Ownership Structure, Incentives, and Asset Prices*

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Abstract

We show how firms' ownership structures, managerial contracts, and asset prices interact in a dynamic equilibrium model. Our framework distinguishes a firm's small shareholders from its large shareholders, who play the role of mediators by determining managerial contracts, while also influencing asset prices through their dynamic trades. Agency conflicts between large shareholders and managers lead to more volatile and higher expected stock returns. Risk sharing between large shareholders and managers makes block ownership dynamics effectively insulated from fluctuations in firm-specific parameters. We derive a number of testable implications for the endogenous relations among ownership concentration, managerial incentives, and stock returns.

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1 Introduction

Traditional theoretical research in financial economics reveals a fundamental dichotomy between its two cornerstones; asset pricing and corporate finance (Gorton, He, and Huang 2014, hereafter GHH). Corporate finance research broadly focuses on how agency conflicts among a firm's various stakeholders influence its cash flows, but exogenously specifies the pricing kernel that determines the market's valuation of the cash flows. In contrast, traditional asset pricing models usually view corporate cash flows as exogenous, and focus on identifying a pricing kernel to price the assumed cash flows. In reality, however, firms' cash flows and their valuation by market participants are simultaneously and endogenously determined.

A nascent literature examines the feedback between the effects of agency conflicts among firms' stakeholders on cash flows and security prices that represent the values of the stakeholders' claims to the cash flows. One group of studies (e.g., Admati, Pfleiderer, and Zechner 1994, hereafter APZ; Huddart 1993; DeMarzo and Urošević 2006, hereafter DU; GHH) specifically focusses on the role that asset prices play in facilitating risk-sharing between small and large shareholders of firms and, thereby, the incentives of large shareholders to influence firms' earnings through their actions. These studies abstract away from the separation between ownership and control that characterizes modern corporations, as well as the resulting *non-market mechanisms* such as managerial contracts that play a central role in mediating the trade-off between risk-sharing and incentives. Another group of studies examines how contracts interact with asset prices (e.g., Ou-Yang 2005), but abstract away from the distinction between large and small shareholders. These studies confront the conceptual problem that stems from assuming that shareholders are competitive price-takers, but are nevertheless able to coordinate with each other to enforce managerial contracts.[1]

We contribute to the literature by developing a dynamic equilibrium model that bridges the gap between the frameworks analyzed by the aforementioned groups of studies. Our unified framework synthesizes an asset pricing model with a principal-agent model by distinguishing a firm's large shareholders from small shareholders. Large shareholders play the role of mediators who determine managerial contracts, while also influencing asset prices through their dynamic trading decisions.[2] We analytically characterize the equilibrium in which block ownership dynamics, managerial compensation, and asset prices endogenously reflect risk-sharing between large and small shareholders in asset markets, as well as the

[1] Corporate boards, who represent shareholders in negotiatiations with managers, are significantly influenced by large shareholders (see Shleifer and Vishny 1997; Tirole 2006).

[2] Our perspective is motivated by evidence that the vast majority of firms around the world have large shareholders with significant block ownership stakes. Holderness (2009) shows that 96% of randomly selected Compustat- and CRSP-listed firms have shareholders who own at least 5% of the firm's common stock ("blockholders"), and that 89% of S&P 500 firms have blockholders.

tension between risk sharing and incentive provision stemming from manager moral hazard. Our unified model generates novel testable implications for the endogenous relations among firms' ownership concentrations, stock returns, and managerial incentives.

In our continuous-time, infinite-horizon model, the representative all-equity firm has two groups of shareholders: small shareholders, who are competitive price-takers; and a representative large shareholder. The large shareholder hires the firm's manager who influences the firm's earnings through his unobservable effort. All agents save and consume over time with the manager's savings being observable. As in DU, the large shareholder trades the firm's stock at discrete dates, but cannot commit to her future trades, whereas small shareholders trade the stock continuously. The stock price is endogenously determined at each instant of time by the market clearing condition for the firm's equity.

The large shareholder offers the manager a long-term contract that can be renegotiated after each trade by the large shareholder. The large shareholder's trades influence the manager's contract that affects the manager's effort and the firm's output. The firm's earnings, in turn, influence the stock price and, thereby, the large shareholder's trades. In this manner, the large shareholder's trading dynamics, managerial incentives, and the stock price process are simultaneously and endogenously determined. We focus on Strong Markov Perfect Public Equilibria in which the large shareholder's ownership, consumption, and contracting choices are sequentially optimal both on- and off-equilibrium. In particular, the large shareholder's decisions in each period are determined by the state vector that comprises of her ownership at the beginning of the period, her money market account balance, and the manager's *promised payoff* or expected continuation utility from his long-term contract.

We analytically characterize the equilibrium when the firm's earnings process is Gaussian and all agents have CARA preferences as in DU. We compare the equilibrium with the equilibria in two benchmark models; (i) the "first-best" model in which the manager's effort is directly contractible so that his optimal contract purely reflects optimal risk-sharing between the large shareholder and the manager; and (ii) the "owner-manager" model studied by DU in which there is no separation between the large shareholder and the manager, that is, the large shareholder directly influences earnings through her effort choices that are, in turn, affected by her ownership stakes over time.

The equilibrium stock price varies *stochastically* in the first-best and second-best agency models in sharp contrast with the owner-manager model in which the stock price is *deterministic*. The stochastic stock price variation arises due to the random evolution of the manager's promised payoff in the agency models that reflects the separation between ownership and control, as well as risk-sharing between the large shareholder and the manager via his long-term contract.

For a given ownership stake of the large shareholder, the expected *dollar* return of the stock and the *dollar* stock return volatility are lower in our second-best agency model relative to the benchmark first-best agency model. A comparison between the second-best and first-best agency models isolates the effects of the tension between risk-sharing and incentive provision due to manager moral hazard. The expected dollar return and dollar return volatility are both lower in the second-best model because optimal incentive provision to the manager requires him to bear a greater portion of firm risk relative to the first-best scenario. As the stock price is determined by the residual earnings (that is, total earnings net of managerial pay), the higher volatility of managerial compensation payments in the second-best scenario effectively lowers the volatility of the dollar stock return, which also lowers the expected dollar return. The owner-manager model studied by DU differs significantly from the agency models that we analyze in that there is no separation between ownership and control. The presence of nontrivial risk-sharing between the large shareholder and the manager in the agency models implies that the residual risk borne by shareholders is highest in the owner-manager model. Consequently, the expected dollar return and dollar return volatility of the stock are highest in the owner-manager model.

We analytically characterize the large shareholder's ownership dynamics in the main second-best agency model and the two benchmark models. Interestingly, although the time-paths of the large shareholder's ownership stake differ across the three models, they converge to the same "steady state" limit. In all three models, the dynamics of the large shareholder's ownership stake reflect the interplay between two forces: risk sharing between the large and small shareholders, and the effects of the large shareholder's ownership on the manager's effort (in the two agency models) or on her own effort (in the owner-manager model). As one approaches the steady state in the three models, the large shareholder's incentives to influence effort provision through changes in her stake disappear. Consequently, the steady state ownership level purely reflects risk-sharing between the large and small shareholders.

We derive testable implications for the relations among the large shareholder's ownership dynamics, managerial incentives, and stock return characteristics. We analytically show that the sensitivity of the manager's pay to the stock return is inversely related to the expected dollar stock return, dollar stock return volatility, and Sharpe ratio. To obtain further implications of our model that cannot be pinned down analytically for general parameter values, we calibrate the model to match key relevant moments in the data that pertain to the main endogenous variables in our model; large shareholder (or block) ownership, stock return characteristics, and managerial pay.

The agency costs of risk sharing between the large shareholder and the manager have a negative impact on the expected dollar stock return and volatility, but also have a negative impact on the stock price relative to the benchmark owner-manager model. In the calibrated model, we show that the negative impact on the stock price actually outweighs the negative effects on the expected dollar return and volatility as compared to the owner-manager model. Consequently, the expected *percentage* stock return (the dollar stock return normalized by the stock price) and *percentage* stock return volatility are *higher* in our second-best agency model relative to the owner-manager model.

Our analysis of the calibrated model further shows that an increase in block ownership has a negative effect on the expected percentage stock return and stock return volatility (hereafter, the expected stock return and volatility). The large shareholder's ownership has *direct* and *indirect* effects on the expected stock return. The direct effect stems from the fact that an increase in the large shareholder's ownership stake reduces the stock's liquidity available to small shareholders, thereby increasing the current stock price and, thus, lowering the expected stock return. The indirect effect arises from the fact that an increase in the large shareholder's ownership stake increases the incentive power of the manager's compensation contract, thereby lowering the volatility of residual cash flows to shareholders and the expected stock return.

The result that an increase in the large shareholder's ownership stake increases the manager's pay-performance sensitivity suggests that block ownership and incentive contracts complement each other in corporate governance, which is consistent with empirical evidence (e.g., Hartzell and Starks 2003; Kim 2010). Further, the equilibrium correlations of block ownership with stock return characteristics as well as managerial incentives lead to the testable implications that the manager's pay-performance sensitivity is negatively related to the expected stock return and volatility. Our results highlight that empirical investigations of the relations among block ownership, managerial incentives, and stock returns should appropriately account for the fact that these variables are simultaneously and endogenously determined in equilibrium.

We also obtain intuitive results concerning the impact of underlying firm-specific parameters—the productivity and cost of managerial effort as well as firm risk—on the equilibrium. In our second-best agency model, the dynamics of the large shareholder's ownership stake are insensitive to the firm-specific parameters. That is, the separation of ownership and control, and the facilitation of risk-sharing through the manager's incentive contract, effectively insulate the large shareholder from variations in firm-specific parameters.In contrast, the large shareholder's ownership dynamics are significantly influenced by the firm-specific parameters in the benchmark owner-manager model. Our prediction that block ownership dynamics are insensitive to variations in firm-specific parameters due to the insulation provided by managerial incentive contracts potentially reconciles the empirical findings of Helwege, Pirinsky, and Stulz (2007). They find no evidence that variation in the intensity of agency conflicts, which are driven by variation in firm-specific parameters, influences the diffusion of block ownership stakes in the U.S.

As mentioned earlier, we contribute to the burgeoning literature that studies the interactions between agency conflicts and asset prices. APZ, DU, and GHH examine the interactions among the ownership decisions of large shareholders, their effort choices, and asset prices. They abstract away from the separation between ownership and control, and the role of non-marketed managerial incentive contracts in moderating the tradeoff between risk sharing and incentive provision. Ou-Yang (2005) examines how managerial contracts interact with asset prices in a contracting framework with lump-sum compensation at the terminal date as in Holmstrom and Milgrom (1987). We complement the above studies by incorporating the separation between large and small shareholders in asset markets as well as between large shareholders and managers through incentive contracts.

Our perspective on the role of large shareholders as mediators is motivated by prior theoretical research that highlights the importance of large shareholders in influencing corporate governance (e.g., Shleifer and Vishny 1986; Holmstrom and Tirole 1993; Burkart, Gromb, and Panunzi 1997). We build on the contributions of these studies by showing how governance through managerial incentive contracts influences the ownership stakes of large shareholders that are, in turn, affected by risk sharing between large and small shareholders in asset markets. Parigi, Pelizzon, and von Thadden (2015) theoretically and empirically show that the quality of corporate governance, which is endogenously chosen by the firm, correlates positively with the CAPM beta and idiosyncratic volatility, and negatively with returns on assets. Our study complements theirs by showing how two important governance mechanisms—block ownership and optimal contracting—endogenously affect stock prices and returns in an equilibrium framework.

Lastly, a branch of the asset pricing literature examines the optimal trading decisions of large shareholders, but takes the effects of their trades on asset prices as exogenous (e.g., Cuoco and Cvitanic 1998; Almgren and Chriss 2000; Subramanian and Jarrow 2001; Huberman and Stanzl 2005; Schied and Schöneborn 2008). These studies employ the optimal trading strategies of blockholders to characterize the *liquidity risk* of their holdings that stems from the price impact of their trades. In our framework, the effect of blockholder trades on asset prices is endogenous, and is determined not just by risk sharing between large and small shareholders, but also by the tension between risk sharing and incentive provision as embedded in managerial contracts. Our analysis, thereby, shows how the liquidity risk of large shareholders' holdings is endogenously influenced by manager moral hazard.

2 The Model

We consider an economy with a representative all-equity firm. The firm's shareholders are comprised of two groups: "large"shareholders or blockholders who hold a block of shares in the firm and a continuum of homogeneous small shareholders who collectively hold the remaining equity stake. Both groups of shareholders are risk averse with potentially differing risk aversions. Based on prior research on firm ownership (e.g., DeMarzo and Urošević [DU] 2006), and empirical evidence (e.g., Brav, Jiang and Thomas 2008; Cella, Ellul, and Giannetti 2013), we consider different holding periods for the two groups of investors. Small shareholders trade shares of the firm continuously, while the large shareholder trades the firm's shares at a discrete set of dates that are equally spaced for notational convenience.[3] Both types of investors also trade a risk-free bond (savings or money market account) continuously.

In reality, major strategic corporate decisions require the approval of corporate boards that are significantly influenced by large shareholders (e.g., see Shleifer and Vishny 1997; Tirole 2006). Cronqvist and Fahlenbrach (2009) document evidence for significant blockholder effects on investment, financial, and executive compensation policies. For simplicity, we ignore strategic behavior among different blockholders—that is, they behave as a monolithic unit in their collective interest—so that we refer to them as a single representative blockholder or large shareholder. The large shareholder hires a risk-averse manager to operate the firm who affects the firm's output through his costly, unobservable effort. The blockholder influences the manager's effort choices through a long-term incentive contract that is contingent on the firm's contractible output process. We now describe the various elements of the model in detail.

2.1 Firm Output Process

We consider a continuous-time model over the time horizon $[0, \infty)$ in which uncertainty is generated by a standard Brownian motion Z_t on a probability space $(\Omega, \mathcal{F}, P)$. The firm's cumulative output or cash flow X_t evolves as follows.

$$dX_t = \mu(a_t, t)dt + \sigma dZ_t, \tag{1}$$

[3] Cella, Ellul, and Giannetti (2013) show that institutional investors hold their positions in a company for periods that range between 14 months and 40 months with an average holding period of about 24 months. Brav, Jiang, and Thomas (2008) show that hedge fund activists, who hold more than 5% of a publicly traded company's shares with the intention of influencing its management, have an average holding period of 22 months.

where $a_t \in [0, A]$ with $0 < A < \infty$ is the manager's effort at date t and the constant $\sigma > 0$ is the output volatility. The output process is publicly observable. We denote the (complete and augmented) information filtration generated by the cumulative output process, X_t, by $\{\mathcal{F}_t\}$. All instantaneous cash flows net of the manager's compensation are paid out to the firm's shareholders as dividends.

2.2 Large and Small Shareholders

The firm has a representative large shareholder, L, and a continuum of small, dispersed shareholders, S (uniformly indexed over the unit interval, that is, $S \in [0, 1]$). We normalize the firm's total outstanding shares to one. As in DU, small shareholders trade continuously, while the large shareholder trades on the discrete set of dates $\{t_i;\ i = 1, 2, \dots, N\}$ with $t_1 = 0$ and $t_i - t_{i-1} = \Delta_t$.[4] We denote $t_{N+1} = \infty$. A small shareholder S's shareholding process, $\theta^S = \{\theta^S_t\}$, is an $\{\mathcal{F}_t\}$-adapted process, where θ^S_t is the number of shares held by S at date t. The large shareholder L's ownership process, $\Theta^L = \{\Theta^L_t\}$, is also an $\{\mathcal{F}_t\}$-adapted process, where Θ^L_t is L's holding at date t. L's and S's ownership levels are publicly observable.

L's shareholding process is piecewise constant (that is, it is constant between successive trading dates), and is right continuous with left limits. If L's shareholdings just prior to her trade at date t_i are $\Theta^L_{t_i^-}$, the difference $\Theta^L_{t_i} - \Theta^L_{t_i^-} > 0$ (< 0) represents the number of shares that she purchases (sells) at date t_i. As in DU, L *cannot commit* to her trading policy. Due to the market clearing condition for the firm's shares, the total number of shares collectively held by small shareholders at any date $t \in [t_i, t_{i+1})$ is $\int_0^1 \theta^S_t dS = 1 - \Theta^L_t = 1 - \Theta^L_{t_i}$.

Investors also have access to a risk-free bond or money market account that is in perfectly elastic supply so that it pays a continuously compounded constant return $r > 0$.[5] The investors are initially endowed with wealth, $Y^j_0 \geq 0$ for $j \in \{L, S\}$. Both groups of investors are risk averse and their preferences are described by the utility function, $u^j(c^j_t)$, that is twice continuously differentiable, strictly increasing, and strictly concave in the instantaneous consumption rate, c^j_t, at date t.

[4] As in DU, we assume a finite number (N) of trading dates for L in our analysis to facilitate a direct comparison with their results. We can extend our analysis to incorporate an infinite number of trading dates. Under additional transversality conditions that guarantee the finiteness of the agents' value functions, our main implications remain unchanged.

[5] We can alternatively assume that the risk-free bond is in zero net supply, which would endogenously determine the risk-free rate over time without altering our main results.

2.3 Contracting

The large shareholder hires a manager, M, to operate the firm and offers him a long-term contract. The contract can be renegotiated by both parties at any of L's trading dates $\{t_i;\, i = 1, 2, \dots, N\}$, provided that it is in the interests of both parties to do so. Both parties, however, commit to the terms of the contract over the trading period, $[t_i, t_{i+1})$. The time line of events in any trading period $[t_i, t_{i+1})$ is as follows. At trading date t_i, given her prior shareholdings $\Theta^L_{t_i^-} = \Theta^L_{t_{i-1}}$, L makes a trading decision $\Theta^L_{t_i}$. L and M then renegotiate the terms of their existing contract. In making her trade, L rationally anticipates its effects on the terms of the renegotiated contract, the manager's effort, as well as the firm's output and stock price. However, L cannot commit to the terms of the renegotiated contract *before* she makes her trade. Small shareholders S competitively and continuously trade shares of the firm—that is, they take the stock price as given—by rationally anticipating L's trading and contracting decisions. The stock price, P_t, at any time t is determined by market clearing for the firm's shares.

Because any contractual terms that are renegotiated in the future can be rationally incorporated in the original contract, we can, without loss of generality, restrict consideration to long-term contracts that are *renegotiation-proof* (RP) at each trading date t_i for $i = 1, 2, 3, \dots, N$ (see also Laffont and Martimort 2002). A long-term contract is RP at each date t_i if it is weakly Pareto optimal among the set of continuation contracts that are themselves RP at future trading dates $\{t_{i+1}, t_{i+2}, ...\}$ (see, for example, Wang 2000).

A long-term contract Π can be formally expressed as $\Pi \equiv \{c^M, a^M\}$, where we augment the definition of the manager's contract to include the recommended effort process, $\{a^M\}$, and his compensation process, $\{c^M\}$. M's effort and compensation processes are $\{\mathcal{F}_t\}$-adapted stochastic processes. The manager continuously chooses an unobservable effort level $a^M_t \in [0, A]$, given his contractual compensation, which affects the firm's output X_t as shown in (1). He incurs the instantaneous effort cost, $\Psi(a_t, t)$, that is twice continuously differentiable, increasing, and convex in the effort level, a^M_t. Throughout, we assume that the upper bound on M's effort, A, is large enough that M's optimal effort choices take interior values. M can also invest in the risk-free bond and the stock, but his savings are observable. Without loss of generality, therefore, we can restrict consideration to "savings proof" contracts in which M's contract directly specifies his consumption at each date. Therefore, the process c^M specified by the contract is M's consumption process.

