac{(1-\alpha)}{2+\gamma(1-\alpha)^2\sigma^2} = \alpha$ is $\Delta = \left(-8\left(\gamma\sigma^2\right)^2 + 9\gamma\sigma^2 - 108\right)\gamma\sigma^2 < 0$, and thus the equation has exactly one real root. Let $\tilde{\alpha}$ be this root. It follows that

$$Q = \begin{cases} \frac{(1-\alpha)\mu}{2+\gamma(1-\alpha)^2\sigma^2} & \text{if } \alpha \geq \tilde{\alpha}, \\ \alpha\mu & \text{otherwise} \end{cases}.$$

Case 1: $\alpha \geq \tilde{\alpha}$. Plugging (23) into (22), we obtain

$$\bar{\alpha} = \arg\max_{\alpha\in(0,1)} \left[\frac{\mu^2}{2}\frac{1-\alpha^2}{2+(1-\alpha)^2\gamma\sigma^2}\right].$$

The objective is unimodal and its solution is

$$\bar{\alpha} = \frac{\sigma^2\gamma + 1 - \sqrt{2\sigma^2\gamma+1}}{\sigma^2\gamma} \in (0,1).$$

This solution satisfies $\bar{\alpha} \geq \tilde{\alpha}$ iff $\frac{(1-\bar{\alpha})}{2+\gamma(1-\bar{\alpha})^2\sigma^2} \leq \bar{\alpha}$ iff $\sigma^2\gamma \geq \frac{5}{16}+\frac{3}{16}\sqrt{17}$. Thus, there are two subcases.

Subcase 1a: If $\sigma^2\gamma \geq \frac{5}{16}+\frac{3}{16}\sqrt{17}$, the optimal α such that $\alpha \geq \tilde{\alpha}$ is $\bar{\alpha}$.

Subcase 1b: If $\sigma^2\gamma < \frac{5}{16}+\frac{3}{16}\sqrt{17}$, the optimal α such that $\alpha \geq \tilde{\alpha}$ is $\tilde{\alpha}$.

Case 2: $\alpha < \tilde{\alpha}$. Plugging $\alpha\mu$ for Q in (22), we obtain

$$\hat{\alpha} = \arg\max_{\alpha\in(0,1)} \left[\mu^2\left(\left(\alpha-\alpha^2\right) - \frac{\gamma}{2}\left(\alpha-\alpha^2\right)^2\sigma^2\right)\right].$$

There are two subcases.

Subcase 2a: If $\gamma\sigma^2 \leq 4$, the objective is unimodal and $\hat{\alpha} = \frac{1}{2}$. However, because $\frac{1}{2} > \tilde{\alpha}$, the optimal α such that $\alpha \leq \tilde{\alpha}$ is $\tilde{\alpha}$.

Subcase 2b: If $\gamma\sigma^2 > 4$, the objective is bimodal and $\hat{\alpha} \in \left\{\frac{1}{2} - \sqrt{\frac{\sigma^2\gamma-4}{4\sigma^2\gamma}}, \frac{1}{2} + \sqrt{\frac{\sigma^2\gamma-4}{4\sigma^2\gamma}}\right\}$. However, because $\frac{1}{2} - \sqrt{\frac{\sigma^2\gamma-4}{4\sigma^2\gamma}} > \tilde{\alpha}$, the optimal α such that $\alpha \leq \tilde{\alpha}$ is $\tilde{\alpha}$.

To sum up Case 2, the optimal α such that $\alpha \leq \tilde{\alpha}$ is always $\tilde{\alpha}$. Therefore, if $\sigma^2\gamma \geq \frac{5}{16}+\frac{3}{16}\sqrt{17}$, then $\alpha^{ico} = \frac{\sigma^2\gamma+1-\sqrt{2\sigma^2\gamma+1}}{\sigma^2\gamma}$, $Q^{ico} = \frac{\mu}{2}\frac{1}{\sqrt{2\sigma^2\gamma+1}}$, and $U^{ico} = \mu^2\frac{\sqrt{2\sigma^2\gamma+1}-1}{4\sigma^2\gamma}$. If $\sigma^2\gamma < \frac{5}{16}+\frac{3}{16}\sqrt{17}$, then $\alpha^{ico} = \tilde{\alpha}$, $Q^{ico} = \tilde{\alpha}\mu$, and $U^{ico} = \frac{\mu^2}{2}\left(\tilde{\alpha}+\tilde{\alpha}^2\right)$. If we let $s = \sqrt{\frac{5}{16}+\frac{3}{16}\sqrt{17}} < 2$, the result follows. ■