The manager's total utility function, $u^M(c^M_t, a^M_t)$, is increasing and concave in consumption c^M_t, and decreasing and concave in effort a^M_t. The cost of effort is a monetary cost at the same unit of his consumption. The manager's *promised payoff* or *continuation value* at any date t—that is, his expected utility from the future consumption and effort under the contract Π—is given by

$$W_t^M(\Pi) \equiv E_t^{a^M}\left[\int_t^\infty e^{-\delta^M(\tau-t)}u^M(c_\tau^M, a_\tau^M)d\tau\right], \tag{2}$$

where $E_t^{a^M}[\cdot]$ denotes the conditional expectation at time t with respect to the probability distribution induced by M's effort choices a^M, and δ^M is M's time discount rate.

The contract must be incentive compatible for the manager at each time t. In other words, given the stream of future compensation c^M, it is optimal for the manager to choose the effort levels a^M specified by the contract. By (2),

$$\begin{aligned} a^M &= \arg\max_{\Pi' \equiv (c^M, a')} W_t^M(\Pi') \\ &= \arg\max_{a'} E_t^{a'}\left[\int_t^\infty e^{-\delta^M(\tau-t)}u^M(c_\tau^M, a'_\tau)d\tau\right]. \end{aligned} \tag{3}$$

Next, the contract must satisfy the manager's participation constraint at date zero, that is,

$$W_0^M(\Pi) \geq W_0, \tag{4}$$

where W_0 is M's reservation utility at date zero. A contract is *incentive feasible* if it is incentive compatible and satisfies the manager's participation constraint at date zero.

Finally, we characterize the renegotiation-proofness (RP) constraints. Let $\Pi_{[t_i,\infty)}$ be the restriction of the long-term contract Π to the period after date t_i, where t_i is a trading date for L. The RP constraints can be recursively characterized as follows (e.g., see Wang 2000; Sannikov 2008).

Renegotiation Proofness:

1. The contract, $\Pi_{[t_N,\infty)}$, over the last trading period $[t_N, \infty)$ is weakly Pareto optimal among all incentive compatible, continuation contracts, $\Pi'_{[t_N,\infty)}$.

2. At any trading date t_i $(i < N)$, the contract, $\Pi_{[t_i,\infty)}$, is weakly Pareto optimal among all incentive compatible, continuation contracts, $\Pi'_{[t_i,\infty)}$, that are themselves RP at future trading dates $\{t_{i+1}, ..., t_N\}$.

The key consequence of renegotiation proofness for our analysis is *sequential optimality,* that is, the manager's contract must be sequentially optimal at each trading date t_i for $i = 1, 2, 3, \ldots, N$.

To solve for the equilibrium fully, we assume the following for the rest of the paper. First, all the players have constant absolute risk aversion (CARA) preferences over their consumption, and their risk aversion parameters are γ^L, γ^M, and γ^S, respectively. More specifically, their utility functions are given by

$$
\begin{aligned}
u^M(c_t^M, a_t^M) &= -\frac{1}{\gamma^M} e^{-\gamma^M\left(c_t^M - \Psi(a_t^M)\right)}, \\
u^j(c_t^j) &= -\frac{1}{\gamma^j} e^{-\gamma^j c_t^j}\ , \quad \text{for } j = L, S.
\end{aligned} \tag{5}
$$

Second, the firm's mean cash flow and the manager's cost of exerting effort are given by the following stationary functions,

$$
\mu(a_t^M, t) = \mu(a_t^M) = \mu_0 + \mu_1 a_t^M;\ \Psi(a_t^M, t) = \Psi(a_t^M) = \frac{1}{2}\psi(a_t^M)^2, \tag{6}
$$

where the constants μ_0, μ_1, and ψ are positive.

2.4 The Objectives of Large and Small Shareholders

L chooses her ownership policy, Θ^L, her consumption policy, c^L, and the manager's incentive feasible RP contract, Π, to maximize her expected utility, that is, L solves

$$
\max_{(\Theta^L, c^L, \Pi)} E_0^{a^M}\left[\int_0^\infty e^{-\delta^L \tau} u^L(c_\tau^L) d\tau\right], \tag{7}
$$

where δ^L is L's time discount rate, and $E_0^{a^M}[\cdot]$ denotes the expectation with respect to the probability measure induced by the manager's effort process a^M specified by his contract. The above optimization program is subject to L's budget constraint.

To characterize L's budget constraint, we need to specify L's proceeds from trading at a trading date t_i *both on and off-equilibrium*. Let $\Theta_{t_i^-}^L$ and $\Theta_{t_i}^L$ be L's share ownership before and after trading, where $\Theta_{t_i^-}^L$ and $\Theta_{t_i}^L$ could be on or off the equilibrium path. We follow DU and Gorton et al. [GHH] (2014) by assuming that L's proceeds from trading are given by

$$
L\text{'s trading proceeds } = \left[\Theta_{t_i^-}^L - \Theta_{t_i}^L\right] P_{t_i}(\Theta^L), \tag{8}
$$

where $P_{t_i}(\Theta^L)$ is the stock price after trading. We characterize L's budget constraint by the evolution of her money market account balance as follows.

$$
\begin{aligned}
dB_t^L &= \left(rB_t^L - c_t^L\right) dt + \Theta_t^L \left(dX_t - c_t^M dt\right) - P_t(\Theta^L) d\Theta_t^L \\
&= \left(rB_t^L - c_t^L + \Theta_t^L(\mu(a_t^M) - c_t^M)\right) dt + \Theta_t^L \sigma dZ_t - P_t(\Theta^L) d\Theta_t^L,
\end{aligned} \tag{9}
$$

where the differential, $P_t(\Theta^L)d\Theta_t^L$, is nonzero only at L's trading dates t_i, and is given by

(8). At time $t \in [t_i, t_{i+1})$ between any two consecutive trading dates of L, during which her shareholdings remain the same, the budget constraint reflects a change in her risk-free money market account balance in a time interval $(t, t+dt)$ due to the instantaneous consumption and dividend payment. The dividend payment is the change in the firm's cumulative output net of M's instantaneous compensation payment. When L trades at each trading date t_i (that is, when she changes her shareholdings from $\Theta^L_{t_{i-1}}$ to $\Theta^L_{t_i}$), her money market account balance also changes due to the proceeds from trading shares. Note that the stock price is affected by L's subsequent ownership and contracting decisions, and that L incorporates the effects of her trades on the stock price in making these decisions. We make the following assumption about the observability of L's ownership, contracting, and consumption decisions.

Assumption 1 *L's ownership process, Θ^L, money market account balance, B^L, consumption process, c^L, and the manager's contract, Π, are publicly observable.*

As mentioned earlier, L cannot commit to her ownership policy. Further, the manager's contract must be sequentially optimal for L at each trading date t_i. In addition, as in GHH, we restrict consideration to *Strong Markov Perfect Public Equilibria (SMPPE)* by imposing the stronger restriction that L's ownership, contracting, and consumption decisions must be sequentially optimal at each trading date t_i both on and off the equilibrium path. In particular, the stock price, $P_t(\Theta^L)$, in (9) is the stock price at L's possibly off-equilibrium ownership choice, Θ^L, incorporating the fact that L's consumption policy c^L and continuation of M's contract Π are sequentially optimal given the ownership level, Θ^L. Consequently, L's ownership, contracting, and consumption decisions must solve

$$\max_{(\Theta^L_{[t_i,\infty)}, c^L_{[t_i,\infty)}, \Pi_{[t_i,\infty)})} E^{a^M}_{t_i}\left[\int_{t_i}^{\infty} e^{-\delta^L(\tau - t_i)} u^L(c^L_\tau) d\tau\right]. \tag{10}$$

In the above, $(\Theta^L_{[t_i,\infty)}, c^L_{[t_i,\infty)}, \Pi_{[t_i,\infty)})$ denotes the restriction of the vector of processes, (Θ^L, c^L, Π), to the interval $[t_i, \infty)$ with the understanding that L's decisions must be sequentially optimal *on or off the equilibrium path*, that is, for *any* past history.

Each small shareholder, S, chooses its consumption and ownership policies to maximize its expected utility taking the stock price process as given, and rationally anticipating L's trading and contract choices, that is, S solves

$$\max_{(\theta^S_{[t,\infty)}, c^S_{[t,\infty)})} E^{a^M}_{t}\left[\int_{t}^{\infty} e^{-\delta^S(\tau - t)} u^S(c^S_\tau) d\tau\right], \tag{11}$$

where δ^S is S's time discount rate. S's money market balance evolves according to

$$dB_t^S = (rB_t^S - c_t^S)dt + \theta_t^S(dX_t - c_t^M dt) - P_t d\theta_t^S. \tag{12}$$

As S changes its shareholdings continuously, its total wealth process, $Y_t^S = B_t^S + \theta_t^S P_t$, evolves as

$$dY_t^S = (rY_t^S - c_t^S)dt + \theta_t^S(dX_t - c_t^M dt + dP_t - rP_t dt). \tag{13}$$

In the above, we explicitly indicate the fact that S, unlike L, takes the stock price process P as given in making its trading decisions.

2.5 Equilibrium Characterization

An equilibrium of the model is described by the vector of processes

$$\left\{\left(\Theta^{L*}, B^{L*}, c^{L*}\right); \left(\theta^{S*}, Y^{S*}, c^{S*}\right); \Pi^*; P^*\right\} \tag{14}$$

where $\left(\Theta^{L*}, B^{L*}, c^{L*}\right)$ is the vector of processes representing L's share ownership, money market account balance, and consumption, respectively; $\left(\theta^{S*}, Y^{S*}, c^{S*}\right)$ is the vector of processes representing a small shareholder S's share ownership, total wealth, and consumption; Π^* is the manager's contract; and P^* is the stock price process.

The processes must satisfy the following equilibrium conditions.

1. L's share ownership, money market account balance, and consumption, $\left(\Theta^{L*}, B^{L*}, c^{L*}\right)$, as well as the manager's incentive feasible RP contract, Π^*, solve (10) subject to the budget constraint (9). In particular, L's decisions are sequentially optimal at each trading date t_i on or off the equilibrium path.

2. Each small shareholder S's ownership, total wealth, and consumption, $\left(\theta^{S*}, Y^{S*}, c^{S*}\right)$, solve (11) subject to the budget constraint (13).

3. The stock price process P^* clears the market at each time t, that is, $\int_0^1 \theta_t^{S*} dS = 1 - \Theta_t^{L*}$. In particular, the stock price process must clear the market on or off the equilibrium path.

3 The Equilibrium

We now derive the Strong Markov Perfect Public Equilibrium (SMPPE) of the economy. Because the agents' decisions are sequentially optimal given any past history, we can use dynamic programming to derive the equilibrium by backward induction as we discuss in detail in the Main Appendix. Specifically, we first characterize the equilibrium value functions and policies for the last trading period, $[t_N, t_{N+1}) = [T, \infty)$. We then proceed to the derivation of the equilibrium value functions and policies for earlier trading periods $[t_i, t_{i+1})$ with $i < N$.

3.1 Optimal Contracting

Consider any period $[t_i, t_{i+1})$. We first characterize L's optimal contracting and consumption decisions for the period for any level of L's ownership, Θ, chosen at the beginning of the period. L's consumption and contracting decisions solve (10) with $\Theta^L_{t_i} = \Theta$. As noted earlier in Section 2, we consider renegotiation-proof contracting so that L's contracting decisions must be sequentially optimal at each trading date t_i. The incorporation of the possibility of renegotation is consistent with the assumption that L cannot commit to her future trading policy at the outset.

The optimal contract for the manager over the period specifies the manager's instantaneous compensation and recommended effort level. Consequently, we can rewrite L's optimization program as

$$W^L_{t_i} = \max_{c^L_{[t_i,\infty)}, c^M_{[t_i,\infty)}, a^M_{[t_i,\infty)}} E^{a^M}_{t_i}\left[\int_{t_i}^{\infty} e^{-\delta^L(\tau - t_i)} u^L(c^L_\tau) d\tau\right], \tag{15}$$

$$= \max_{c^L_{[t_i,t_{i+1})}, c^M_{[t_i,t_{i+1})}, a^M_{[t_i,t_{i+1})}} E^{a^M}_{t_i}\left[-\frac{1}{\gamma^L}\int_{t_i}^{t_{i+1}} e^{-\delta^L(\tau - t_i) - \gamma^L c^L_\tau} d\tau + W^L_{t_{i+1}}\right], \tag{16}$$

where $W^L_{t_{i+1}}$is L's maximum expected utility or continuation value at t_{i+1} given that she makes future decisions optimally. The above optimization program is subject to L's budget constraint (9), M's incentive compatibility constraint (3), and the renegotiation-proofness constraint,

$$(RP):\ W^M_{t_i}(\Pi_{[t_i,\infty)}) \geq W^M_{t_i}, \tag{17}$$

where $W^M_{t_i}(\Pi_{[t_i,\infty)})$ is the manager's expected utility or continuation value at date t_i from the contract, $\Pi_{[t_i,\infty)} = (c^M_{[t_i,\infty)}, a^M_{[t_i,\infty)})$, as defined in (2), and $W^M_{t_i}$ is the manager's promised payoff from the original contract (that is subject to renegotiation) at the beginning of the period. The above constraint at date 0 is simply M's participation constraint (4), that is, W^M_0 on the R.H.S. above is simply W_0.

Following the dynamic contracting literature (see the survey by Cvitanic and Zhang 2013), the contracting problem can be characterized recursively if we take the manager's promised payoff, W^M_t, as an additional state variable whose value evolves according to the following stochastic process:

$$dW_t^M = \left(\delta^M W_t^M - u^M(c_t^M, a_t^M)\right) dt + \chi_t^M \sigma dZ_t. \tag{18}$$

In the above, χ_t^M is a $\{\mathcal{F}_t\}$-adapted process that represents the sensitivity of the manager's promised payoff to the exogenous shock in the firm's output and plays a key role in the provision of incentives.

Under technical conditions that are satisfied when L and M have CARA preferences, the incentive compatibility constraint, (3), for the manager can be replaced by the following local incentive compatibility constraint (e.g., see Sannikov 2008, He 2009, Williams 2009):

$$\chi_t^M = -\frac{u_a^M(c_t^M, a_t^M)}{\mu'(a_t^M)} = \frac{\Psi'(a_t^M)}{\mu'(a_t^M)} H(c_t^M, a_t^M) > 0, \tag{19}$$

where $H(c_t^M, a_t^M) \equiv e^{-\gamma^M(c_t^M - \Psi(a_t^M))}$. Specifically, the above condition is the manager's first-order condition for his optimal effort. Increasing the effort level negatively affects his instantaneous utility captured by $u_a^M(c_t^M, a_t^M)$, but positively affects the drift of the firm earnings, which in turn increases his promised payoff by $\chi_t^M \mu'(a_t^M)$. Incentive compatibility requires that the marginal benefit must equal the marginal effort cost. We combine the local IC condition (19) with (18) for the dynamics of M's promised payoff, W_t^M. The manager's contract is publicly observable so that we can assume (without loss of generality) that his promised payoff, W^M, is publicly observable.

As we now show in the following proposition, L's optimal consumption and contracting problem is completely characterized by the state vector, (t, B_t^L, W_t^M). We provide the detailed proofs of all propositions in the main Appendix and the Internet Appendix.

Proposition 1 (Large Shareholder's Optimal Policies for Given Ownership Level) *Let Θ be any (on- or off-equilibrium) level of L's ownership at $t \in [t_i, t_{i+1})$. Suppose that M's promised payoff at date t is W_t^M. L's maximum expected utility or value function at time t is given by*

$$W_t^L = -\frac{1}{\gamma^L r} e^{-\gamma^L \left[r(B_t^L + G(t,\Theta)) + \frac{\Theta}{\gamma^M} \ln(-W_t^M) + \frac{\delta^L - r}{\gamma^L r}\right]}. \tag{20}$$

L's optimal consumption as well as M's optimal effort and compensation are

$$\begin{aligned} c_t^L &= r(B_t^L + G(t,\Theta)) + \frac{\Theta}{\gamma_M} \ln(-W_t^M) + \frac{\delta^L - r}{\gamma^L r}, \\ a_t^M &= \alpha, \\ c_t^M &= -\frac{1}{\gamma^M} \ln \beta - \frac{1}{\gamma^M} \ln(-W_t^M) + \Psi(a_t^M), \end{aligned} \tag{21}$$

where α and β are determined by the following system of equations:

$$\beta = \gamma^M r \left[\gamma^M \alpha(\psi\alpha - \mu_1) + 1\right], \tag{22}$$

$$\left(1+\frac{\gamma^L}{\gamma^M}\Theta\right)\left(\frac{\psi}{\mu_1}\alpha\beta\sigma\right)^2 - (\gamma^L r\Theta\sigma)\left(\frac{\psi}{\mu_1}\alpha\beta\sigma\right) - \left(r - \frac{\beta}{\gamma^M}\right) = 0. \tag{23}$$

In (20), the time-deterministic function, G, is L's certainty-equivalent payoff given her ownership policy, and satisfies the recursion,

$$G(t,\Theta) = \phi_i(t)V(\Theta) + (1 - r\phi_i(t))G(t_{i+1},\Theta);\ \lim_{t\to\infty} G(t,\Theta) = 0. \tag{24}$$

In the above, $\phi_i(t)$ is defined by

$$\phi_i(t) = \frac{1}{r}\left(1 - e^{-r(t_{i+1}-t)}\right), \tag{25}$$

and the net benefit flow to L from holding stake Θ, $V(\Theta)$, is given by

$$V(\Theta) = \Theta\left(\mu(\alpha) - \left(-\frac{1}{\gamma^M}\ln(\beta) - \frac{1}{\gamma^M r}\left(\mu_W - \frac{1}{2}\sigma_W^2\right) + \Psi(\alpha)\right)\right) - \frac{1}{2}\gamma^L r\Theta^2\left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)^2, \tag{26}$$

$$\text{with } \ \mu_W \equiv \delta^M - \frac{\beta}{\gamma^M};\ \sigma_W \equiv \frac{\psi}{\mu_1}\alpha\beta\sigma. \tag{27}$$

In the above, μ_W and σ_W are the drift and volatility in M's promised payoff process, W_t^M, specified by (18). The drift and volatility are determined by the contractual parameter, α, which is M's optimal effort level, and the parameter, β, that determines the fixed component of M's compensation (see (21)). These parameters, α and β, are constant over the time period $[t_i, t_{i+1})$, and depend nonlinearly on L's ownership level, Θ, during the period. The evolution of the manager's promised payoff process, (18), is described by the following corollary.

Corollary 1 (Manager's Promised Payoff Process)
M's promised payoff process W_t^M follows a geometric Brownian motion,

$$d\ln(-W_t^M) = \left(\mu_W - \frac{1}{2}\sigma_W^2\right)dt - \sigma_W dZ_t, \tag{28}$$

where μ_W and σ_W are defined by (27).

The manager's dollar-dollar pay-performance sensitivity (PPS) with respect to the firm's cash flows, PPS_X, is the dollar change in M's pay for a dollar change in the firm's cumulative cash flows X_t. From (21) and (28), we have

$$\begin{aligned}
dc_t^M &= -\frac{1}{\gamma^M} d\ln(-W_t^M) \\
&= -\frac{1}{\gamma^M}\left(\mu_W - \frac{1}{2}\sigma_W^2\right)dt + \frac{\sigma_W}{\gamma^M} dZ_t \\
&= -\frac{1}{\gamma^M}\left[\frac{\sigma_W}{\sigma}\mu(\alpha) + \left(\mu_W - \frac{1}{2}\sigma_W^2\right)\right]dt + \frac{\sigma_W}{\gamma^M \sigma} dX_t, \qquad (29)
\end{aligned}$$

where we use (1) and (6) to obtain the last equality above. The manager's dollar-dollar PPS with respect to the firm's cash flows is, therefore,

$$PPS_X = \frac{dc_t^M}{dX_t} = \frac{\sigma_W}{\gamma^M \sigma} = \frac{1}{\gamma^M}\left(\frac{\psi}{\mu_1}\alpha\beta\right), \qquad (30)$$

where the last equality follows from the definition of σ_W in (27). The dollar-dollar PPS is positively associated with the volatility term of the manager's promised payoff process, σ_W. As the manager's contract terms α and β depend on L's ownership level, Θ, M's incentives with respect to the firm's cash flows also depend on L's ownership level. Note that M's optimal effort, α, satisfies a sixth degree polynomial equation obtained from (22) and (23). Because it is, in general, impossible to analytically characterize the solutions of a sixth degree polynomial equation by the Abel-Ruffini theorem, the relation between M's incentives and L's ownership level is difficult to pin down analytically. We thus calibrate the model in Section 5 and numerically analyze the relation between L's ownership level and M's incentives.