Proof of Proposition 1: According to Lemma 1, VC financing results in $U^{vc} = \frac{\mu^2}{4}\left(1-\frac{\gamma\sigma^2}{8}\right)$. According to Lemma 2, the solution under ICO financing depends on the value of $\gamma\sigma^2$. However, it follows from the proof of this lemma that the solution in part (a) corresponds to the case in which the unconstrained optimal investment exceeds the ICO proceeds, and the constraint $Q^2 \leq \alpha p$ is binding. If we relax this constraint (by allowing the entrepreneur to borrow at the risk-free rate), the optimal solution will have the same functional form as the one in part (b). Thus, the solution in part (b) is an upper bound on the optimal solution for any $\gamma\sigma^2$, i.e., $U^{ico} \leq \mu^2 \frac{\sqrt{2\sigma^2\gamma+1}-1}{4\sigma^2\gamma}$. Thus, to show that $U^{vc} \geq U^{ico}$, it is enough to show that

$$\frac{\mu^2}{4}\left(1-\frac{\gamma\sigma^2}{8}\right) \geq \mu^2 \frac{\sqrt{2\sigma^2\gamma+1}-1}{4\sigma^2\gamma}, \tag{24}$$

which is clearly true.■

Proof of Lemma 3: Using $\Pi = \xi Q$, the entrepreneur's problem is

$$Q^{vc}, \alpha^{vc} = \arg\max_{Q,\alpha} \left[(1-\alpha)\,\mu Q - \frac{\gamma\sigma^2}{2}(1-\alpha)^2 Q^2\right] \text{ such that } Q^2 \leq \alpha Q\mu - \frac{\delta}{2}\alpha^2 Q^2\sigma^2. \tag{25}$$

The assumption of normal ξ requires that we impose some restrictions on model parameters. Namely, we require that at the optimal solution, (i) the entrepreneur's objective is decreasing in α, i.e., $\gamma\sigma^2(1-\alpha)\,Q < \mu$, and (ii) the VC's objective (the RHS of his participation constraint) is increasing in α, i.e., $\delta\sigma^2\alpha Q < \mu$. We assume for now that these inequalities hold at the optimum, and later verify that that is indeed the case as long as $\gamma\sigma^2 < 4$. Conditions (i) and (ii) imply that the VC's participation constraint must be binding at the optimal solution and, therefore, $Q^{vc}(\alpha) = \frac{\alpha\mu}{1+\frac{\delta}{2}\alpha^2\sigma^2}$. The entrepreneur's problem (25) then becomes

$$\alpha^{vc} = \arg\max_{\alpha} \left[(1-\alpha)\,\mu\frac{\alpha\mu}{1+\frac{\delta}{2}\alpha^2\sigma^2} - \frac{\gamma\sigma^2}{2}(1-\alpha)^2\left(\frac{\mu\alpha}{1+\frac{\delta}{2}\alpha^2\sigma^2}\right)^2\right]. \tag{26}$$

The first-order optimality condition for (26) is

$$\left(2-\delta\alpha^2\sigma^2-4\alpha\right)\left(1-\frac{\gamma\sigma^2(1-\alpha)\,\alpha}{1+\frac{\delta}{2}\alpha^2\sigma^2}\right) = 0. \tag{27}$$

Condition (i) above and the fact that $Q^{vc}(\alpha) = \frac{\alpha\mu}{1+\frac{\delta}{2}\alpha^2\sigma^2}$ together imply that at the optimal solution, $1-\frac{\gamma\sigma^2(1-\alpha)\alpha}{1+\frac{\delta}{2}\alpha^2\sigma^2} > 0$. Thus, the optimality condition (27) becomes $2-\delta\alpha^2\sigma^2-4\alpha = 0$. Using some algebra, we can obtain the expressions for α^{vc}, Q^{vc}, and U^{vc} given in Lemma 3. Finally, it is straightforward to verify that if $\gamma\sigma^2 < 4$, conditions (i) and (ii) are satisfied at α^{vc} and Q^{vc}.■

Proof of Proposition 2: Let $z \equiv \sigma^2\gamma \in (0,4)$ and let $\psi \equiv \delta/\gamma \in (0,1]$. We know from Lemma 3 that $U^{vc} = \frac{\mu^2}{2}\frac{\sqrt{2z\psi+4}-2}{z\psi}\left(1-\frac{\sqrt{2z\psi+4}-2}{4\psi}\right)$. We also know from Lemma 2 and its proof that $U^{ico} = \mu^2\frac{\sqrt{2z+1}-1}{4z}$if $z \geq \frac{5}{16}+\frac{3}{16}\sqrt{17}$ and $U^{ico} < \mu^2\frac{\sqrt{2z+1}-1}{4z}$ otherwise. Therefore, if we define

$$X_\psi(z) = \frac{\sqrt{2z+1}-1}{4} - \frac{\sqrt{2z\psi+4}-2}{2\psi}\left(1-\frac{\sqrt{2z\psi+4}-2}{4\psi}\right), \tag{28}$$

we have

$$\begin{aligned} X_\psi(z) < 0 &\Longleftrightarrow U^{ico} < U^{vc} \quad \text{if } z \geq \tfrac{5}{16}+\tfrac{3}{16}\sqrt{17}, \text{ and} \\ X_\psi(z) < 0 &\Longrightarrow U^{ico} < U^{vc} \quad \text{if } z < \tfrac{5}{16}+\tfrac{3}{16}\sqrt{17}. \end{aligned} \tag{29}$$

The equation $X_\psi(z) = 0$ has exactly one root in $(0,4)$, which is

$$\bar{z} = 2\left(4+5\psi+\psi^2-2\sqrt{1+5\psi+8\psi^2+4\psi^3}\right).$$

Furthermore, $X_\psi(0) = 0$, $X'_\psi(0) = 0$, $X''_\psi(0) < 0$, and $X_\psi(4) > 0$, so $X_\psi(z) > 0 \Longleftrightarrow z > \bar{z}$. Because $\bar{z} > \frac{5}{16}+\frac{3}{16}\sqrt{17}$, we have $U^{ico} > U^{vc} \Longleftrightarrow z > \bar{z}$. Therefore, for any $\delta \in (0,\gamma]$, $U^{ico} > U^{vc} \Longleftrightarrow \sigma > \sqrt{\bar{z}/\gamma}$.■

Proof of Lemma 4: Using the mean-variance form of the entrepreneur's expected utility, problem (11) can be written as

$$x^{vc}, Q^{vc}, \alpha^{vc} = \arg\max_{x,\alpha,Q}\left[\mathbb{E}\left((1-\alpha)\Pi + r_m x\right) - \frac{\gamma}{2}\mathrm{Var}\left((1-\alpha)\Pi + r_m x\right)\right], \tag{30}$$

It is straightforward to show that $x^{vc}(Q,\alpha) = \frac{\bar{r}_m}{\gamma\sigma_m^2} - (1-\alpha)Q\rho\sigma/\sigma_m$. Substituting $x^{vc}(Q,\alpha)$ into (30) and applying some algebra, the entrepreneur's choice of the optimal VC contract becomes

$$\begin{aligned} Q^{vc}, \alpha^{vc} &= \arg\max_{\alpha,Q}\left[(1-\alpha)Q\hat{\mu} - \frac{\gamma}{2}\hat{\sigma}^2(1-\alpha)^2Q^2 + \frac{1}{2}\frac{\bar{r}_m^2}{\gamma\sigma_m^2}\right] \\ \text{such that } Q^2 &\leq \alpha Q\hat{\mu} - \frac{\delta}{2}\alpha^2Q^2\hat{\sigma}^2. \end{aligned} \tag{31}$$