3.2 Large Shareholder Ownership, and Stock Prices and Returns

We now analyze a representative small shareholder's portfolio choice problem and, thereby, derive the stock price for a given level of L's ownership. Note that, as small shareholders are homogeneous, each small shareholder's problem is identical. The representative small shareholder, S, makes its portfolio choice by taking the stock price as given, but rationally anticipates L's ownership stake and her optimal decision on the manager's contract. The following proposition characterizes the stock market equilibrium derived from S's optimal portfolio and consumption problem and the stock market clearing condition.

Proposition 2 (Stock Price for Given Large Shareholder Ownership)
Given any (on- or off-equilibrium) level of L's ownership, Θ, and M's promised utility, W_t^M,

at time $t \in [t_i, t_{i+1})$, the stock price is given by

$$P(t, \Theta, W_t^M) = \Lambda(t, \Theta) + \frac{1}{\gamma^M r} \ln(-W_t^M), \tag{31}$$

where

$$\Lambda(t, \Theta) = \phi_i(t) k(\Theta) + (1 - r\phi_i(t)) \Lambda(t_{i+1}, \Theta); \ \lim_{t \to \infty} \Lambda(t, \Theta) = 0, \tag{32}$$

$$k(\Theta) = \mu(\alpha) - \left(-\frac{1}{\gamma^M} \ln \beta - \frac{1}{\gamma^M r} \left(\mu_W - \frac{1}{2} \sigma_W^2 \right) + \Psi(\alpha) \right) - \gamma^S r (1 - \Theta) \left(\sigma - \frac{\sigma_W}{\gamma^M r} \right)^2. \tag{33}$$

The excess dollar return for holding a share of the firm's stock within the time interval $(t, t + dt)$, which includes the instantaneous dividend payment and capital gain, is

$$dR_t = dX_t - c_t^M dt + dP_t - rP_t dt = \mu_R(\Theta) dt + \sigma_R(\Theta) dZ_t, \tag{34}$$

where the expected dollar return and dollar return volatility are

$$\mu_R(\Theta) = (1 - \Theta) \gamma^S r \left(\sigma - \frac{\sigma_W}{\gamma^M r} \right)^2; \ \sigma_R(\Theta) = \left(\sigma - \frac{\sigma_W}{\gamma^M r} \right) > 0. \tag{35}$$

The Sharpe ratio is, by definition,

$$\Sigma_R(\Theta) = \frac{\mu_R(\Theta)}{\sigma_R(\Theta)} = (1 - \Theta) \gamma^S r \left(\sigma - \frac{\sigma_W}{\gamma^M r} \right). \tag{36}$$

From (28) and (31), we see that the stock price is stochastic as it depends on the manager's promised payoff that evolves stochastically. By (35), there is a positive relation between the stock return characteristics,

$$\mu_R(\Theta) = (1 - \Theta) \gamma^S r \sigma_R^2(\Theta). \tag{37}$$

The above relation arises because the collective demand for the stock by small shareholders with CARA preferences is competitively determined by the expected stock returns adjusted for their risk premium, $\mu_R / (\gamma^S r \sigma_R^2)$, which must equal the supply of the shares available to small shareholders, $1 - \Theta$, to ensure stock market clearing.

Note from (37) that L's ownership level, Θ, has a *direct* and *indirect* influence on the expected dollar return, μ_R. Her ownership stake reduces the stock's liquidity available to small shareholders, thereby increasing the current stock price by lowering the supply of shares. The decline in the supply of shares lowers the expected dollar return as shown by the term $(1 - \Theta)$. By Proposition 1 and Corollary 1, however, L's ownership also affects the mean and volatility of the manager's promised payoff process and compensation payments that, in turn, influence the volatilities of the dividend payments and stock returns. The net effects are difficult to pin down analytically for general parameter values so that we explore them by numerically analyzing the calibrated model in Section 5.

As shown by Propositions 1 and 2, the manager's optimal compensation payment c_t^M and the equilibrium stock price P_t are negatively and positively related to the state variable, $\ln(-W_t^M)$, respectively. These relations, in turn, suggest a negative relation between CEO pay and the contemporaneous stock price. The intuition is that an increase in the manager's compensation lowers the residual payoffs to shareholders that, in turn, decreases the stock price.

Combining (34) with (29), we obtain the sensitivity of managerial pay to the dollar return:

$$PPS_R = \frac{dc_t^M}{dR_t} = \frac{(\sigma_W/\gamma^M)dZ_t}{\sigma_R(\Theta)dZ_t} = \frac{r}{\gamma^M r(\sigma/\sigma_W) - 1}. \tag{38}$$

By (35), (36), and (38), the expected excess dollar return, dollar return volatility, and Sharpe ratio of the stock decrease with the volatility of M's promised payoff, σ_W, but the managerial pay-stock return sensitivity increases. We immediately obtain the following corollary.

Corollary 2 (Pay-Performance Sensitivity and Stock Return)
If $\sigma_W'(\Theta) \geq 0$, the sensitivities of managerial pay to the firm's cash flows and to the dollar return are negatively correlated with the expected excess dollar return, return volatility, and Sharpe ratio of the stock.

As we discussed earlier, the volatility of M's promised payoff, σ_W, depends on L's ownershp level Θ. If σ_W increases with L's ownership level, then an increase in L's ownership imposes more risk on the manager. In this scenario, the sensitivity of M's pay to firm's cash flows and to the dollar return, PPS_X and PPS_R, are negatively correlated with the expected excess dollar return, return volatility, and Sharpe ratio of the stock as the corollary states. In Section 5, we confirm that σ_W, indeed, increases with L's ownership level in the calibrated model.

3.3 Large Shareholder's Optimal Ownership Policy

We now discuss L's optimal ownership choice at each trading date t_i. Recall that L's ownership process is piecewise constant, that is, her holdings in the firm are constant over a trading period, $[t_i, t_{i+1})$. L's ownership decision for period $[t_i, t_{i+1})$ is made at t_i^- when she holds $\Theta_{t_i^-}^L = \Theta_{t_{i-1}}^L$ shares in the firm and her money market balance is $B_{t_i^-}^L$. By (20), L's value function after she chooses the new equity stake Θ at t_i is

$$W_{t_i}^L = -\frac{1}{\gamma^L r} e^{-\gamma^L \left[r(B_{t_i}^L + G(t_i,\Theta)) + \frac{\Theta}{\gamma^M} \ln(-W_{t_i}^M) + \frac{\delta^L - r}{\gamma^L r} \right]}, \tag{39}$$

so that L's optimal ownership choice Θ maximizes $B_{t_i}^L + G(t_i,\Theta) + \frac{\Theta}{\gamma^M r} \ln(-W_{t_i}^M)$. As noted earlier, her money market account balance changes from $B_{t_i^-}^L$ to $B_{t_i}^L$ at t_i by the proceeds from trading shares as below:

$$B_{t_i}^L = B_{t_i^-}^L + (\Theta_{t_{i-1}}^L - \Theta) P(t_i, \Theta, W_{t_i}^M). \tag{40}$$

L's optimal ownership choice for the period $[t_i, t_{i+1})$, given her current equity stake, $\Theta_{t_{i-1}}^L$, thus solves

$$\Theta_{t_i}^L = \arg\max_{\Theta} \left[B_{t_i^-}^L + (\Theta_{t_{i-1}}^L - \Theta) P(t_i, \Theta, W_{t_i}^M) + G(t_i,\Theta) + \frac{\Theta}{\gamma^M r} \ln(-W_{t_i}^M) \right]. \tag{41}$$

The following proposition describes L's optimal ownership path.

Proposition 3 (Large Shareholder's Ownership Path)
Given her current shareholdings, $\Theta_{t_i^-}^L = \Theta_{t_{i-1}}^L$, L chooses her optimal ownership level, $\Theta_{t_i}^{L*}$, for period $[t_i, t_{i+1})$ that satisfies the following first order condition:

$$FOC: \ (\Theta_{t_{i-1}}^L - \Theta_{t_i}^{L*}) \frac{\partial \Lambda(t_i, \Theta_{t_i}^{L*})}{\partial \Theta} + \phi_i(t_i) \left[\gamma^S - (\gamma^L + \gamma^S) \Theta_{t_i}^{L*} \right] r \sigma_R^2(\Theta_{t_i}^{L*}) = 0, \tag{42}$$

where Λ and σ_R are given by (32) and (35).

From the above, we see that L's optimal ownership choices are determined by two "forces." The first force is L's risk-sharing with S, which causes L's ownership choice to depend on the risk aversions of L and S. The second force is L's costly risk-sharing with M through M's optimal incentive contract. In particular, as shown by Proposition 1, L's ownership stake reflects its effects on the power of incentives provided to the manager and, thereby, the effort exerted by the manager. The manager's effort and compensation in turn influence the expected residual cash flows to shareholders and, therefore, the stock price via the time-deterministic component, Λ, as shown by (31), (32), and (33). In particular, Λ represents the present value of the marginal benefit flows, $k(\Theta)$ in (33), to small shareholders from holding an additional share of the firm. The marginal benefit flow incorporates the agency costs (captured by the expected CEO pay), as well as the effects of the incentive intensity of the manager's contract on the volatility of the residual cash flows to shareholders and, therefore, on the variance of the dollar return of the stock, σ_R^2.

As L's ownership stake converges to its steady state level, however, the sizes of L's trades decline. Hence, the effects of the second force above—incentive provision to M—on L's ownership choices decline. In the steady state, L's ownership level is then determined solely by the first force above; namely, risk-sharing between L and S.

Corollary 3 (Steady State Equilibrium)
In the steady state, L's optimal ownership stake is

$$\Theta_{ss}^{L*} = \frac{\gamma^S}{\gamma^S + \gamma^L}. \tag{43}$$

L's optimal consumption and contract choices are given by Proposition 1 with L's ownership level equal to Θ_{ss}^{L*}. The stock price is given by

$$P(t, \Theta_{ss}^{L*}, W_t^M) = \frac{1}{r} k(\Theta_{ss}^{L*}) + \frac{1}{\gamma^M r} \ln(-W_t^M). \tag{44}$$

From (44), we note that the stock price in the steady state equilibrium is stochastic through its dependence on the manager's promised payoff, W_t^M. In the steady state, L's ownership level is constant through time, but incentive provision to M causes the manager's promised payoff to evolve stochastically.

4 Benchmark Models

In this section, we compare our main model with two benchmark models to further understand how the agency conflict between L and M affects the properties of the equilibrium.

4.1 Benchmark One: The First-Best Scenario

We begin by analyzing the first-best benchmark in which the manager's effort choices are observable and contractible. As in the main model, the Strong Markov Public Perfect Equilibria (SMPPE) of the first-best benchmark can be expressed in terms of the state vector, (t, B_t^L, W_t^M). Because M's effort level is directly contractible, there is no tension between risk-sharing and incentive provision in the contract between L and M, that is, M's contract purely reflects optimal risk-sharing between L and M. Comparing the first-best benchmark with our second-best agency contracting model, therefore, highlights the effects of optimal incentive provision due to the non-contractibility of managerial effort in the contracting model.

As M is always committed to exert the effort level desired by L, L can freely choose the sensitivity of M's promised payoff, χ_t^M, without being subject to any incentive compatibility constraints for M. The following proposition characterizes L's value function as well as her optimal consumption and contracting decisions given her ownership stake.

Proposition 4 (Large Shareholder's Optimal Policy in First-Best Benchmark) *Given any (on- or off-equilibrium) level of L's ownership stake Θ, her optimal value function at any date t is*

$$W_t^L = -\frac{1}{\gamma^L r} e^{-\gamma_L \left[r(B_t^L + \widehat{G}(t,\Theta)) + \frac{\Theta}{\gamma^M} \ln(-W_t^M) + \frac{\delta^L - r}{\gamma^L r} \right]}. \tag{45}$$

The optimal policy variables are given by

$$\begin{aligned}
\hat{c}_t^L &= r(B_t^L + \widehat{G}(t,\Theta)) + \frac{\Theta}{\gamma^M} \ln(-W_t^M) + \frac{\delta^L - r}{\gamma^L r}, \\
\hat{a}_t^M &= \frac{\mu_1}{\psi}, \\
\hat{c}_t^M &= -\frac{1}{\gamma^M} \ln(\gamma^M r) - \frac{1}{\gamma^M} \ln(-W_t^M) + \Psi(\hat{a}_t^M), \\
\hat{\chi}_t^M &= \frac{\gamma^L r \Theta}{1 + \frac{\gamma^L}{\gamma^M}\Theta} (-W_t^M).
\end{aligned} \tag{46}$$

In the above, L's time-deterministic certainty-equivalent payoff $\widehat{G}$ satisfies the recursion

$$\widehat{G}(t,\Theta) = \phi_i(t)\widehat{V}(\Theta) + (1 - r\phi_i(t))\widehat{G}(t_{i+1},\Theta); \ \lim_{t\to\infty} \widehat{G}(t,\Theta) = 0, \tag{47}$$

where $\phi_i(t)$ is defined in (25), and the net benefit flow to L from holding Θ is

$$\widehat{V}(\Theta) = \Theta \left(\mu(\hat{a}_t^M) - \left(-\frac{1}{\gamma^M} \ln(\gamma^M r) - \frac{1}{\gamma^M r} \left(\hat{\mu}_W - \frac{1}{2}\hat{\sigma}_W^2 \right) + \Psi(\hat{a}_t^M) \right) \right) - \frac{1}{2}\gamma^L r \Theta^2 \left(\sigma - \frac{\hat{\sigma}_W}{\gamma^M r} \right)^2. \tag{48}$$

In the above,

$$\hat{\mu}_W \equiv \delta^M - r; \ \hat{\sigma}_W \equiv \frac{\gamma^L r \Theta}{1 + \frac{\gamma^L}{\gamma^M}\Theta} \sigma. \tag{49}$$

Comparing Proposition 4 with Proposition 1, we see that L's value function has the same functional form as in the second-best agency contracting case, but with a different certainty equivalent payoff that satisfies the recursion (47) with (48). Note that, given the direct contractibility of the manager's effort, its first-best optimal level is determined by two parameters; the productivity of the manager's effort, μ_1, and the unit cost of effort, ψ. By comparing the second-best and first-best volatilities of M's promised payoff, σ_W in (27) and $\hat{\sigma}_W$ in (49), we can see that the first-best volatility, unlike the second-best volatility, does not depend on M's effort level. This reflects the fact that there is no tradeoff between risk-sharing and incentive provision in the first-best setting. The contract is, therefore, determined to achieve optimal risk-sharing between L and M as reflected in their respective relative risk aversions in the volatility $\hat{\sigma}_W$. In particular, the first-best volatility incereases with L's ownership level Θ, that is, L's risk sharing incentives increase as L holds a higher equity stake.

As in our main model, M's promised payoff process W_t^M follows a geometric Brownian motion,

$$d\ln(-W_t^M) = \left(\hat{\mu}_W - \frac{1}{2}\hat{\sigma}_W^2\right)dt - \hat{\sigma}_W dZ_t, \tag{50}$$

where $\hat{\mu}_W$ and $\hat{\sigma}_W$ are given by (49).

The following proposition characterizes the equilibrium stock price in the first-best scenario. As in the analysis of the main agency contracting model in Section 3, we derive the equilibrium stock price and return characteristics by analyzing a representative small shareholder's optimal portfolio and consumption problem as well as the stock market clearing condition.

Proposition 5 (Stock Price Process in First-Best Benchmark)
Given any (on- or off-equilibrium) level of L's ownership, Θ, and M's promised utility, W_t^M, at time $t \in [t_i, t_{i+1})$, the stock price is given by

$$\widehat{P}(t, \Theta, W_t^M) = \widehat{\Lambda}(t, \Theta) + \frac{1}{\gamma^M r}\ln(-W_t^M), \tag{51}$$

where $\phi_i(t)$ is defined in (25), and

$$\widehat{\Lambda}(t, \Theta) = \phi_i(t)\widehat{k}(\Theta) + (1 - r\phi_i(t))\widehat{\Lambda}(t_{i+1}, \Theta); \ \lim_{t\to\infty}\widehat{\Lambda}(t, \Theta) = 0, \tag{52}$$

$$\widehat{k}(\Theta) = \mu(\widehat{a}_t^M) - \left(-\frac{1}{\gamma^M}\ln(\gamma^M r) - \frac{1}{\gamma^M r}\left(\widehat{\mu}_W - \frac{1}{2}\widehat{\sigma}_W^2\right) + \Psi(\widehat{a}_t^M)\right) - \gamma^S r(1-\Theta)\left(\sigma - \frac{\widehat{\sigma}_W}{\gamma^M r}\right)^2. \tag{53}$$

The first-best excess dollar return for holding a share of the firm's stock within the time interval $(t, t + dt)$ is

$$d\widehat{R}_t = dX_t - \hat{c}_t^M dt + d\widehat{P}_t - r\widehat{P}_t dt = \widehat{\mu}_R(\Theta)dt + \widehat{\sigma}_R(\Theta)dZ_t, \tag{54}$$

where

$$\widehat{\mu}_R(\Theta) = (1-\Theta)\,\gamma^S r\left(\sigma - \frac{\hat{\sigma}_W}{\gamma_M r}\right)^2;\ \widehat{\sigma}_R(\Theta) = \left(\sigma - \frac{\hat{\sigma}_W}{\gamma^M r}\right), \tag{55}$$

and the Sharpe ratio is

$$\widehat{\Sigma}_R(\Theta) = \frac{\widehat{\mu}_R(\Theta)}{\widehat{\sigma}_R(\Theta)} = (1-\Theta)\,\gamma^S r\left(\sigma - \frac{\hat{\sigma}_W}{\gamma_M r}\right). \tag{56}$$

The following proposition describes L's optimal ownership path as well as her steady state ownership level in the first-best case.

Proposition 6 (Large Shareholder's Ownership Path in First-Best Benchmark) *L's optimal ownership level, $\widehat{\Theta}^L_{t_i}$, in period $[t_i, t_{i+1})$, given her current holdings $\Theta^L_{t_{i-1}}$, satisfies the following equation:*

$$FOC:\ (\Theta^L_{t_{i-1}} - \widehat{\Theta}^L_{t_i})\frac{\partial\Lambda(t_i, \widehat{\Theta}^L_{t_i})}{\partial\Theta} + \phi_i(t_i)\left[\gamma^S - (\gamma^L + \gamma^S)\widehat{\Theta}^L_{t_i}\right] r\widehat{\sigma}^2_R(\widehat{\Theta}^L_{t_i}) = 0. \tag{57}$$

In the steady state, L's ownership stake is

$$\widehat{\Theta}^L_{ss} = \frac{\gamma^S}{\gamma^S + \gamma^L}, \tag{58}$$

and the steady state equilibrium stock price is

$$\widehat{P}(t, \widehat{\Theta}^L_{ss}, W^M_t) = \frac{1}{r}\widehat{k}(\widehat{\Theta}^L_{ss}) + \frac{1}{\gamma^M r}\ln(-W^M_t), \tag{59}$$

where $\widehat{k}(\cdot)$ is given by (53) with L's optimal policies in Proposition 4 and L's ownership level equal to $\widehat{\Theta}^L_{ss}$.

Comparing Propositions 2 and 3 with Propositions 5 and 6, respectively, we see that the equilibrium stock price and L's ownership dynamics have similar functional forms in the main second-best agency contracting model and the first-best benchmark model, but the constants specifying these variables are determined differently. As we discussed earlier, in the first-best benchmark, the optimal contract purely reflects optimal risk-sharing between L and M. In other words, L's first-best optimal ownership choices are determined by optimal risk sharing with the manager through his contract and optimal risk sharing with small shareholders through trading. As in the second-best, only the latter force remains as L's ownership stake converges to its steady state level.

We can obtain the sensitivities of CEO pay to the firm's cash flows and to its stock return in the first-best case by substituting the first-best volatility of M's promised payoff, $\hat{\sigma}_W$ in

(49), for σ_W in (30) and (38), respectively. It is clear that these PPS measures increase with L's risk aversion, γ^L, but decrease with M's risk aversion, γ^M in the first-best case. The following proposition summarizes the results of the analytical comparison between the first-best and second-best equilibrium solutions.

Proposition 7 (Comparison Between First-Best and Second-Best Agency Models)

- *The manager's optimal effort level in the second-best is lower than his first-best effort level.*

- *The drift of the manager's promised payoff process is higher in the second-best case.*

- *Suppose L's ownership is Θ. Compared to the first-best benchmark,*

  - *the volatility of the manager's promised payoff process is greater,*
  - *the manager's pay-performance sensitivity measures are higher,*
  - *the expected excess dollar return and the dollar return volatility of the stock are lower in the second-best agency contracting case.*

The intuition for the above results hinges on the fact that the optimal contract between L and M in the first-best benchmark purely reflects optimal risk-sharing between L and M. In contrast, in the main agency model (second-best case), the optimal contract reflects the tradeoff between risk-sharing and incentive provision. The agency costs of risk-sharing between L and M in the main model cause the manager's effort level to be lower than in the first-best benchmark. The need for incentive provision to the manager causes him to bear greater risk in the second-best agency model. As a result, the manager's pay-performance sensitivity measures as well as the volatility of his promised payoff process are higher than in the first best benchmark scenario. To compensate for the increased risk the manager needs to bear, his expected promised payoff–as embodied in the drift of his promised payoff process—is higher in the second-best case than in the first-best benchmark scenario. The dollar return volatility of the stock is lower in the second-best case because the manager bears greater risk so that the volatility of the residual cash flows to shareholders is lower. The lower dollar return volatility of the stock in the second-best case causes the expected excess dollar return to also be lower because of the positive relation between the dollar return volatility and the expected excess dollar return as shown by (55).

4.2 Benchmark Two: The Owner-Manager Scenario

We now analyze another benchmark model, which corresponds to the framework studied by DU. In this benchmark, L directly runs the firm by exerting costly effort herself so that there is no separation between ownership and control as in our main model. In other words, as in DU, there are only two sets of agents: the large shareholder L and the small shareholders S.

As in our analysis of the main agency model in Section 3, L's consumption and ownership decisions must be sequentially optimal given any past history. Suppose that L's ownership policy in the firm is given by $\Theta^L = \{\Theta^L_{t_i}; i = 1, \ldots, N\}$. Adapting (10) to the case where L also makes effort choices, her problem is to solve

$$\max_{(\Theta^L_{[t_i,\infty)}, c^L_{[t_i,\infty)}, a^L_{[t_i,\infty)})} \quad E^{a^L}_{t_i}\left[\int_{t_i}^{\infty} e^{-\delta^L(\tau - t_i)} u^L(c^L_\tau, a^L_\tau) d\tau\right] = E^{a^L}_{t_i}\left[-\int_{t_i}^{\infty} \frac{1}{\gamma^L} e^{-\delta^L(\tau - t_i) - \gamma^L(c^L_\tau - \Psi(a^L_\tau))} d\tau\right], \tag{60}$$

subject to the budget constraint

$$dB^L_t = (rB^L_t - c^L_t)dt + \Theta^L_t dX_t - P_t(\Theta^L_t)d\Theta^L_t. \tag{61}$$

In (60), $E^{a^L}_{t_i}[\cdot]$ signifies the expectation with respect to the probability distribution induced by L's effort choice process.

A small shareholder S chooses its ownership and consumption policies rationally anticipating L's ownership, effort, and consumption decisions to solve

$$\max_{(\theta^S_{[t,\infty)}, c^S_{[t,\infty)})} E^{a^L}_t\left[\int_t^{\infty} e^{-\delta^S(\tau - t)} u^S(c^S_\tau) d\tau\right] = E^{a^L}_t\left[-\int_t^{\infty} \frac{1}{\gamma^S} e^{-\delta^S(\tau - t) - \gamma^S c^S_\tau} d\tau\right], \tag{62}$$

subject to the budget constraint

$$dY^S_t = (rY^S_t - c^S_t)dt + \theta^S_t(dX_t + dP_t - rP_t dt). \tag{63}$$

As in the analysis of our main model, we consider the SMPPE of this benchmark model. Because there is no separation between L and M, the public state vector is now (t, B^L_t). The following proposition describes L's optimal consumption and effort choices. Although the results in this section are the same as those derived by DU, we provide detailed proofs in the online Appendix to keep our discussion self-contained.

Proposition 8 (Large Shareholder's Optimal Policy in Owner-Manager Benchmark) *Let Θ be any (on- or off-equilibrium) level of L's ownership at time $t \in [t_i, t_{i+1})$. L's value function at time t has the form of*

$$W^L_t = -\frac{1}{\gamma^L r} e^{-\gamma^L\left[r(B^L_t + \widetilde{G}(t,\Theta)) + \frac{\delta^L - r}{\gamma^L r}\right]}. \tag{64}$$

L's optimal effort and consumption policies are

$$\widetilde{a}_t^L = \frac{\mu_1}{\psi}\Theta, \tag{65}$$

$$\widetilde{c}_t^L = r(B_t^L + \widetilde{G}(t,\Theta)) + \frac{\delta^L - r}{\gamma^L r} + \Psi(\widetilde{a}_t^L). \tag{66}$$

In the above, L's time-deterministic certainty-equivalent payoff, $\widetilde{G}(t,\Theta)$, satisfies the recursion

$$\widetilde{G}(t,\Theta) = \phi_i(t)\widetilde{V}(\Theta) + (1 - r\phi_i(t))\widetilde{G}(t_{i+1},\Theta);\ \lim_{t\to\infty}\widetilde{G}(t,\Theta) = 0, \tag{67}$$

where $\widetilde{V}(\Theta)$ is the net benefit flow to L from holding Θ,

$$\widetilde{V}(\Theta) = \Theta\mu(\widetilde{a}_t^L) - \Psi(\widetilde{a}_t^L) - \frac{1}{2}\gamma_L r\Theta^2\sigma^2 = \mu_0\Theta + \frac{1}{2}\left[\frac{\mu_1^2}{\psi} - \gamma_L r\sigma^2\right]\Theta^2. \tag{68}$$

As in the analysis of the main model and the first-best benchmark model, L's value function has an exponential functional form with a time-deterministic certainity-equivalent payoff, $\widetilde{G}$. By (65), we note that, not surprisingly, L exerts greater effort when she holds a larger block of shares in the firm, and when the productivity of effort is higher or the unit cost of effort is lower. L's net benefit flow from holding Θ, $\widetilde{V}(\Theta)$, captures the trade-off between the expected payoff from holding Θ in the firm, which is reduced by the effort cost solely borne by L in the owner-manager case, and the cost of the risk that she bears from her ownership stake in the firm.

The following proposition describes the stock price process in the owner-manager scenario. As in the analysis of the main model and the first-best benchmark model, we derive the stock price process via the analysis of the representative small shareholder S's optimal portfolio and consumption choices as well as the stock market clearing condition.

Proposition 9 (Stock Price Process in Owner-Manager Benchmark)
Given any (on- or off-equilibrium) level of L's ownership, Θ, the stock price process is given by

$$\widetilde{P}(t,\Theta) = \phi_i(t)\widetilde{k}(\Theta) + (1 - r\phi_i(t))\widetilde{P}(t_{i+1},\Theta);\ \lim_{t\longrightarrow\infty}\widetilde{P}(t,\Theta) = 0, \tag{69}$$

$$\widetilde{k}(\Theta) = \mu(\widetilde{a}_t^L) - \gamma^S r(1-\Theta)\sigma^2 = \mu_0 + \frac{\mu_1^2}{\psi}\Theta - \gamma^S r(1-\Theta)\sigma^2. \tag{70}$$

The excess dollar return for holding a share of the firm's stock within the time interval

$(t, t + dt)$ *is defined as*

$$d\widetilde{R}_t = dX_t + d\widetilde{P}_t - r\widetilde{P}_t dt = \widetilde{\mu}_R(\Theta)dt + \widetilde{\sigma}_R(\Theta)dZ_t, \tag{71}$$

where the expectation and volatility of the excess dollar return of the stock are

$$\widetilde{\mu}_R(\Theta) = (1 - \Theta)\,\gamma^S r\sigma^2;\ \widetilde{\sigma}_R(\Theta) = \sigma. \tag{72}$$

The Sharpe ratio is

$$\widetilde{\Sigma}_R(\Theta) = \frac{\widetilde{\mu}_R(\Theta)}{\widetilde{\sigma}_R(\Theta)} = (1 - \Theta)\,\gamma^S r\sigma. \tag{73}$$

The following corollary summarizes how L's ownership level Θ is related to the dollar stock return characteristics.

Corollary 4 (Large Shareholder Ownership and Stock Returns)
The expected excess dollar return of the stock and its Sharpe ratio decline with the large shareholder's ownership.

By (72) and (73), the mean excess dollar return and the Sharpe ratio of the stock decline with L ownership level. As L holds a larger block of the company's stock, the stock's liquidity available to small shareholders is lower, thereby increasing the current stock price due to the lower supply of shares and, thus, lowering the expected stock return as well as the Sharpe ratio.

We now discuss L's optimal ownership choice at each trading date t_i. As L's value function after she chooses the new equity stake Θ is given by (64), L's optimal ownership choice maximizes $B^L_{t_i} + \widetilde{G}(t_i, \Theta) = B^L_{t_i^-} + \widetilde{P}(t_i, \Theta)(\Theta^L_{t_{i-1}} - \Theta) + \widetilde{G}(t_i, \Theta)$, which includes the proceeds from trading. The following proposition characterizes L's optimal ownership path, as well as her steady state ownership level.

Proposition 10 (Large Shareholder's Ownership Path in Owner-Manager Benchmark)
L's optimal ownership level, $\widetilde{\Theta}^L_{t_i}$, for period $[t_i, t_{i+1})$, given her current holdings, $\Theta^L_{t_{i-1}}$, satisfies the following equation:

$$FOC:\ (\Theta^L_{t_{i-1}} - \widetilde{\Theta}^L_{t_i})\frac{\partial \widetilde{P}(t_i, \widetilde{\Theta}^L_{t_i})}{\partial \Theta} + \phi_i(t_i)\left[\widetilde{V}'(\widetilde{\Theta}^L_{t_i}) - \widetilde{k}(\widetilde{\Theta}^L_{t_i})\right] = 0. \tag{74}$$

L's optimal ownership stake in the firm, determined by the above necessary condition, evolves over time as follows:

$$\widetilde{\Theta}^L_{t_i}(\Theta^L_{t_{i-1}}) = \frac{\eta_{1i}\Theta^L_{t_{i-1}} + \phi_i(t_i)\gamma^S r\sigma^2}{\eta_{1i} + \phi_i(t_i)(\gamma^S + \gamma^L)r\sigma^2}, \tag{75}$$

where the coefficient η_{1i} evolves as

$$\eta_{1i} = \phi_i(t_i)\nu + (1 - r\phi_i(t_i))\frac{\eta_{1i+1}^2}{\eta_{1i+1} + \phi_{i+1}(t_{i+1})(\gamma^S + \gamma^L)r\sigma^2}, \tag{76}$$

and $\nu = \frac{\mu_1^2}{\psi} + \gamma^S r\sigma^2$.

In the steady state, L's ownership stake is

$$\widetilde{\Theta}_{ss}^L = \frac{\gamma^S}{\gamma^S + \gamma^L}, \tag{77}$$

and the steady state equilibrium stock price is

$$\widetilde{P}(t, \widetilde{\Theta}_{ss}^L) = \frac{1}{r}\widetilde{k}(\widetilde{\Theta}_{ss}^L), \tag{78}$$

where $\widetilde{k}(\cdot)$ is given by (70) with L's optimal policies in Proposition 8 and L's ownership level equal to $\widetilde{\Theta}_{ss}^L$.

By (75) and (76), we see that L's ownership path is a deterministic function of time. It then follows from Proposition 9 that the equilibrium stock price is also a *deterministic* function of time. Because L's ownership level and effort are constant between successive trading dates, the firm's expected output and S's cost of risk from its ownership stake are also constant between L's successive trading dates. As shown by (69) and (70), the stock price is determined by the present value of the expected earnings net of S's cost of risk and is, therefore, a *deterministic* function of time. The deterministic evolution of the stock price stands in sharp contrast with the stock price process in the main model and first-best benchmark model as described by Propositions 2 and 5, respectively.[6] The stochastic evolution of the stock price in the second-best and first-best agency contracting cases arises due to its dependence on the manager's promised payoff that evolves stochastically. In other words, the separation of ownership and control in the first-best and second-best agency contracting models makes the stock price evolve stochastically. The following corollary compares the stock return characteristics in the benchmark owner-manager scenario and in our main model.

Corollary 5 (Stock Returns in Agency and Owner-Manager Benchmark Models) *For a given level of L's ownership, Θ, the expected excess dollar return, dollar return volatility, and Sharpe ratio of the stock are lower in the second-best agency model than in the*

[6]We, however, should note that, even with the time deterministic capital gain of the stock, its excess dollar return is still stochastic in the owner-manager benchmark case as shown by (71) because the firm's dividend payment is stochastic.

benchmark owner-manager model.

The possibility of risk-sharing between the large shareholder and the manager through the manager's incentive contract in our main model makes the manager's compensation payments volatile. As shareholders receive the dividend streams net of the manager's compensation payments, the sharing of risk with the manager makes the firm's residual cash flows less volatile and, thereby, lowers the dollar return volatility than in the benchmark owner-manager case. The expected excess return of the stock and its Sharpe ratio are also lower in the main second-best agency model than in the owner-manager scenario due to the lower dollar return volatility.

Comparing (43) with (58) and (77), we see that L's "steady state" ownership level is the same in the main model and both the benchmark models. In the steady state, L's ownership level equals the competitive equilibrium level that depends only on the risk aversions of L and S. By Propositions 9 and 10, L's ownership level and the stock price converge deterministically to their competitive equilibrium values in the owner-manager scenario. The separation of ownership and control and agency conflicts arising from the separation, therefore, affect the convergence of L's ownership level to the competitive equilibrium level as we explore numerically in the next section. Further, as is apparent from (44), (59), and (78), the stock price evolves stochastically in the steady state equilibria of the first-best and second-best agency contracting cases because of the stochastic evolution of the manager's promised payoff process, but is constant in the benchmark owner-manager scenario. The following corollary compares the steady-state dollar return characteristics of the stock in the main model and two benchmark models.

Corollary 6 (Comparison of Steady State Equilibria)
In the steady state, the expected excess dollar return, dollar return volatility, and Sharpe ratio of the stock are lowest in the second-best agency model, followed by the first-best agency model, and then by the owner-manager model.

As emphasized above, the optimal incentive provision to the manager plays a significant role of risk-sharing, which effectively lowers the firm's dollar return volatility and thereby the expected excess dollar return.

5 Numerical Analysis

In this section, we numerically derive additional implications of the second-best agency model that are difficult to derive analytically. We also carry out additional comparisons of the predictions of the second-best agency model with those of the benchmark models.

5.1 Baseline Parametrization

We first determine the baseline parameters for our numerical analysis by setting some of the parameters directly using guidance from previous studies and calibrating the remaining parameters to match salient variables in the steady state equilibrium of our main agency model that we described in Section 3. Table 1 reports the baseline parameters of the model. We set the annual risk-free rate, r, to 1.94%, and each economic agent's time discount rate, δ^j, to 0.0404 so that the annual discount factor is $e^{-\delta^j} = 0.96$ for $j = L, M, S$ (Guvenen 2009). We choose the time interval between the large shareholder's successive trading dates, Δ_t, to be one year.[7]

We calibrate the remaining parameters to match selected variables in the steady state equilibrium of the main, second-best agency model. These parameters include those that determine the firm's mean cash flow (output) including the constant term and the productivity of effort, μ_0 and μ_1; the firm's cash flow volatility, σ; the unit cost of the manager's effort, ψ; the coefficients of absolute risk aversion for the manager, the large shareholder, and small shareholders, γ^j for $j = L, M, S$; and the manager's mean promised payoff in the steady state, $\ln(-W_{ss}^M)$.

We match the following empirical moments whose values are reported in Table 2: (i) the median block ownership in firms that have been public for at least five years as reported in Foley and Greenwood (2010); (ii) the mean volatility of percentage stock returns; (iii) the mean Sharpe ratio; (iv) the mean market to book ratio; (v) the mean dividend-price ratio; (vi) the mean sensitivity of the sum of the top five executives' pay to shareholder value; and (vii) the mean ratio of the sum of the top five executives' pay to market value. The stock-related moments (volatility, Sharp ratio, and dividend-price ratio) are the values used in Guvenen (2009). Although our analytical results in Sections 3 and 4 focus on the dollar return characteristics of the stock, we match the statistics of percentage stock returns (the dollar stock return normalized by the stock price) in our calibration as we do not observe excess dollar returns of stocks in the data.

The model counterpart of L's ownership is given by (43). As it is the competitive equilibrium level determined by the risk aversion parameters of L and S, matching the observed block ownership helps to identify the ratio of the two risk aversion parameters. We define the model-predicted market-to-book ratio (MB_{ss}) in the steady state using the fact that the firm's stock price represents its total market value as the number of shares outstanding is normalized to one. The model-predicted firm market value (MV_{ss}) is thus the mean stock

[7] Our main numerical results are not sensitive to the choice of Δ_t.

price in the steady-state equilibrium, which is given by

$$MV_{ss} = \frac{1}{r}k(\Theta_{ss}^{L*}) + \frac{1}{\gamma^M r}\ln(-W_{ss}^M). \tag{79}$$

The model-predicted firm book value (BV_{ss}) is the stock price of the hypothetical firm in the absence of the manager's human capital inputs (e.g., Ou-Yang 2005):

$$BV_{ss} = \frac{1}{r}\overline{k}(\Theta_{ss}^{L*}) \text{ where } \overline{k}(\Theta_{ss}^{L*}) = \mu_0 - \gamma^S r(1-\Theta_{ss}^{L*})\sigma^2. \tag{80}$$

We set μ_0, which is a parameter in the firm's mean cash flow, $\mu(\cdot)$, such that the firm's steady state book value, BV_{ss}, is normalized to one. We match the mean market-to-book ratio from the data to the ratio of the model-predicted market and book values specified by the above equations. To match the mean percentage stock return volatility from the data, we divide the dollar return volatility σ_R in (35) by the steady state mean stock price (MV_{ss}). We compute the model-predicted manager pay and dividend payments by (1) and (21):

$$c_{ss}^M = -\frac{1}{\gamma^M}\ln\beta - \frac{1}{\gamma^M}\ln(-W_{ss}^M) + \Psi(\alpha);\ d_{ss} = \mu(\alpha) - c_{ss}^M, \tag{81}$$

where α and β are obtained by (22) and (23) with L's steady state ownership Θ_{ss}^{L*}. Table 2 reports the actual and model-predicted values of the moments. We see that the baseline model matches the moments reasonably well. According to the baseline parameter values in Table 1, small shareholders are less risk averse than the large shareholder and the manager. This result conforms to the intuition that outside shareholders are well-diversified relative to blockholders who have a concentrated ownership in firms and managers who have significant human capital invested in their firms. We should note, however, that the calibration exercise is only intended to provide a reasonable set of baseline parameter values for our numerical analysis.