The problem (31) is the same as (25) except that μ and σ^2 are replaced by $\hat{\mu}$ and $\hat{\sigma}^2$, respectively, and the entrepreneur's objective function is increased by the additive constant $\frac{1}{2}\frac{\bar{r}_m^2}{\gamma\sigma_m^2}$.■

Proof of Lemma 5: Using the mean-variance form of the entrepreneur's expected utility, problem

(13) can be written as

$$x^{ico}(\alpha), Q^{ico}(\alpha) = \arg\max_{x,Q} \left[\mathbb{E}\left((1-\alpha)\Pi + x r_m + \alpha p - Q^2 \right) - \frac{\gamma}{2} \mathrm{Var}\left((1-\alpha)\Pi + x r_m \right) \right], \quad (32)$$

such that $Q^2 \leq \alpha p$. It is straightforward to show that $x^{ico}(Q,\alpha) = \frac{\bar{r}_m}{\gamma \sigma_m^2} - Q(1-\alpha)\rho\sigma/\sigma_m$. Substituting $x^{ico}(Q,\alpha)$ back into (32) and applying some algebra, the entrepreneur's choice of the optimal production investment becomes

$$Q^{ico}(\alpha) = \arg\max_{Q} \left[\hat{\mu} Q (1-\alpha) - Q^2 - \frac{\gamma}{2} Q^2 (1-\alpha)^2 \hat{\sigma}^2 + \alpha p + \frac{1}{2} \frac{\bar{r}_m^2}{\gamma \sigma_m^2} \right], \quad (33)$$

such that $Q^2 \leq \alpha p$. The optimal α maximizes the same objective function, except that p is now given by (12) and Q is given by (33), i.e.,

$$\alpha^{ico} = \arg\max_{\alpha} \left[\hat{\mu} Q - Q^2 - \frac{\gamma}{2} Q^2 (1-\alpha)^2 \hat{\sigma}^2 + \frac{1}{2} \frac{\bar{r}_m^2}{\gamma \sigma_m^2} \right], \quad (34)$$

where $Q = Q^{ico}(\alpha)$ given by (33). The problem formulation in (33)–(34) is the same as the one in (21) and (22), except that μ and σ^2 are replaced by $\hat{\mu}$ and $\hat{\sigma}^2$, respectively, and the entrepreneur's objective function is increased by the additive constant $\frac{1}{2} \frac{\bar{r}_m^2}{\gamma \sigma_m^2}$.■

Proof of Proposition 3: It follows from Lemmata 4 and 5, and the proof of Proposition 2 that for any $\delta \in (0,\gamma]$,

$$U^{ico} > U^{vc} \iff \hat{\sigma} > \sqrt{\bar{z}/\gamma},$$

where $\bar{z} = 2\left(4 + 5\psi + \psi^2 - 2\sqrt{1 + 5\psi + 8\psi^2 + 4\psi^3}\right)$ and $\psi \equiv \delta/\gamma$. Using $\hat{\sigma}^2 = \left(1-\rho^2\right)\sigma^2$, we can write

$$U^{ico} > U^{vc} \iff \rho^2 < 1 - \frac{\bar{z}}{\sigma^2 \gamma},$$

which completes the proof.■

Proof of Lemma 6: Let p_j be the token price during the ICO if investors believe the entrepreneur is of type j. We first consider the second-stage problem of choosing the optimal output contingent on the optinal investment in the market portfolio. Let Q_{ij} be the optimal output of type i that is perceived as type j by investors. Similarly to (33), we have

$$Q_{ij}(\alpha) = \arg\max_{Q} \left[(1-\alpha)\hat{\mu}_i Q - Q^2 - \frac{\gamma}{2}(1-\alpha)^2 Q^2 \hat{\sigma}^2 + \alpha p_j + \frac{1}{2}\frac{\bar{r}_m^2}{\gamma \sigma_m^2} \right], \tag{35}$$

such that $Q^2 \leq \alpha p_j$. Focusing on the interior solution, we have $Q_{ij} = \frac{\hat{\mu}_i(1-\alpha)}{2+\gamma(1-\alpha)^2\hat{\sigma}^2}$. Because the optimal output depends only on the firm's true type i, we can simplify notation by dropping the j-subscript from Q_{ij}, i.e., we let $Q_i = \frac{\hat{\mu}_i(1-\alpha)}{2+\gamma(1-\alpha)^2\hat{\sigma}^2}$ be the optimal output of type i.