Table 3 shows key equilibrium variables in the steady state in the main, second-best agency contracting model and the two benchmark models that are computed using the baseline parameter values in Table 1. As discussed in Section 4, we notice that the steady state ownership stakes of the large shareholder are identical in the three scenarios. We first confirm the analytical comparison between the first-best and second-best contracting cases in Proposition 7: the manager's effort, the expected excess dollar return of the stock, and the dollar return volatility are lower in the second-best case, but the manager's pay-performance sensitivity measures are higher. The stock price, which is simply the market-to-book ratio as the firm's steady state book value is normalized to one, is lower in the second-best case because of higher agency costs captured by higher expected manager pay. The manager's lower effort and higher expected manager pay also lead to lower expected dividend payments. Even with a lower stock price, the expected percentage stock return and volatility as well as the dividend-price ratio are still lower in the second-best agency contracting case.

We now compare with the owner-manager case. The manager's second-best effort is higher than the large shareholder's optimal effort in the owner-manager case mainly due to the fact that the large shareholder cannot internalize her effort costs with small shareholders in the latter case. As shown analytically in Corollary 6, the expected dollar return, dollar return volatility, and Sharpe ratio of the stock are all lower in the agency contracting case because of the risk-sharing effects of the manager's contract. Even with the higher effort of the manager, dividends are lower in the agency contracting case because of the agency costs incurred through the compensation paid to the manager. Hence, the stock price (or market value of the firm) is much higher in the owner-manager case. The difference in the stock price relative to the owner-manager case outweighs the differences in expected dollar returns and in dividends so that the expected percentage stock return and percentage stock return volatility as well as the dividend-price ratio are higher in the agency contracting case than in the owner-manager case.

5.2 Large Shareholder Ownership, Stock Returns, and Incentives

Figure 1 shows the effects of varying the large shareholder's ownership level on the steady-state equilibrium variables in the main agency model: (i) the manager's effort and two PPS measures (PPS_X and PPS_R); (ii) the expected dollar return (μ_R) and dollar return volatility (σ_R); (iii) the expected percentage stock return ($\mu_R^{\%}$), percentage stock return volatility ($\sigma_R^{\%}$), and Sharpe ratio (Σ_R). First, the large shareholder's ownership stake has a positive impact on the two PPS measures, thereby inducing a higher effort level. This finding suggests that block ownership and incentive contracting are complementary mechanisms in corporate governance. The positive relation between L's ownership level and PPS measures is consistent with the empirical evidence in Hartzell and Starks (2003), Almazan, Hartzell and Starks (2005), and Kim (2010) that the sensitivity of CEO compensation to firm performance is positively associated with the level of outside block ownership.

The expected percentage return, percentage return volatility and Sharpe ratio of the stock all decline with the large shareholder's ownership level. As discussed in the previous sections, the large shareholder's ownership has direct and indirect effects on the expected dollar return of the stock. The direct effect stems from the fact that her ownership stake reduces the stock's liquidity available to small shareholders, thereby increasing the current stock price and, thus, lowering the expected stock return. The indirect effect stems from the positive effects of the large shareholder's ownership stake on the manager's incentive compensation. Greater incentive compensation for the manager reduces the volatility of the firm's residual earnings and, therefore, the dollar return volatility. The expected *percentage* return and volatility of the stock are given by their *dollar* counterparts normalized by the stock price. The stock price increases with L's ownership level as it results in greater managerial effort and, thus, greater earnings. Consequently, the expected percentage return and volatility of the stock (hereafter, the expected stock return and volatility) decline more sharply with L's ownership level.

5.3 Firm Characteristics, Stock Returns, and Incentives

Figure 2 shows the effects of the productivity of effort, μ_1, on the expected stock return, volatility, Sharpe Ratio, as well as the manager's PPS measures in the steady state. The expected return, return volatility and Sharpe ratio of the stock decline with the productivity of effort, while the PPS measures increase. An increase in the effort productivity increases the power of incentives provided to the manager and, therefore, the manager's effort. Consequently, the stock price increases, thereby lowering the expected percentage stock return. The return volatility declines because of the increase in the power of incentives to the manager so that the volatility of the residual cash flows to shareholders declines. The Sharpe ratio barely varies due to the offsetting effects.

Figures 3 and 4 show the effects of the firm's cash flow volatility (firm risk), σ, and the unit cost of the manager's effort, ψ, respectively. The expected stock return, return volatility, and Sharpe ratio all increase with σ and ψ, whereas the PPS measures decline. An increase in the output volatility or the unit effort cost increases the costs of risk-sharing, thereby lowering the power of incentives and lowering the manager's effort. Consequently, the stock price decreases so that the expected percentage return of the stock increases. The volatility of the stock also increases because of the decline in the incentive power of the manager's contract.

5.4 Firm Characteristics and Large Shareholder Ownership Dynamics

Figure 5 shows the dynamics of the large shareholder's ownership stake in the agency contracting case and the two benchmark cases. We set L's initial ownership stake to 60% that is the median ownership of blockholders for IPO firms around their IPO time as reported in Foley and Greenwood (2010). We find that the large shareholder adjusts her equity stake toward the competitive equilibrium level more slowly in the owner-manager benchmark case than in the first-best and second-best agency contracting cases in which there is separation of ownership and control.

In the owner-manager benchmark case (as studied in DU), the trade-off the large shareholder faces in divesting her stake in the firm is that decreasing her stake will lower the share price as small investors anticipate a reduction in her effort that affects the firm's cash flows, but holding a higher stake increases her exposure to the firm-specific risk. The key distinction of the first-best and second-best contracting cases relative to the owner-manager case is the large shareholder's additional risk sharing with the manager through optimal contracting which reduces the firm's stock return volatility. Hence, in the first-best and second-best agency contracting cases, decreasing her stake in the firm will effectively reduce the risk-sharing with the manager and thus increase her risk premium she needs to bear, which increases her desire to sell. We also need to note that the large shareholder's ownership dynamics are barely affected by whether it is a first-best or second-best agency contracting case (that is, whether or not agency conflicts exist due to moral hazard). Accordingly, it is the separation of ownership and control, rather than moral hazard, that influences the large shareholder ownership dynamics.

Figures 6-8 show the effects of effort productivity, firm risk, and effort cost on the dynamics of the large shareholder's ownership stake in the main agency contracting case and the benchmark cases. These parameters have a significant impact on the large shareholder's optimal ownership choices in the benchmark owner-manager scenario. As DU highlight, for example, if the manager's effort is more productive or less costly, there is a larger price impact from selling shares, which reduces L's desire to sell so that L's ownership reaches the competitive level more slowly. In sharp contrast, the large shareholder's ownership choices are largely insensitive to the parameters in the first-best and second-best agency contracting scenarios. Indeed, the separation of ownership and control, and the facilitation of risk-sharing through contracting with the manager, effectively insulates the large shareholder from fluctuations in firm-specific parameters. Consequently, the large shareholder's ownership choices are largely determined by the risk aversions of the large and small shareholders as well as the manager. In other words, in the contracting case, risk-sharing between the large and small shareholders that determines the stock price is effectively separated from the contracting problem that determines the manager's effort and the firm's earnings. In sharp contrast with the benchmark owner-manager scenario, the large shareholder's ownership stake plays less of an incentive role so that it is largely determined by risk-sharing with small shareholders.

5.5 Empirical Implications

The results of the analytical and quantitative analyses of the model generate empirically testable implications for the relations among block ownership, stock return characteristics, and managerial contracts. We summarize them below as a guide for future empirical research. In particular, our framework and analysis highlight the importance of empirically investigating the relations among block ownership, stock returns, and managerial contracts by appropriately incorporating the fact that these variables are simultaneously and endogenously determined in equilibrium.

1. *Ceteris paribus*, managerial pay-performance sensitivities are negatively associated with expected stock returns, return volatilities, and Sharpe ratios.

2. *Ceteris paribus*, block ownership (or large shareholder ownership) is negatively associated with expected stock returns, volatilities and Sharpe ratios.

3. *Ceteris paribus*, block ownership (or large shareholder ownership) is positively related to managerial incentives.

4. *Ceteris paribus*, the shareholder ownership of owner-managed firms becomes dispersed more slowly than the ownership of firms characterized by significant separation between ownership and control.

5. *Ceteris paribus*, block ownership dynamics are more sensitive to firm-specific characteristics such as firm productivities, cash flow volatilities, and effort costs in owner-managed firms than firms characterized by significant separation between ownership and control.

6 Conclusions

We develop a tractable dynamic equilibrium framework to investigate the interactions among the ownership dynamics of large shareholders, managerial incentive contracts, and asset returns. Our framework synthesizes an asset pricing model with a dynamic principal-agent model. Large shareholders serve as mediators who determine optimal incentive contracts for firm managers, while also influencing asset prices through their dynamic trading decisions. In equilibrium, block ownership dynamics, managerial compensation, and asset prices endogenously reflect risk-sharing between large and small shareholders in asset markets as well as the tension between risk sharing and incentive provision stemming from manager moral hazard. Our unified model generates novel testable implications for the equilibrium relations among firms' ownership concentrations, earnings and stock return characteristics, as well as managerial incentives. We also show how the separation between ownership and control as well as manager moral hazard affect the equilibrium by relating our main model with two benchmark models: the "first-best" model in which there is no manager moral hazard, and the "owner-manager" model in which there is no separation between large shareholders and managers.

In future research, it would be interesting to consider a more general framework with multiple firms and imperfect correlations among the firms' earnings. Such a model would generate cross-sectional implications for block ownership, managerial incentives, and stock return characteristics (expected returns, volatilities, and stock betas) that would conform more closely to the data.

Our adoption of a "CARA-normal" setting facilitates an analytical characterization of the equilibrium. It would be interesting to consider more general preferences, but such an extension is likely to be analytically intractable. Even in the simpler principal-agent setting without the endogenous determination of asset prices via the trading decisions of shareholders, only numerical solutions are feasible for optimal contracts with more general preferences (see Sannikov 2008; Cvitanic and Zhang 2013).

Lastly, as in APZ and DU, we do not consider an endogenous distinction between the large shareholder and small shareholders. It would be interesting to endogenize the distinction between large and small shareholders by modeling the ex ante decision of an investor to become a large shareholder. Such a decision would reflect the tradeoff between the benefits of supra-marginal rents from a large ownership stake that provides the opportunity to influence the terms of managerial contracts (and, thereby, asset prices); and the costs stemming from lack of diversification. We leave the exploration of these extensions for future research.

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Main Appendix

Proofs for Main Model in Section 3: The Second-Best Agency Contracting Model

We proceed to derive the equilibrium of the agency contracting model in Section 3 using backward induction. We first characterize the equilibrium value functions and policies for the last trading period, $[t_N, t_{N+1}) = [T, \infty)$. We then proceed to the derivation of the equilibrium value functions and policies for earlier trading periods $[t_i, t_{i+1})$ with $i < N$.

Proof of Proposition 1: Last Period

Consider the last period $t \in [t_N, t_{N+1}) = [T, \infty)$ where L's ownership level is $\Theta_t^L = \Theta$. L's optimal choices of consumption and the terms of M's contract solve

$$W_t^L = \max_{c_t^L, c_t^M, a_t^M} E_t^{a^M}\left[\int_t^\infty e^{-\delta^L(\tau - t)} u^L(c_\tau^L) d\tau\right] = \max_{c_t^L, c_t^M, a_t^M} E_t^{a^M}\left[-\frac{1}{\gamma^L}\int_t^\infty e^{-\delta^L(\tau-t)-\gamma^L c_\tau^L} d\tau\right], \tag{A1}$$

subject to the budget constraint characterized by L's money market account balance,

$$dB_t^L = \left(rB_t^L - c_t^L\right) dt + \Theta\left(dX_t - c_t^M dt\right) = \left(rB_t^L - c_t^L + \Theta\left(\mu(a_t^M) - c_t^M\right)\right) dt + \Theta\sigma dZ_t, \tag{A2}$$

and M's incentive compatibility (IC) constraint,

$$\begin{aligned}(IC):\ dW_t^M &= \left(\delta^M W_t^M - u^M(c_t^M, a_t^M)\right) dt - \frac{u_a^M(c_t^M, a_t^M)}{\mu'(a_t^M)}\sigma dZ_t \\ &= \left(\delta^M W_t^M + \frac{1}{\gamma^M} H(c_t^M, a_t^M)\right) dt + \frac{\psi}{\mu_1} a_t^M H(c_t^M, a_t^M)\sigma dZ_t. \end{aligned} \tag{A3}$$

In the above, $H(c_t^M, a_t^M) \equiv e^{-\gamma^M(c_t^M - \frac{1}{2}\psi(a_t^M)^2)}$, and the second equation follows from the specifications of the firm's cash flow process, $\mu(a) = \mu_0 + \mu_1 a$, and the manager's effort cost function, $\Psi(a) = \frac{1}{2}\psi a^2$. Since the range of the manager's effort choice, a_t^M, is bounded, and $\mu(\cdot)$ is continuously differentiable, we can check that the linear growth and Lipschitz conditions for the drift and volatility terms of the SDE above are satisfied so that it has a unique solution (See, for example, Williams 2009 for more details).

Using the standard "guess and verify" approach to solve L's stochastic control problem, we conjecture that L's value function is a function of time t, L's money market balance B_t^L, and M's promised payoff W_t^M, and has the following exponential form:

$$W_t^L = F(t, B_t^L, W_t^M) = -\frac{1}{\gamma^L r} e^{-\gamma^L\left[r(B_t^L + G(t,\Theta)) + \frac{\Theta}{\gamma^M}\ln(-W_t^M) + \frac{\delta^L - r}{\gamma^L r}\right]}. \tag{A4}$$

The Hamilton-Jacobi-Bellman (HJB) equation for the above problem is

$$F_t + \max_{c_t^L, c_t^M, a_t^M}\left[F_B(rB_t^L - c_t^L + \Theta(\mu(a_t^M) - c_t^M)) + F_W\left(\delta^M W_t^M + \frac{1}{\gamma^M} H(c_t^M, a_t^M)\right) + \frac{1}{2}F_{BB}\Theta^2\sigma^2 +\right.$$

$$+F_{BW}\Theta\sigma\left(\frac{\psi}{\mu_1}a_t^M H(c_t^M,a_t^M)\sigma\right)+\frac{1}{2}F_{WW}\left(\frac{\psi}{\mu_1}a_t^M H(c_t^M,a_t^M)\sigma\right)^2+u^L(c_t^L)-\delta^L F\Bigg]=0. \tag{A5}$$

The first order conditions (FOCs) with respect to (c_t^L, c_t^M, a_t^M) are then

$$\begin{aligned}
&c_t^L:\ -F_B+u_c^L(c_t^L)=0,\\
&c_t^M:\ F_B\Theta+F_W H+F_{BW}\Theta\gamma^M\left(\frac{\psi}{\mu_1}a_t^M H\right)\sigma^2+F_{WW}\gamma^M\left(\frac{\psi}{\mu_1}a_t^M H\right)^2\sigma^2=0,\\
&a_t^M:\ F_B\Theta\mu_1+F_W(\psi a_t^M H)+F_{BW}\Theta\left(1+\gamma^M\psi(a_t^M)^2\right)\left(\frac{\psi}{\mu_1}H\right)\sigma^2\\
&\quad+F_{WW}\left(1+\gamma^M\psi(a_t^M)^2\right)\left(\frac{\psi}{\mu_1}H\right)^2 a_t^M\sigma^2=0,
\end{aligned} \tag{A6}$$

The above equations, along with the derivatives of $F(t,B_t^L,W_t^M)$, lead to the following optimal policies:

$$\begin{aligned}
c_t^L &= r(B_t^L+G(t,\Theta))+\frac{\Theta}{\gamma_M}\ln(-W_t^M)+\frac{\delta^L-r}{\gamma^L r},\\
a_t^M &= \alpha,\\
c_t^M &= -\frac{1}{\gamma^M}\ln\beta-\frac{1}{\gamma^M}\ln(-W_t^M)+\Psi(a_t^M),
\end{aligned} \tag{A7}$$

where the contractual parameters, α and β, are determined by

$$\begin{aligned}
\beta &= \gamma^M r\left[\gamma^M\alpha(\psi\alpha-\mu_1)+1\right],\\
\left(1+\frac{\gamma^L}{\gamma^M}\Theta\right)\left(\frac{\psi}{\mu_1}\alpha\beta\sigma\right)^2 &- (\gamma^L r\Theta\sigma)\left(\frac{\psi}{\mu_1}\alpha\beta\sigma\right)-\left(r-\frac{\beta}{\gamma^M}\right)=0.
\end{aligned} \tag{A8}$$

By (A3) and the optimal policies in (A7), M's promised payoff follows a geometric Brownian motion process:

$$\begin{aligned}
dW_t^M &= \mu_W W_t^M dt-\sigma_W W_t^M dZ_t,\\
\Rightarrow\ d\ln(-W_t^M) &= \left(\mu_W-\frac{1}{2}\sigma_W^2\right)dt-\sigma_W dZ_t,
\end{aligned} \tag{A9}$$

where

$$\mu_W\equiv\delta^M-\frac{\beta}{\gamma^M};\ \ \sigma_W\equiv\frac{\psi}{\mu_1}\alpha\beta\sigma. \tag{A10}$$

Substituting the optimal policies (A7) and the derivatives of $F(t,B_t^L,W_t^M)$ into the HJB equation (A5) yields

$$G_t(t,\Theta)=rG(t,\Theta)-V(\Theta), \tag{A11}$$

where $V(\Theta)$ is given by:

$$V(\Theta)=\Theta\left(\mu(\alpha)-\left(-\frac{1}{\gamma^M}\ln\beta-\frac{1}{\gamma^M r}\left(\mu_W-\frac{1}{2}\sigma_W^2\right)+\Psi(\alpha)\right)\right)-\frac{1}{2}\gamma^L r\Theta^2\left(\sigma-\frac{\sigma_W}{\gamma^M r}\right)^2. \tag{A12}$$

By integrating (A11) over the period $[t, t_{i+1})$, we obtain

$$G(t,\Theta)=\phi_i(t)V(\Theta)+(1-r\phi_i(t))G(t_{i+1},\Theta)=\frac{1}{r}V(\Theta), \tag{A13}$$

where $\phi_i(t)=\frac{1}{r}\left(1-e^{-r(t_{i+1}-t)}\right)$, and the second equation follows from the assumptions that $t_{N+1}=\infty$ and that the terminal payoff to L is zero: $G(t_{N+1})=\lim_{t\to\infty}G(t,\Theta)=0$. We can check that the function W_t^L, (A4), satisfies the conditions for the dynamic programming verification theorem: (i) it is twice continuously differentiable in its arguments; and (ii) it satisfies a polynomial growth condition.[8] Thus, it is, indeed, L's value function for the last period.

Proof of Proposition 2: Last Period

We now consider small shareholders. Unlike L, small shareholders trade competitively as price takers, that is, they make optimal consumption and portfolio choices by taking the stock price as given. Let Θ denote any (on or off-equilibrium) ownership level of L that (we recall) is publicly observable, and a_t^M denote M's corresponding optimal effort choice derived above. Given L's ownership level and M's contract, small shareholders rationally anticipate M's optimal effort process.

Because small shareholders continuously trade in the market, the stock price is determined by the risk-adjusted discounted value of future dividend payments to them. If their pricing kernel between τ and t is $\xi_{t,\tau}^S$, the stock price function is defined as follows:

$$P_t=E_t^{a^M}\left[\int_t^\infty \xi_{t,\tau}^S(1-\Theta)(dX_\tau-c_\tau^M d\tau)\right], \tag{A14}$$

where $(1-\Theta)(dX_\tau-c_\tau^M d\tau)$ represents the proportion of the dividend payment to small shareholders in each point in time, which excludes M's compensation payment, c_τ^M, from the firm's incremental cash flow dX_τ. As we showed in (A7), c_t^M is an affine function of $\ln(-W_t^M)$. We thus conjecture and verify that, for any given level of L's ownership Θ, the equilibrium stock price is linear in $\ln(-W_t^M)$ and has the form of

$$P(t,\Theta,W_t^M)=\Lambda(t,\Theta)+\frac{1}{\gamma^M r}\ln(-W_t^M). \tag{A15}$$

The excess dollar return of the stock, which includes both the dividend payment and capital

[8] For detailed proofs of the relevant verification theorem, see Theorem IV 3.1 for the finite horizon case and Theorem IV.5.1. for the infinite horizon case in Fleming and Soner (2006).

gain from the stock, is defined as

$$dR_t \equiv dX_t - c_t^M dt + dP_t - rP_t dt. \tag{A16}$$

Using the guessed form of the stock price function (A15), the optimal solution for c_t^M in (A7), and M's promised payoff process (A9), we can show that the excess dollar return process is linear in the underlying stochastic process in the firm's earnings with the drift term, $\mu_R(t,\Theta)$, and the diffusion term, $\sigma_R(t,\Theta)$:

$$dR_t = \mu_R(t,\Theta)dt + \sigma_R(t,\Theta)dZ_t, \tag{A17}$$

where

$$\mu_R(t,\Theta) \;=\; \mu(\alpha) + \frac{1}{\gamma^M}\ln\beta + \frac{1}{\gamma^M r}\left(\mu_W - \frac{1}{2}\sigma_W^2\right) - \Psi(\alpha) + \Lambda_t(t,\Theta) - r\Lambda(t,\Theta), \tag{A18}$$

$$\sigma_R(t,\Theta) \;=\; \left(\sigma - \frac{\sigma_W}{\gamma^M r}\right). \tag{A19}$$

Given the stock price process (A15), the representative small shareholder S's optimal consumption and portfolio problem is specified by

$$W_t^S = \max_{c_t^S,\theta_t^S} E_t^{a^M}\left[\int_t^\infty e^{-\delta^S(\tau-t)}u^S(c_\tau^S)d\tau\right] = E_t^{a^M}\left[-\frac{1}{\gamma^S}\int_t^\infty e^{-\delta^S(\tau-t)-\gamma^S c_\tau^S}d\tau\right], \tag{A20}$$

subject to the budget constraint that captures the evolution of S's stock and money market account balances. S's money market balance evolves according to

$$dB_t^S = (rB_t^S - c_t^S)dt + \theta_t^S(dX_t - c_t^M dt) - P_t d\theta_t^S. \tag{A21}$$

S's total wealth Y_t^S from his portfolio holdings is $Y_t^S = B_t^S + \theta_t^S P_t$, which evolves according to

$$dY_t^S = (rY_t^S - c_t^S)dt + \theta_t^S(dX_t - c_t^M dt + dP_t - rP_t dt) = (rY_t^S - c_t^S)dt + \theta_t^S dR_t, \tag{A22}$$

where the second equation follows from the definition of the excess dollar return of the stock, (A16).

In a similar manner to the proof of Proposition 1, we guess and verify S's value function, W_t^S, as an exponential function of time t and S's total weath, Y_t^S:

$$W_t^S = Q(t, Y_t^S) = -\frac{1}{\gamma^S r}e^{-\gamma^S\left[r(Y_t^S + J(t,\Theta)) + \frac{\delta^S - r}{\gamma^S r}\right]}. \tag{A23}$$

The HJB equation is then

$$Q_t + \max_{c_t^S,\theta_t^S}\left[Q_Y\left(rY_t^S - c_t^S + \theta_t^S\mu_R(t,\Theta)\right) + \frac{1}{2}Q_{YY}\left(\theta_t^S\sigma_R(t,\Theta)\right)^2 + u^S(c_t^S) - \delta^S Q\right] = 0, \tag{A24}$$

where we use the form of the excess dollar return process, (A17). The FOCs with respect to

c_t^S and θ_t^S are given by

$$Q_Y = e^{-\gamma^S c_t^S}; \ \theta_t^S = -\frac{Q_Y \mu_R(t,\Theta)}{Q_{YY}\sigma_R^2(t,\Theta)}. \tag{A25}$$

The above equations, together with the derivatives of $Q(t, Y_t^S)$, yield the following optimal decisions for S :

$$c_t^S = r(Y_t^S + J(t,\Theta)) + \frac{\delta^S - r}{\gamma^S r}; \tag{A26}$$

$$\theta_t^S = \frac{\mu_R(t,\Theta)}{\gamma^S r \sigma_R^2(t,\Theta)}. \tag{A27}$$

In equilibrium, the stock market clears ($\int_S \theta_t^S dS = 1 - \Theta$). Hence, (A27) leads to the following relation between the expected excess dollar return of the stock and the volatility of the excess dollar return:

$$\mu_R(t,\Theta) = \gamma^S r(1-\Theta)\sigma_R^2(t,\Theta). \tag{A28}$$

By combining the above with (A18) and (A19), we obtain the following ODE for the time deterministic function $\Lambda(t,\Theta)$ in the stock price process (A15):

$$\Lambda_t(t,\Theta) = r\Lambda(t,\Theta) - k(\Theta), \tag{A29}$$

where

$$k(\Theta) = \mu(\alpha) - \left(-\frac{1}{\gamma^M}\ln\beta - \frac{1}{\gamma^M r}\left(\mu_W - \frac{1}{2}\sigma_W^2\right) + \Psi(\alpha)\right) - \gamma^S r(1-\Theta)\left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)^2. \tag{A30}$$

By integrating (A29) over period $[t, t_{i+1})$, we express $\Lambda(t,\Theta)$ recursively as follows.

$$\Lambda(t,\Theta) = \phi_i(t)k(\Theta) + (1 - r\phi_i(t))\Lambda(t_{i+1},\Theta) = \frac{1}{r}k(\Theta), \tag{A31}$$

where $\phi_i(t) = \frac{1}{r}\left(1 - e^{-r(t_{i+1}-t)}\right)$, and the second equation follows as it is the last period ($t_{N+1} = \infty$) so that $\Lambda(t_{N+1},\Theta) = \lim_{t\to\infty}\Lambda(t,\Theta) = 0$.

By plugging the optimal policies of S, (A26) and (A27), and the derivatives of Q into the HJB equation (A24), we obtain

$$J_t(t,\Theta) = rJ(t,\Theta) - \theta_t^S \mu_R(t,\Theta) + \frac{1}{2}\gamma^S r\left(\theta_t^S \sigma_R(t,\Theta)\right)^2, \tag{A32}$$

$$= rJ(t,\Theta) - \frac{1}{2}\gamma^S r(1-\Theta)^2\left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)^2, \tag{A33}$$

where the second equation follows from the stock market clearing condition as well as (A28) and (A19). Integrating (A33) over the period $[t, t_{i+1})$ yields

$$J(t,\Theta) = \phi_i(t)V^S(\Theta) + (1 - r\phi_i(t))J(t_{i+1},\Theta) = \frac{1}{r}V^S(\Theta), \tag{A34}$$

where $V^S(\Theta) = \frac{1}{2}\gamma^S r(1-\Theta)^2\left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)^2$ and $\phi_i(t) = \frac{1}{r}\left(1 - e^{-r(t_{i+1}-t)}\right)$. The second equation follows from the assumption on the terminal payoff to S, $J(t_{N+1}) = \lim_{t\to\infty} J(t,\Theta) = 0$. Again, it is easy to check that the function W_t^S, (A23), satisfies the conditions for the verification theorem of dynamic programming so that it is, indeed, S's value function for the last period.

Proof of Proposition 3: Last Period

We now consider L's optimal ownership choice for the last period, $[t_N, t_{N+1})$, which is made at date t_N^-. By (A4) in the proof of Proposition 1, L's value function at t_N right after she chooses the ownership level Θ is

$$W_{t_N}^L = -\frac{1}{\gamma^L r}e^{-\gamma^L\left[r(B_{t_N}^L + G(t_N,\Theta)) + \frac{\Theta}{\gamma^M}\ln(-W_{t_N}^M) + \frac{\delta^L - r}{\gamma^L r}\right]}. \tag{A35}$$

We denote L's money market account balance and shareholdings at t_N^- by $B_{t_N^-}^L$ and $\Theta_{t_N^-}^L = \Theta_{t_{N-1}}^L$, respectively. The new ownership level Θ at t_N^- maximizes $B_{t_N}^L + G(t_N,\Theta) + \frac{\Theta}{\gamma^M r}\ln(-W_{t_N}^M)$ in the value function above. By considering the proceeds from the new trade, $(\Theta_{t_{N-1}}^L - \Theta)P(t_N,\Theta,W_{t_N}^M)$, it maximizes $B_{t_N^-}^L + (\Theta_{t_{N-1}}^L - \Theta)P(t_N,\Theta,W_{t_N}^M) + G(t_N,\Theta) + \frac{\Theta}{\gamma^M r}\ln(-W_{t_N}^M)$, which, along with (A15), derives the time-deterministic function G in L's value function at t_N^- as

$$\begin{aligned} G(t_N^-,\Theta_{t_{N-1}}^L) &= \max_\Theta\ (\Theta_{t_{N-1}}^L - \Theta)\Lambda(t_N,\Theta) + G(t_N,\Theta) \\ &= \max_\Theta\ (\Theta_{t_{N-1}}^L - \Theta)\frac{1}{r}k(\Theta) + \frac{1}{r}V(\Theta). \end{aligned} \tag{A36}$$

In the above, the second equation follows from the time-deterministic functions G and Λ in L's value function and in the stock price process, (A13) and (A31). The FOC is then given by

$$\text{FOC}: \ (\Theta_{t_{N-1}}^L - \Theta)\frac{1}{r}k'(\Theta) + \frac{1}{r}[V'(\Theta) - k(\Theta)] = 0. \tag{A37}$$

By the definitions of $V(\Theta)$ and $k(\Theta)$ in (A12) and (A30),

$$V(\Theta) = \Theta\Omega_1(\Theta) + \frac{\Theta}{\gamma^M r}\Omega_2(\Theta) - \frac{1}{2}\gamma^L r\Theta^2\Omega_3(\Theta), \tag{A38}$$

$$k(\Theta) = \Omega_1(\Theta) + \frac{1}{\gamma^M r}\Omega_2(\Theta) - \gamma^S r(1-\Theta)\Omega_3(\Theta), \tag{A39}$$

where $\Omega_1(\Theta) \equiv (\mu(\alpha) + \frac{1}{\gamma^M}\ln(\beta) - \Psi(\alpha))$, $\Omega_2(\Theta) \equiv \left(\mu_W - \frac{1}{2}\sigma_W^2\right)$, and $\Omega_3(\Theta) \equiv \left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)^2$.

From (A8), note that the manager's contractual parameters, α and β, also depend on Θ and that β is directly obtained by its relation to α. Also note that the variables μ_W and σ_W defined in (A10) are functions of α.

The derivative of $V(\Theta)$ with respect to Θ is

$$V'(\Theta) = \Omega_1(\Theta) + \Theta\Omega_1'(\Theta) + \frac{1}{\gamma^M r}\left(\Omega_2(\Theta) + \Theta\Omega_2'(\Theta)\right) - \gamma^L r\Theta\Omega_3(\Theta) - \frac{1}{2}\gamma^L r\Theta^2\Omega_3'(\Theta) \quad \text{(A40)}$$

Consider the three derivative terms above:

$$\begin{aligned} & \Theta\Omega_1'(\Theta) + \frac{1}{\gamma^M r}\Theta\Omega_2'(\Theta) - \frac{1}{2}\gamma^L r\Theta^2\Omega_3'(\Theta) \\ = \; & \Theta\left(\mu_1\alpha'(\Theta) + \frac{1}{\gamma^M}\frac{\beta'(\alpha)\alpha'(\Theta)}{\beta(\alpha)} - \psi\alpha(\Theta)\alpha'(\Theta)\right) \\ + \; & \frac{1}{\gamma^M r}\Theta\left(\mu_W'(\alpha)\alpha'(\Theta) - \sigma_W(\alpha)\sigma_W'(\alpha)\alpha'(\Theta)\right) - \gamma^L r\Theta^2\left(\sigma - \frac{\sigma_W(\alpha)}{\gamma^M r}\right)\left(-\frac{1}{\gamma^M r}\sigma_W'(\alpha)\alpha'(\Theta)\right) = 0. \end{aligned} \quad \text{(A41)}$$

By the envelope theorem, the sum of the above three terms that arise from the dependence of Θ through α must be zero. To see this clearly, recall that we chose the manager's optimal effort α to maximize L's value function in (A4) when L's ownership level is Θ and M's promised payoff is W_t^M, which in fact maximizes $G(t,\Theta)=\frac{1}{r}V(\Theta)$ at $t \in [t_N, \infty)$:

$$\begin{aligned} \frac{\partial V'(\alpha)}{\partial\alpha} \; = \; & \Theta\left(\mu_1 + \frac{1}{\gamma^M}\frac{\beta'(\alpha)}{\beta(\alpha)} - \psi\alpha(\Theta)\right) \\ + \; & \frac{1}{\gamma^M r}\Theta\left(\mu_W'(\alpha) - \sigma_W(\alpha)\sigma_W'(\alpha)\right) - \gamma^L r\Theta^2\left(\sigma - \frac{\sigma_W(\alpha)}{\gamma^M r}\right)\left(-\frac{1}{\gamma^M r}\sigma_W'(\alpha)\right) = 0. \end{aligned}$$

As a result, we see that the sum of the three derivative terms in (A41) is zero so that the derivative of $V(\Theta)$, (A40), is

$$V'(\Theta) = \Omega_1(\Theta) + \frac{1}{\gamma^M r}\Omega_2(\Theta) - \gamma^L r\Theta\Omega_3(\Theta), \quad \text{(A42)}$$

and the FOC (A37) reduces to

$$\text{FOC}: \; (\Theta_{t_{N-1}}^L - \Theta)\frac{1}{r}k'(\Theta) + \left[\gamma^S - (\gamma^L + \gamma^S)\Theta\right]\Omega_3(\Theta) = 0, \quad \text{(A43)}$$

where, by (A19), we note that $\Omega_3(\Theta) = \left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)^2 = \sigma_R^2(\Theta)$.

We solved for L's optimal ownership choice and optimal contracting with M and derived the equilibrium stock price from S's optimal consumption and portfolio problem for the last trading period $[t_N, \infty)$. We now complete the backward induction by deriving the equilibrium value functions and policies for an arbitrary earlier period $[t_i, t_{i+1})$ with $i < N$.

Proof of Proposition 1: General Period

Suppose that L holds an equity stake Θ in the firm at $t \in [t_i, t_{i+1})$ with $i < N$. L's optimal choices of consumption and M's contractual parameters solves

$$W_t^L = \max_{c_t^L, c_t^M, a_t^M} E_t^{a^M}\left[\int_t^\infty e^{-\delta^L(\tau-t)} u^L(c_\tau^L) d\tau\right] = \max_{c_t^L, c_t^M, a_t^M} E_t^{a^M}\left[-\frac{1}{\gamma^L}\int_t^{t_{i+1}} e^{-\delta^L(\tau-t)-\gamma^L c_\tau^L} d\tau + W_{t_{i+1}}^L\right], \tag{A44}$$

subject to M's incentive compatibility (IC) constraint:

$$\begin{aligned}(IC):\ dW_t^M &= \left(\delta^M W_t^M - u^M(c_t^M, a_t^M)\right) dt - \frac{u_a^M(c_t^M, a_t^M)}{\mu'(a_t^M)}\sigma dZ_t \\ &= \left(\delta^M W_t^M + \frac{1}{\gamma^M} H(c_t^M, a_t^M)\right) dt + \frac{\psi}{\mu_1} a_t^M H(c_t^M, a_t^M)\sigma dZ_t,\end{aligned} \tag{A45}$$

where $H(c_t^M, a_t^M) = e^{-\gamma^M(c_t^M - \frac{1}{2}\psi(a_t^M)^2)}$. Given that the range of M's possible effort choices, a_t^M, is bounded and the firm's cash flow function, $\mu(\cdot)$, is linear, the above SDE has a unique solution. L's money market account balance evolves according to

$$dB_t^L = \left(rB_t^L - c_t^L\right) dt + \Theta\left(dX_t - c_t^M dt\right) = \left(rB_t^L - c_t^L + \Theta(\mu(a_t^M) - c_t^M)\right) dt + \Theta\sigma dZ_t. \tag{A46}$$

As in the proof for the last period, we conjecture and verify that L's value function has the following exponential form:

$$W_t^L = F(t, B_t^L, W_t^M) = -\frac{1}{\gamma^L r} e^{-\gamma^L\left[r(B_t^L + G(t,\Theta)) + \frac{\Theta}{\gamma^M}\ln(-W_t^M) + \frac{\delta^L - r}{\gamma^L r}\right]}. \tag{A47}$$

The HJB equation is then given by

$$\begin{aligned}F_t + \max_{c_t^L, c_t^M, a_t^M}\Bigg[&F_B(rB_t^L - c_t^L + \Theta(\mu(a_t^M) - c_t^M)) + F_W\left(\delta^M W_t^M + \frac{1}{\gamma^M} H(c_t^M, a_t^M)\right) + \frac{1}{2} F_{BB}\Theta^2\sigma^2 + \\ &+ F_{BW}\Theta\sigma\left(\frac{\psi}{\mu_1} a_t^M H(c_t^M, a_t^M)\sigma\right) + \frac{1}{2} F_{WW}\left(\frac{\psi}{\mu_1} a_t^M H(c_t^M, a_t^M)\sigma\right)^2 + u^L(c_t^L) - \delta^L F\Bigg] = 0.\end{aligned} \tag{A48}$$

The FOCs with respect to (c_t^L, c_t^M, a_t^M) are then

$$\begin{aligned}&c_t^L:\ -F_B + u_c^L(c_t^L) = 0, \\ &c_t^M:\ F_B\Theta + F_W H + F_{BW}\Theta\gamma^M\left(\frac{\psi}{\mu_1} a_t^M H\right)\sigma^2 + F_{WW}\gamma^M\left(\frac{\psi}{\mu_1} a_t^M H\right)^2\sigma^2 = 0, \\ &a_t^M:\ F_B\Theta\mu_1 + F_W(\psi a_t^M H) + F_{BW}\Theta\left(1 + \gamma^M\psi(a_t^M)^2\right)\left(\frac{\psi}{\mu_1} H\right)\sigma^2 \\ &\qquad + F_{WW}\left(1 + \gamma^M\psi(a_t^M)^2\right)\left(\frac{\psi}{\mu_1} H\right)^2 a_t^M\sigma^2 = 0.\end{aligned} \tag{A49}$$

The above equations, along with the derivatives of $F(t, B_t^L, W_t^M)$, lead to the following optimal policies

$$\begin{aligned} c_t^L &= r(B_t^L + G(t,\Theta)) + \frac{\Theta}{\gamma_M}\ln(-W_t^M) + \frac{\delta^L - r}{\gamma^L r}, \\ a_t^M &= \alpha, \\ c_t^M &= -\frac{1}{\gamma^M}\ln\beta - \frac{1}{\gamma^M}\ln(-W_t^M) + \Psi(a_t^M), \end{aligned} \tag{A50}$$

where α and β are determined by

$$\begin{aligned} \beta &= \gamma^M r\left[\gamma^M \alpha(\psi\alpha - \mu_1) + 1\right], \\ \left(1 + \frac{\gamma^L}{\gamma^M}\Theta\right)\left(\frac{\psi}{\mu_1}\alpha\beta\sigma\right)^2 &- (\gamma^L r\Theta\sigma)\left(\frac{\psi}{\mu_1}\alpha\beta\sigma\right) - \left(r - \frac{\beta}{\gamma^M}\right) = 0. \end{aligned} \tag{A51}$$

By M's IC constraint (A45) and the optimal policies in (A50), M's promised payoff follows a geometric Brownian motion process:

$$\begin{aligned} dW_t^M &= \mu_W W_t^M dt - \sigma_W W_t^M dZ_t, \\ \Rightarrow d\ln(-W_t^M) &= \left(\mu_W - \frac{1}{2}\sigma_W^2\right)dt - \sigma_W dZ_t, \end{aligned} \tag{A52}$$

where

$$\mu_W = \delta^M - \frac{\beta}{\gamma^M};\ \sigma_W = \frac{\psi}{\mu_1}\alpha\beta\sigma. \tag{A53}$$

Substituting the optimal policies in (A50) and the derivatives of $F(t, B_t^L, W_t^M)$ into the HJB equation (A48) yields

$$G_t(t,\Theta) = rG(t,\Theta) - V(\Theta), \tag{A54}$$

where $V(\Theta)$ is

$$V(\Theta) = \Theta\left(\mu(\alpha) - \left(-\frac{1}{\gamma^M}\ln\beta - \frac{1}{\gamma^M r}\left(\mu_W - \frac{1}{2}\sigma_W^2\right) + \Psi(\alpha)\right)\right) - \frac{1}{2}\gamma^L r\Theta^2\left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)^2. \tag{A55}$$

By integrating (A54) over the period $[t, t_{i+1})$, we obtain

$$G(t,\Theta) = \phi_i(t)V(\Theta) + (1 - r\phi_i(t))G(t_{i+1},\Theta), \tag{A56}$$

where $\phi_i(t) = \frac{1}{r}\left(1 - e^{-r(t_{i+1}-t)}\right)$. Again, one can verify that the above W_t^L process, (A47), satisfies the conditions for the verification theorem of dynamic programming so that it is, indeed, L's value function.