The expected utility of type i that is perceived as type j is

$$U_{ij}(\alpha) = (1-\alpha)\hat{\mu}_i Q_i - Q_i^2 - \frac{\gamma}{2}(1-\alpha)^2 Q_i^2 \hat{\sigma}^2 + \alpha p_j + \frac{1}{2}\frac{\bar{r}_m^2}{\gamma \sigma_m^2}. \tag{36}$$

Using the fact that in equilibrium $p_j = \hat{\mu}_j Q_j$, and applying some algebra, this can be written as

$$U_{ij}(\alpha) = \frac{1}{2}\frac{(1-\alpha)}{2+\gamma\hat{\sigma}^2(1-\alpha)^2}\left(\hat{\mu}_i^2(1-\alpha) + 2\hat{\mu}_j^2\alpha\right) + \frac{1}{2}\frac{\bar{r}_m^2}{\gamma \sigma_m^2}. \tag{37}$$

Next, we show that $U_{LH}(\alpha)$ is unimodal in α, i.e., $\frac{d}{d\alpha}U_{LH}(\alpha)$ changes the sign from plus to minus exactly once in $(0,1)$. Because

$$\text{sign}\left(\frac{d}{d\alpha}U_{LH}(\alpha)\right) = \text{sign}\left((1-\alpha)\left(\gamma\hat{\sigma}^2\hat{\mu}_H^2 + 2\hat{\mu}_H^2 - 2\hat{\mu}_L^2 - \gamma\hat{\sigma}^2\alpha\hat{\mu}_H^2\right) - 2\alpha\hat{\mu}_H^2\right), \tag{38}$$

it is enough to show that the RHS of (38) is positive at $\alpha = 0$, negative at $\alpha = 1$, and decreasing in α, which is all clearly true. It can be similarly shown that $U_{HH}(\alpha)$ is unimodal.

Next, we prove that $\alpha_L = \alpha^{ico}$ and $\alpha_H = t$, where t is given by (17), is a SE, i.e., we show that t satisfies conditions (15)-(16), and $\alpha_L = \alpha^{ico}$ and $\alpha_H = t$ are the optimal actions given t. Let t be the smallest α that satisfies

$$U_{LH}(t) = U_{LL}\left(\alpha^{ico}\right) \tag{39}$$

$$\Leftrightarrow \frac{1}{2}\frac{(1-t)}{2+\gamma(1-t)^2\hat{\sigma}^2}\left(\hat{\mu}_L^2(1-t) + 2\hat{\mu}_H^2 t\right) = U_{LL}\left(\alpha^{ico}\right) - \frac{1}{2}\frac{\bar{r}_m^2}{\gamma\sigma_m^2}. \tag{40}$$

It is straightforward to verify that t given in (17) is the smaller of the two roots of (40), and that

$0 < t < \alpha^{ico}$. Because the unimodal function $U_{LH}(\alpha)$ is increasing in α for $\alpha \leq t$, we have

$$\max_{\alpha \leq t} U_{LH}(\alpha) = U_{LH}(t) = U_{LL}\left(\alpha^{ico}\right) = \max_{\alpha > t} U_{LL}(\alpha). \tag{41}$$

Thus, condition (16) is satisfied as equality, and $\alpha_L = \alpha^{ico}$ is an optimal action for the low type.