Proof of Corollary 1: General Period

See M's promised payoff process, specified by (A52) and (A53), in the above proof.

Proof of Proposition 2: General Period

The equilibrium stock price is determined by small shareholders' competitive trading in the stock market. In a similar manner to the proof for the last trading period, we conjecture and verify that, for any given level of L's ownership Θ, the equilibrium stock price is linear in $\ln(-W_t^M)$, and has the form of:

$$P(t,\Theta,W_t^M) = \Lambda(t,\Theta) + \frac{1}{\gamma^M r}\ln(-W_t^M). \tag{A57}$$

By taking the stock price given, the representative small shareholder S makes its optimal consumption and portfolio choices to maximize:

$$W_t^S = \max_{c_t^S,\theta_t^S} E_t^{a^M}\left[\int_t^\infty e^{-\delta^S(\tau-t)}u^S(c_\tau^S)d\tau\right] = E_t^{a^M}\left[-\frac{1}{\gamma^S}\int_t^{t_{i+1}} e^{-\delta^S(\tau-t)-\gamma^S c_\tau^S}d\tau + W_{t_{i+1}}^S\right], \tag{A58}$$

subject to its total wealth process,

$$dY_t^S = (rY_t^S - c_t^S)dt + \theta_t^S(dX_t - c_t^M dt + dP_t - rP_t dt) = (rY_t^S - c_t^S)dt + \theta_t^S dR_t. \tag{A59}$$

In the above, R_t represents the excess dollar return of the stock that, given the guessed form of the stock price (A57), follows

$$dR_t = \mu_R(t,\Theta)dt + \sigma_R(t,\Theta)dZ_t, \tag{A60}$$

where the drift term, $\mu_R(t,\Theta)$, and the diffusion term, $\sigma_R(t,\Theta)$, are given by

$$\mu_R(t,\Theta) = \mu(\alpha) + \frac{1}{\gamma^M}\ln\beta + \frac{1}{\gamma^M r}\left(\mu_W - \frac{1}{2}\sigma_W^2\right) - \Psi(\alpha) + \Lambda_t(t,\Theta) - r\Lambda(t,\Theta), \tag{A61}$$

$$\sigma_R(t,\Theta) = \left(\sigma - \frac{\sigma_W}{\gamma^M r}\right). \tag{A62}$$

Using the definition of σ_W (A53) and the equations for M's contractual parameters α and β in (A51), we now show that $\sigma_R(\Theta)$ defined above is greater than zero. The first equation in (A51) implies that if $\alpha < (>)\frac{\mu_1}{\psi}$, then $\beta < (>)\gamma_M r$. By the second equation in (A51) and (A53),

$$\sigma_W = \frac{\gamma_L r\Theta\sigma \pm \left[(\gamma_L r\Theta\sigma)^2 + 4\left(1+\frac{\gamma_L}{\gamma_M}\Theta\right)\left(r - \frac{\beta}{\gamma_M}\right)\right]^{\frac{1}{2}}}{2\left(1+\frac{\gamma_L}{\gamma_M}\Theta\right)}. \tag{A63}$$

Suppose that $\alpha > \frac{\mu_1}{\psi}$. As noted, we then have $\beta > \gamma_M r$ and, therefore, $\sigma_W = \frac{\psi}{\mu_1}\alpha\beta\sigma > \gamma_M r\sigma$. If $\beta > \gamma_M r$, (A63) implies that $\sigma_W < \frac{\gamma_L r\Theta\sigma}{\left(1+\frac{\gamma_L}{\gamma_M}\Theta\right)}$. By comparing $\gamma_M r\sigma$ and $\frac{\gamma_L r\Theta\sigma}{\left(1+\frac{\gamma_L}{\gamma_M}\Theta\right)}$, we see that the former is greater than the latter so that $\sigma_W > \gamma_M r\sigma$ and $\sigma_W < \frac{\gamma_L r\Theta\sigma}{\left(1+\frac{\gamma_L}{\gamma_M}\Theta\right)}$ are not consistent with each other. Accordingly, it must be the case that $\alpha < \frac{\mu_1}{\psi}$. Then, $\beta < \gamma_M r$

and $\sigma_W = \frac{\psi}{\mu_1}\alpha\beta\sigma < \gamma_M r\sigma$. As σ_W needs to be non-negative (by its definition),

$$\sigma_W = \frac{\gamma_L r\Theta\sigma + \left[(\gamma_L r\Theta\sigma)^2 + 4\left(1+\frac{\gamma_L}{\gamma_M}\Theta\right)\left(r-\frac{\beta}{\gamma_M}\right)\right]^{\frac{1}{2}}}{2\left(1+\frac{\gamma_L}{\gamma_M}\Theta\right)} > \frac{\gamma_L r\Theta\sigma}{\left(1+\frac{\gamma_L}{\gamma_M}\Theta\right)}, \tag{A64}$$

which is consistent with $\sigma_W < \gamma_M r\sigma$ as $\gamma_M r\sigma > \frac{\gamma_L r\Theta\sigma}{\left(1+\frac{\gamma_L}{\gamma_M}\Theta\right)}$. Therefore, $\sigma_R(\Theta) = \left(\sigma - \frac{\sigma_W}{\gamma^M r}\right) > 0$.

As we did in the proof for the last trading period, we conjecture S's value function as a negative exponential form and solve for its optimal consumption and demand for the stock. We then impose the stock market clearing condition, $\int_S \theta_t^S dS = 1-\Theta$, from which we obtain

$$\mu_R(\Theta) = (1-\Theta)\,\gamma^S r\sigma_R^2(\Theta). \tag{A65}$$

Note that the mean and volatility terms in the excess dollar return process, (A62) and (A65), are time invariant and dependent upon L's ownership level Θ. By combining (A65) with (A61) and (A62), we obtain the following ODE for the time-deterministic function $\Lambda(t,\Theta)$ in the stock price process (A57):

$$\Lambda_t(t,\Theta) = r\Lambda(t,\Theta) - k(\Theta), \tag{A66}$$

where

$$k(\Theta) = \mu(\alpha) - \left(-\frac{1}{\gamma^M}\ln\beta - \frac{1}{\gamma^M r}\left(\mu_W - \frac{1}{2}\sigma_W^2\right) + \Psi(\alpha)\right) - \gamma^S r(1-\Theta)\left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)^2. \tag{A67}$$

By integrating (A66) over period $[t, t_{i+1})$, we express $\Lambda(t,\Theta)$ recursively as follows.

$$\Lambda(t,\Theta) = \phi_i(t)k(\Theta) + (1 - r\phi_i(t))\Lambda(t_{i+1},\Theta), \tag{A68}$$

where $\phi_i(t) = \frac{1}{r}\left(1 - e^{-r(t_{i+1}-t)}\right)$.

Proof of Corollary 2: General Period

As specified by (30) and (38), the sensitivities of managerial pay to the firm's output and to the excess dollar return of the stock, PPS_X and PPS_R, both increase with σ_W. By (A62), the dollar return volatility of the stock decreases with σ_W so that we immediately obtain the negative relation between the PPS measures and the dollar return volatility σ_R. As noted in the earlier proofs, σ_W varies with L's ownership level Θ. The expected excess dollar return $\mu_R(\Theta)$ and the Sharp ratio $\Sigma_R(\Theta)$ of the stock, which is the ratio of the expected dollar return to the dollar return volatility $\left(\Sigma_R(\Theta) = \frac{\mu_R(\Theta)}{\sigma_R(\Theta)} = (1-\Theta)\,\gamma^S r\sigma_R(\Theta)\right)$, also decrease with σ_W and are therefore indirectly affected by Θ. However, these two stock variables are also directly affected by Θ through the additional term $1-\Theta$. By taking the derivatives of $\mu_R(\Theta)$ and $\Sigma_R(\Theta)$ with respect to Θ, we have

$$\frac{d\mu_R(\Theta)}{d\Theta} = -\gamma^S r\left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)\left[\left(\sigma - \frac{\sigma_W}{\gamma^M r}\right) + 2(1-\Theta)\frac{\sigma_W'(\Theta)}{\gamma^M r}\right], \tag{A69}$$

$$\frac{d\Sigma_R(\Theta)}{d\Theta} = -\gamma^S r\left[\left(\sigma - \frac{\sigma_W}{\gamma^M r}\right) + (1-\Theta)\frac{\sigma_W'(\Theta)}{\gamma^M r}\right]. \tag{A70}$$

From the above, it is clear that as long as $\sigma_W'(\Theta) \geq 0$, the effects of Θ on the expected dollar return and the Sharpe ratio are negative. Because the manager's PPS measures are positively related to σ_W and, therefore, to Θ when $\sigma_W'(\Theta) \geq 0$, the expected dollar return and the Sharpe ratio are also unambiguously negatively associated with the manager's PPS measures.

Proof of Proposition 3: General Period

We now consider L's optimal ownership choice for period $[t_i, t_{i+1})$ with $i < N$, which is made at date t_i^-. By (A47) in the proof of Proposition 1, L's value function at t_i right after she chooses the ownership level Θ is

$$W_{t_i}^L = -\frac{1}{\gamma^L r}e^{-\gamma^L\left[r(B_{t_i}^L + G(t_i,\Theta)) + \frac{\Theta}{\gamma^M}\ln(-W_{t_i}^M) + \frac{\delta^L - r}{\gamma^L r}\right]}. \tag{A71}$$

We denote L's money market account balance and shareholdings at t_i^- (before her new trade) by $B_{t_i^-}^L$ and $\Theta_{t_i^-}^L = \Theta_{t_{i-1}}^L$, respectively. L's optimal trading decision, that is, the new ownership level Θ at t_i^- maximizes $B_{t_i}^L + G(t_i,\Theta) + \frac{\Theta}{\gamma^M r}\ln(-W_{t_i}^M)$ in the value function above. By considering the proceeds from the new trade, $(\Theta_{t_{i-1}}^L - \Theta)P(t_i,\Theta,W_{t_i}^M)$, it maximizes $B_{t_i^-}^L + (\Theta_{t_{i-1}}^L - \Theta)P(t_i,\Theta,W_{t_i}^M) + G(t_i,\Theta) + \frac{\Theta}{\gamma^M r}\ln(-W_{t_i}^M)$, which, along with (A57), derives the time-deterministic function G in L's value function at t_i^- as below:

$$\begin{aligned} G(t_i^-, \Theta_{t_{i-1}}^L) &= \max_{\Theta}\,(\Theta_{t_{i-1}}^L - \Theta)\Lambda(t_i,\Theta) + G(t_i,\Theta) \\ &= \max_{\Theta}\,(\Theta_{t_{i-1}}^L - \Theta)\Lambda(t_i,\Theta) + \phi_i(t_i)V(\Theta) + (1 - r\phi_i(t_i))G(t_{i+1},\Theta), \end{aligned} \tag{A72}$$

where the second line follows from (A56). The FOC is then derived as

$$\text{FOC}:\ (\Theta_{t_{i-1}}^L - \Theta)\frac{\partial\Lambda(t_i,\Theta)}{\partial\Theta} - \Lambda(t_i,\Theta) + \phi_i(t_i)V'(\Theta) + (1 - r\phi_i(t_i))\frac{\partial G(t_{i+1},\Theta)}{\partial\Theta} = 0, \tag{A73}$$

where the last term is, due to the envelope theorem,

$$\frac{\partial G(t_{i+1},\Theta)}{\partial\Theta} = \Lambda(t_{i+1},\Theta_{t_{i+1}}^L). \tag{A74}$$

By (A68), we rewrite the FOC above as

$$\text{FOC}:\ (\Theta_{t_{i-1}}^L - \Theta)\frac{\partial\Lambda(t_i,\Theta)}{\partial\Theta} + \phi_i(t_i)\left[V'(\Theta) - k(\Theta)\right] = 0. \tag{A75}$$

By the definitions of $V(\Theta)$ and $k(\Theta)$ in (A55) and (A67),

$$\begin{aligned} V(\Theta) &= \Theta\Omega_1(\Theta) + \frac{\Theta}{\gamma^M r}\Omega_2(\Theta) - \frac{1}{2}\gamma^L r\Theta^2\Omega_3(\Theta) \\ k(\Theta) &= \Omega_1(\Theta) + \frac{1}{\gamma^M r}\Omega_2(\Theta) - \gamma^S r(1-\Theta)\Omega_3(\Theta), \end{aligned}$$

where $\Omega_1(\Theta) \equiv (\mu(\alpha) + \frac{1}{\gamma^M}\ln(\beta) - \Psi(\alpha))$, $\Omega_2(\Theta) \equiv \left(\mu_W - \frac{1}{2}\sigma_W^2\right)$, and $\Omega_3(\Theta) \equiv \left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)^2$.

The derivative of $V(\Theta)$ with respect to Θ is

$$V'(\Theta) = \Omega_1(\Theta) + \Theta\Omega_1'(\Theta) + \frac{1}{\gamma^M r}\left(\Omega_2(\Theta) + \Theta\Omega_2'(\Theta)\right) - \gamma^L r\Theta\Omega_3(\Theta) - \frac{1}{2}\gamma^L r\Theta^2\Omega_3'(\Theta) \quad \text{(A76)}$$

By the envelope theorem (and as we showed in the proof for the last period), the sum of the three terms $\Theta\Omega_1'(\Theta) + \frac{1}{\gamma^M r}\Theta\Omega_2'(\Theta) - \frac{1}{2}\gamma^L r\Theta^2\Omega_3'(\Theta)$ in (A76) that arise from the dependence of Θ through α must be zero. We thus have

$$V'(\Theta) = \Omega_1(\Theta) + \frac{1}{\gamma^M r}\Omega_2(\Theta) - \gamma^L r\Theta\Omega_3(\Theta). \quad \text{(A77)}$$

As a result, the FOC (A75) further reduces to

$$\text{FOC}: \ (\Theta^L_{t_{i-1}} - \Theta)\frac{\partial\Lambda(t_i,\Theta)}{\partial\Theta} + \phi_i(t_i)\left[\gamma^S - (\gamma^L + \gamma^S)\Theta\right] r\Omega_3(\Theta) = 0, \quad \text{(A78)}$$

where note that $\Omega_3(\Theta) = \left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)^2 = \sigma_R^2(\Theta) > 0$.

Proof of Corollary 3: General Period

By (A78), it is evident that, in the steady-state equilibrium, L's optimal ownership level solves

$$\text{FOC}: \ \phi_i(t_i)\left[\gamma^S - (\gamma^L + \gamma^S)\Theta\right] r\Omega_3(\Theta) = 0,$$

which, because of $\Omega_3(\Theta) = \left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)^2 = \sigma_R^2(\Theta) > 0$ as shown in the proof of Proposition 2, leads to the steady state L's ownership, $\Theta^{L*}_{ss} = \gamma^S/(\gamma^L + \gamma^S)$.

Table I: Baseline Parameter Values. This table provides the description of model parameters and their baseline values from calibration.

Model parameter	Value
Calibrated outside the model	
Time interval between L's successive trading dates ($\triangle_t$)	1
Annual risk-free rate (r)	0.0194
Time discount rate ($\delta^L = \delta^M = \delta^S$)	0.0404
Calibrated inside the model	
Constant term (μ_0) in the firm's mean cash flow	0.095
(set directly from the normalized book value of the firm ($BV_{ss} = 1$))	
Productivity of effort (μ_1) in the firm's mean cash flow	0.99
Firm cash flow (output) volatility (σ)	0.34
Unit cost of effort (ψ)	10.11
L's absolute risk aversion (γ^L)	133.88
M's absolute risk aversion (γ^M)	108.58
S's absolute risk aversion (γ^S)	44.63
M's mean promised payoff in the steady state ($\ln(-W_{ss}^M)$)	-9.13

Table II: Actual and Model-Predicted Moments.

This table compares the model-predicted moments from the steady-state equlibrium of the main second-best agency contracting model that we described in Section 3 with the actual moments from the data.

	L's ownership	% stock return volatility	Sharpe ratio	Market-to-book ratio	Dividend-price ratio	PPS_R	Executive pay-to-market value ratio
Actual	0.250	0.194	0.320	1.215	0.045	0.017	0.003
Predicted	0.250	0.196	0.154	1.215	0.045	0.008	0.107

Table III: Steady-State Equilibrium Variables in Main and Two Benchmark Models.

This table compares the steady-state equilibrium variables for the main (second-best agency contracting) model with those in the first-best and owner-manager benchmark models.