To show that condition (15) holds, we show that for any $\alpha > t$, we have

$$U_{HH}(t) > U_{HL}(\alpha). \tag{42}$$

Eq. (39) together with the fact that for any α, $U_{LL}\left(\alpha^{ico}\right) \geq U_{LL}(\alpha)$, implies that for any α, $U_{LH}(t) \geq U_{LL}(\alpha)$, i.e.,

$$\frac{1}{2}\frac{(1-t)}{2+\gamma(1-t)^2\hat{\sigma}^2}\left(\hat{\mu}_L^2(1-t)+2\hat{\mu}_H^2 t\right) \geq \frac{1}{2}\frac{(1-\alpha)}{2+\gamma(1-\alpha)^2\hat{\sigma}^2}\left(\hat{\mu}_L^2(1-\alpha)+2\hat{\mu}_L^2\alpha\right). \tag{43}$$

To prove (42), we need to show that for any $\alpha > t$, we have

$$\begin{aligned}
\frac{1}{2}\frac{(1-t)}{2+\gamma(1-t)^2\hat{\sigma}^2}\left(\hat{\mu}_H^2(1-t)+2\hat{\mu}_H^2 t\right) &> \frac{1}{2}\frac{(1-\alpha)}{2+\gamma(1-\alpha)^2\hat{\sigma}^2}\left(\hat{\mu}_H^2(1-\alpha)+2\hat{\mu}_L^2\alpha\right) & (44)\\
\Leftrightarrow \frac{1}{2}\frac{(1-t)}{2+\gamma(1-t)^2\hat{\sigma}^2}\left(\hat{\mu}_L^2(1-t)+2\hat{\mu}_H^2 t\right) &> \frac{1}{2}\frac{(1-\alpha)}{2+\gamma(1-\alpha)^2\hat{\sigma}^2}\left(\hat{\mu}_H^2(1-\alpha)+2\hat{\mu}_L^2\alpha\right) &\\
&\quad -\frac{1}{2}\frac{(1-t)}{2+\gamma(1-t)^2\hat{\sigma}^2}\left(\hat{\mu}_H^2(1-t)-\hat{\mu}_L^2(1-t)\right) & (45)
\end{aligned}$$

Using (43), to show (45), it is enough to show that for any $\alpha > t$, we have

$$\begin{aligned}
\frac{1}{2}\frac{(1-\alpha)}{2+\gamma(1-\alpha)^2\hat{\sigma}^2}\left(\hat{\mu}_L^2(1-\alpha)+2\hat{\mu}_L^2\alpha\right) &\geq \frac{1}{2}\frac{(1-\alpha)}{2+\gamma(1-\alpha)^2\hat{\sigma}^2}\left(\hat{\mu}_H^2(1-\alpha)+2\hat{\mu}_L^2\alpha\right) &\\
&\quad -\frac{1}{2}\frac{(1-t)}{2+\gamma(1-t)^2\hat{\sigma}^2}\left(\hat{\mu}_H^2(1-t)-\hat{\mu}_L^2(1-t)\right) & (46)\\
\Leftrightarrow \frac{(1-\alpha)^2}{2+\gamma(1-\alpha)^2\hat{\sigma}^2} &\leq \frac{(1-t)^2}{2+\gamma(1-t)^2\hat{\sigma}^2}. & (47)
\end{aligned}$$

Inequality (47) follows from the fact that $\frac{(1-\alpha)^2}{2+\gamma(1-\alpha)^2\hat{\sigma}^2}$ is decreasing in α for any $\gamma\hat{\sigma}^2$. Thus, condition (15) holds as a strict inequality, i.e., the high type strictly prefers to choose $\alpha \leq t$. The facts that $U_{HH}(\alpha)$ is unimodal and $t < \alpha^{ico}$ imply that the high type's optimal action is $\alpha_H = \arg\max_{\alpha \leq t} U_{HH}(\alpha) = t$.

Next, we prove that this SE is the unique LCSE. First, any belief threshold $\bar{t} > t$ cannot be a

SE. This is because $U_{LH}(\alpha)$ increases at t and thus $\max_{\alpha\leq\bar{t}} U_{LH}(\alpha) > \max_{\alpha\leq t} U_{LH}(\alpha) = U_{LL}\left(\alpha^{ico}\right)$, i.e., the low type would choose $\alpha \leq \bar{t}$, and the belief threshold $\bar{t}$ would not be self-consistent. Second, because $U_{HH}(\alpha)$ is increasing for $\alpha \leq t$, with any belief threshold $\bar{t} < t$, the high type would be worse off, i.e., $\max_{\alpha\leq\bar{t}} U_{HH}(\alpha) < U_{HH}(t)$. Thus, t is the unique *least-cost* SE belief threshold.■

Proof of Proposition 4: Given that U^{vc} of the high type is independent of θ, it is enough to prove that U^{ico} of the high type, which in the LCSE equals $U_{HH}(t)$, monotonically decreases in θ. Because $\frac{d}{d\theta}U_{HH}(t) = U'_{HH}(t)\frac{\partial t}{\partial\theta}$, it is enough to prove that (i) $\frac{\partial t}{\partial\theta} \leq 0$ and (ii) $U'_{HH}(t) \geq 0$. We prove (i) next. We know from the proof of Lemma 6 that t is the smaller of the two roots of

$$\begin{aligned}
\frac{1}{2}\frac{(1-\alpha)}{2+\gamma(1-\alpha)^2\hat{\sigma}^2}\left(\hat{\mu}_L^2(1-\alpha)+2\hat{\mu}_H^2\alpha\right) &= U_{LL}\left(\alpha^{ico}\right) - \frac{1}{2}\frac{\bar{r}_m^2}{\gamma\sigma_m^2} \Leftrightarrow \\
\frac{1}{2}\frac{(1-\alpha)}{2+\gamma(1-\alpha)^2\hat{\sigma}^2}\left(\hat{\mu}_L^2(1-\alpha)+2\hat{\mu}_H^2\alpha\right) &= \hat{\mu}_L^2\frac{\sqrt{2\hat{\sigma}^2\gamma+1}-1}{4\hat{\sigma}^2\gamma} \Leftrightarrow \\
\frac{(1-\alpha)(1-\alpha+2\theta\alpha)}{2+\gamma(1-\alpha)^2\hat{\sigma}^2} &= \frac{\sqrt{2\hat{\sigma}^2\gamma+1}-1}{2\hat{\sigma}^2\gamma}. \qquad (48)
\end{aligned}$$

Recall from the proof of Lemma 6 that $U_{LH}(\alpha) = \hat{\mu}_L^2\frac{1}{2}\frac{(1-\alpha)(1-\alpha+2\theta\alpha)}{2+\gamma\hat{\sigma}^2(1-\alpha)^2} + \frac{1}{2}\frac{\bar{r}_m^2}{\gamma\sigma_m^2}$ is unimodal in α, namely, it first increases and then decreases in $(0,1)$. Thus, the LHS of (48) must be similarly unimodal in α and, hence, it must be increasing in α at $\alpha = t$. Because the LHS of (48) is clearly increasing in θ, it follows that $\frac{\partial t}{\partial\theta} \leq 0$.

Next, we prove (ii). Because $U_{HH}(\alpha) = \frac{\hat{\mu}_H^2}{2}\frac{1-\alpha^2}{2+\gamma\hat{\sigma}^2(1-\alpha)^2} + \frac{1}{2}\frac{\bar{r}_m^2}{\gamma\sigma_m^2}$, we have $U'_{HH}(\alpha) \geq 0$ iff

$$\gamma\alpha^2\hat{\sigma}^2 - 2\gamma\alpha\hat{\sigma}^2 - 2\alpha + \gamma\hat{\sigma}^2 \geq 0.$$

Since $\alpha^{ico} = \frac{\hat{\sigma}^2\gamma+1-\sqrt{2\hat{\sigma}^2\gamma+1}}{\hat{\sigma}^2\gamma}$ is the smaller root of $\gamma\alpha^2\hat{\sigma}^2 - 2\gamma\alpha\hat{\sigma}^2 - 2\alpha + \gamma\hat{\sigma}^2 = 0$, we have $\gamma\alpha^2\hat{\sigma}^2 - 2\gamma\alpha\hat{\sigma}^2 - 2\alpha + \gamma\hat{\sigma}^2 \geq 0$ for $\alpha < \alpha^{ico}$. Thus, $U'_{HH}(\alpha) \geq 0$ for $\alpha < \alpha^{ico}$. Because $t < \alpha^{ico}$, we have $U'_{HH}(t) \geq 0$.■