	L's ownership	Effort	PPS_X	PPS_R	Expected dollar return	Dollar return volatility	Expected % return	% return volatility	Sharp ratio	Market-to-book ratio	Dividend-price ratio
Second-Best Case	0.250	0.091	0.006	0.0083	0.037	0.238	0.030	0.196	0.154	1.215	0.045
First-Best Case	0.250	0.098	0.005	0.0060	0.044	0.260	0.034	0.201	0.169	1.292	0.051
Owner-Manager Case	0.250	0.024			0.075	0.340	0.014	0.061	0.221	5.549	0.021

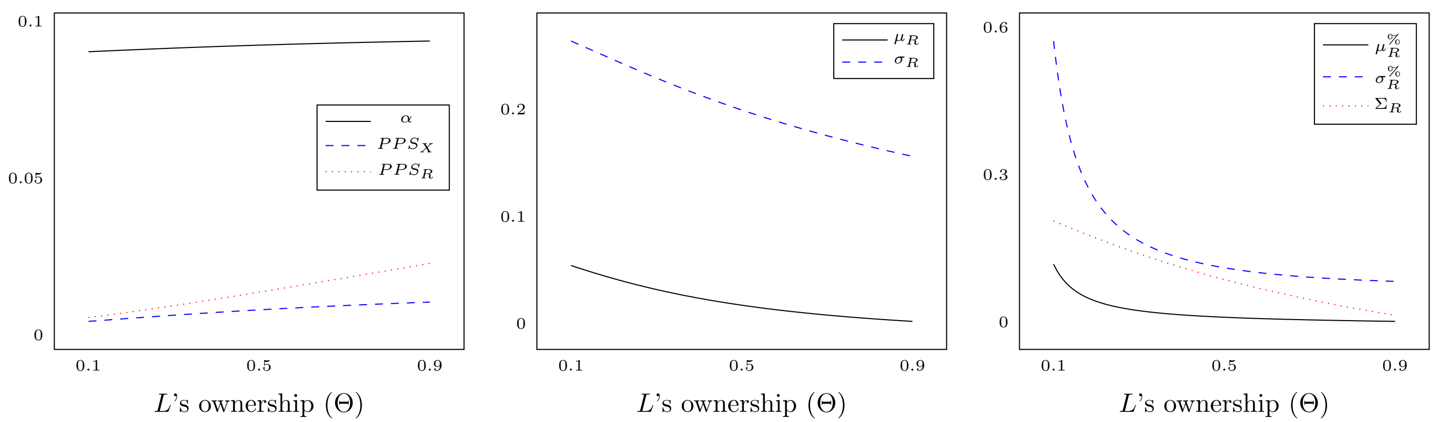

Figure 1: Effects of Large Shareholder Ownership in Main Model

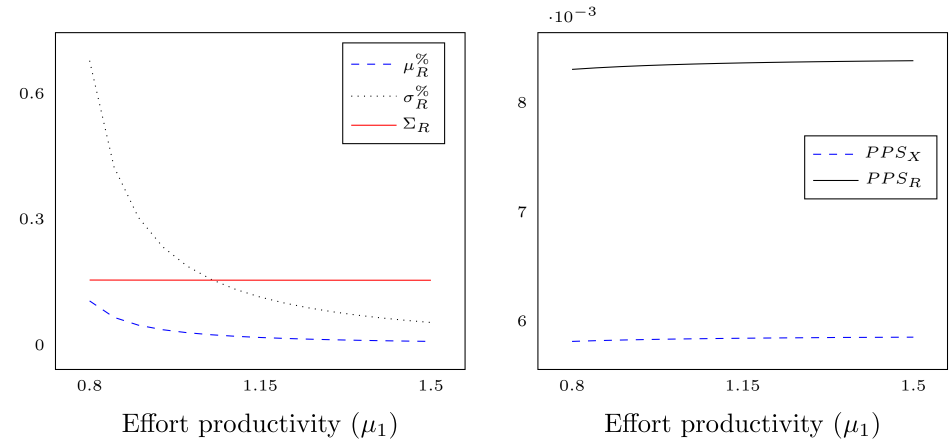

Figure 2: Productivity, Stock Returns, and Incentives in Main Model

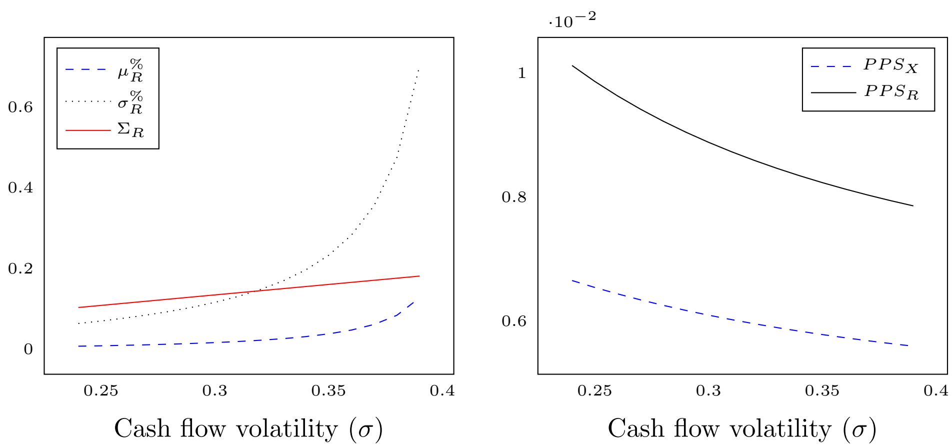

Figure 3: Cash Flow Volatility, Stock Returns, and Incentives in Main Model

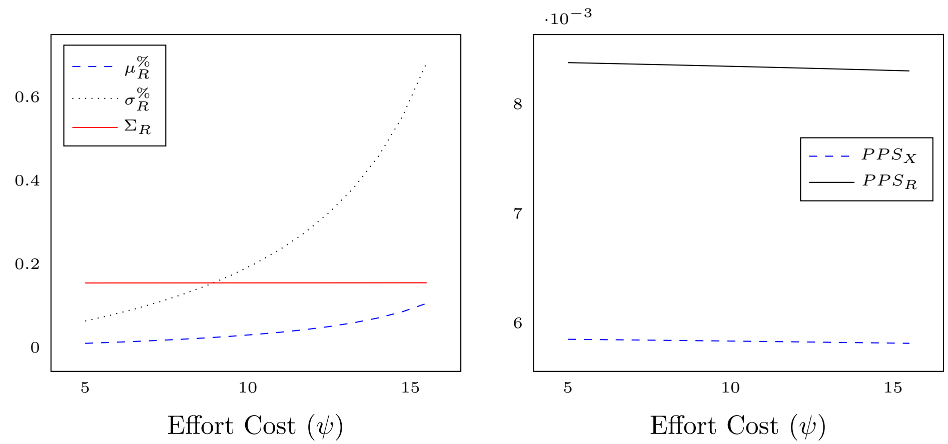

Figure 4: Effort Cost, Stock Returns, and Incentives in Main Model

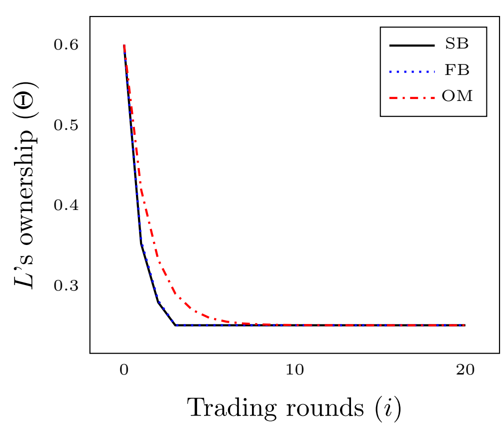

Figure 5: Large Shareholder Ownership Dynamics in Main and Benchmark Models

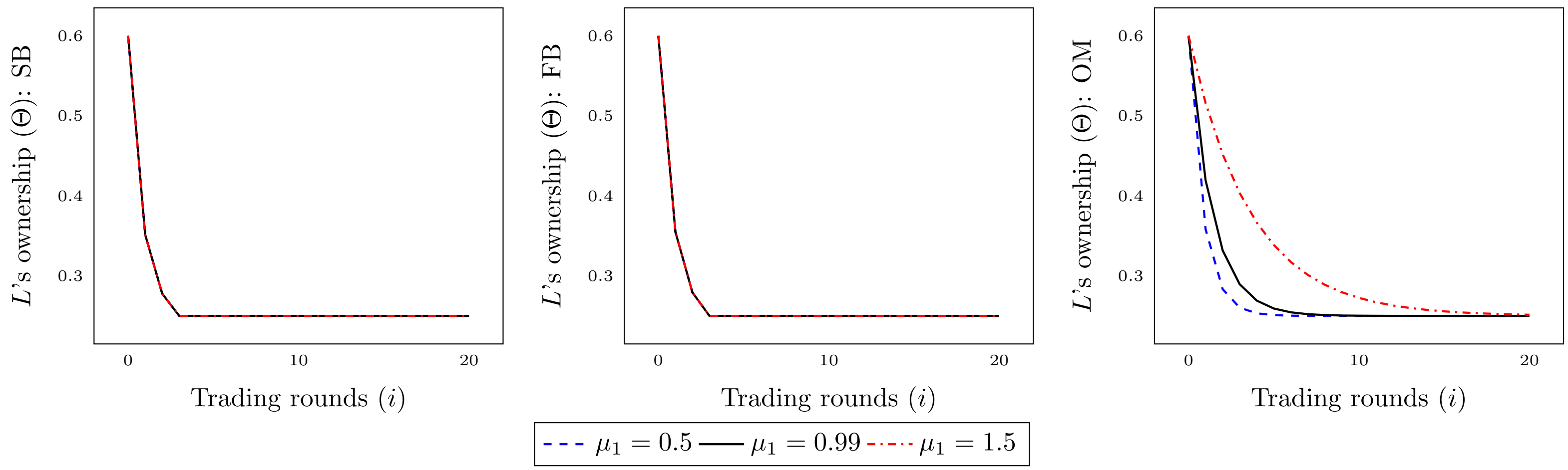

Figure 6: Productivity and Large Shareholder Ownership in Main and Benchmark Models

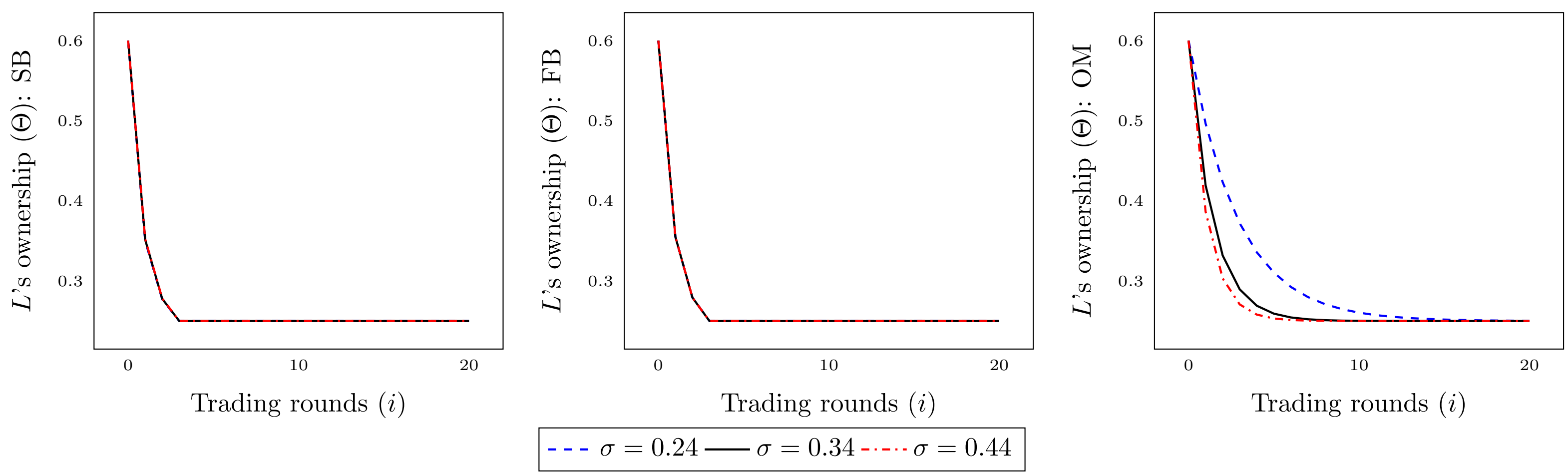

Figure 7: Cash Flow Volatility and Large Shareholder Ownership in Main and Benchmark Models

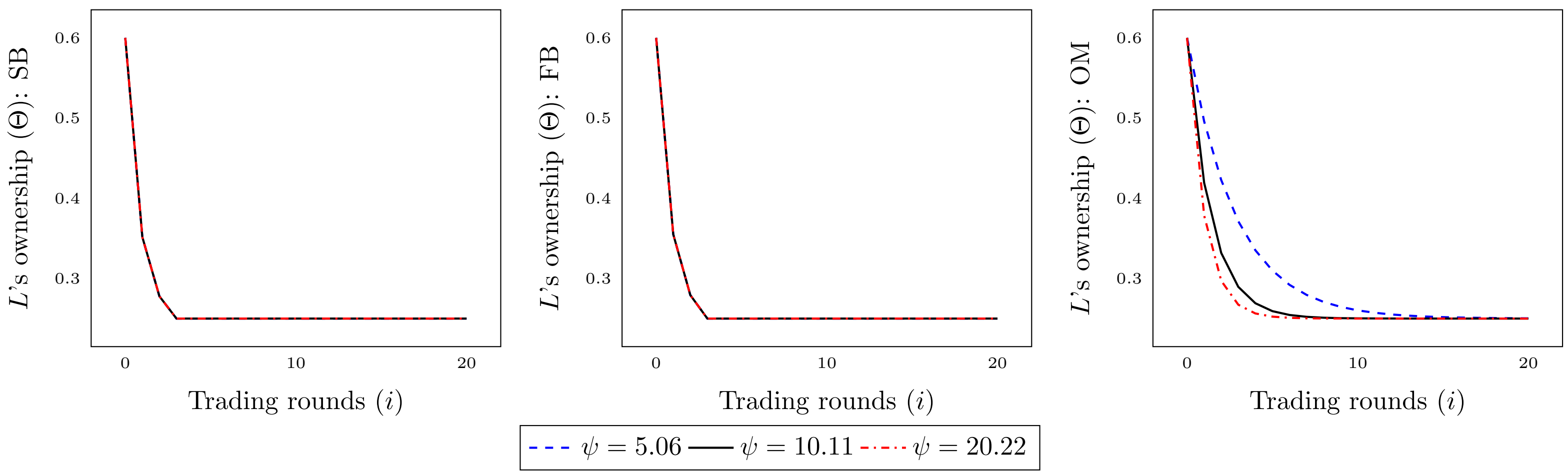

Figure 8: Effort Cost and Large Shareholder Ownership in Main and Benchmark Models

Internet Appendix for "Ownership Structure, Incentives, and Asset Prices"

A Proofs for Benchmark Models in Section 4

A.1 Proofs for the First-Best Benchmark Model

We obtain the first-best equilibrium solution in a similar manner to that shown in the previous proofs for our main model (which is the second-best agency contracting scenario). We first characterize the equilibrium value functions and policies in the last period $[T, \infty)$, and then proceed to earlier periods. For expositional convenience, we show the proofs for an arbitrary period $[t_i, t_{i+1})$ with $i \leq N$.

Proof of Proposition 4

Consider period $t \in [t_i, t_{i+1})$ with $i \leq N$ where L's ownership level is $\Theta_t^L = \Theta$. As M is always committed to exert the effort level desired by L in the first-best case, L can freely choose the sensitivity of M's promised payoff, χ_t^M, as another control variable without being subject to the M's incentive compatibility constraint. M's promised payoff then evolves as

$$dW_t^M = \left(\delta^M W_t^M - u^M(c_t^M, a_t^M)\right) dt + \chi_t^M \sigma dZ_t. \tag{A79}$$

Given L's ownership choice Θ, L's optimal choices of consumption and the terms of M 's contract solve

$$W_t^L = \max_{c_t^L, c_t^M, a_t^M, \chi_t^M} E_t^{a^M} \left[\int_t^\infty e^{-\delta^L(\tau - t)} u^L(c_\tau^L) d\tau \right] = \max_{c_t^L, c_t^M, a_t^M, \chi_t^M} E_t^{a^M} \left[-\frac{1}{\gamma^L} \int_t^{t_{i+1}} e^{-\delta^L(\tau-t) - \gamma^L c_\tau^L} d\tau + W_{t_{i+1}}^L \right], \tag{A80}$$

subject to L's budget constraint,

$$dB_t^L = \left(rB_t^L - c_t^L\right) dt + \Theta\left(dX_t - c_t^M dt\right) = \left(rB_t^L - c_t^L + \Theta\left(\mu(a_t^M) - c_t^M\right)\right) dt + \Theta\sigma dZ_t, \tag{A81}$$

and M's promised payoff process, (A79). In (A80), $W_{t_{i+1}}^L = 0$ when $i = N$ (that is, $t_{i+1} = t_{N+1} = \infty$). Similar to solving our main model, we conjecture and verify that L's value function has the following exponential form:

$$W_t^L = F(t, B_t^L, W_t^M) = -\frac{1}{\gamma^L r} e^{-\gamma^L\left[r(B_t^L + \widehat{G}(t,\Theta)) + \frac{\Theta}{\gamma^M}\ln(-W_t^M) + \frac{\delta^L - r}{\gamma^L r}\right]}. \tag{A82}$$

L's HJB equation is then given by

$$\begin{aligned} F_t + \max_{c_t^L, c_t^M, a_t^M, \chi_t^M} \Bigg[& F_B(rB_t^L - c_t^L + \Theta(\mu(a_t^M) - c_t^M)) + F_W\left(\delta^M W_t^M + \frac{1}{\gamma^M} H(c_t^M, a_t^M)\right) + \frac{1}{2} F_{BB}\Theta^2\sigma^2 \\ & + F_{BW}\Theta\chi_t^M\sigma^2 + \frac{1}{2}F_{WW}\left(\chi_t^M\sigma\right)^2 + u^L(c_t^L) - \delta^L F \Bigg] = 0, \end{aligned} \tag{A83}$$

where $H = e^{-\gamma^M (c_t^M - \frac{1}{2}\psi(a_t^M)^2)}$. The FOCs with respect to $\left(c_t^L, c_t^M, a_t^M, \chi_t^M\right)$ are then

$$
\begin{aligned}
c_t^L &: \ -F_B + u_c^L(c_t^L) = 0,\\
c_t^M &: \ F_B\Theta + F_W H = 0,\\
a_t^M &: \ F_B\Theta\mu_1 + F_W\psi a_t^M H = 0,\\
\chi_t^M &: \ F_{BW}\Theta + F_{WW}\chi_t^M = 0.
\end{aligned}
\tag{A84}
$$

Using the derivatives of $F(t, B_t^L, W_t^M)$ and the FOCs in (A84), we obtain the optimal policies in the first-best case as follow:

$$
\begin{aligned}
\hat{c}_t^L &= r(B_t^L + \widehat{G}(t,\Theta)) + \frac{\Theta}{\gamma^M}\ln(-W_t^M) + \frac{\delta^L - r}{\gamma^L r},\\
\hat{a}_t^M &= \frac{\mu_1}{\psi},\\
\hat{c}_t^M &= -\frac{1}{\gamma^M}\ln(\gamma^M r) - \frac{1}{\gamma^M}\ln(-W_t^M) + \Psi(\hat{a}_t^M),\\
\hat{\chi}_t^M &= \frac{\gamma^L r\Theta}{\left(1 + \frac{\gamma^L}{\gamma^M}\Theta\right)}(-W_t^M).
\end{aligned}
\tag{A85}
$$

By (A79) and the optimal policies in (A85), the log of M's promised payoff process W_t^M follows

$$
d\ln(-W_t^M) = \left(\hat{\mu}_W - \frac{1}{2}\hat{\sigma}_W^2\right)dt - \hat{\sigma}_W dZ_t,
\tag{A86}
$$

where

$$
\hat{\mu}_W \equiv \delta^M - r;\ \ \hat{\sigma}_W \equiv \frac{\gamma^L r\Theta}{\left(1 + \frac{\gamma^L}{\gamma^M}\Theta\right)}\sigma.
\tag{A87}
$$

Substituting the optimal policies and the derivatives of $F(t, B_t^L, W_t^M)$ into the HJB equation (A83) yields

$$
\widehat{G}_t(t,\Theta) = r\widehat{G}(t,\Theta) - \widehat{V}(\Theta),
\tag{A88}
$$

where $\widehat{V}(\Theta)$ is given by:

$$
\widehat{V}(\Theta) = \Theta\left(\mu(\hat{a}_t^M) - \left(-\frac{1}{\gamma^M}\ln(\gamma^M r) - \frac{1}{\gamma^M r}\left(\widehat{\mu}_W - \frac{1}{2}\widehat{\sigma}_W^2\right) + \Psi(\hat{a}_t^M)\right)\right) - \frac{1}{2}\gamma^L r\Theta^2\left(\sigma - \frac{\widehat{\sigma}_W}{\gamma^M r}\right)^2.
\tag{A89}
$$

By integrating (A88) over the period $[t, t_{i+1})$, we obtain

$$
\widehat{G}(t,\Theta) = \phi_i(t)\widehat{V}(\Theta) + (1 - r\phi_i(t))\widehat{G}(t_{i+1},\Theta).
\tag{A90}
$$

where $\phi_i(t) = \frac{1}{r}\left(1 - e^{-r(t_{i+1}-t)}\right)$. Again, one can verify that the above W_t^L process, (A82), satisfies the conditions for the dynamic programming verification theorem so that it is, indeed, L's value function.

Proof of Proposition 5

The equilibrium stock price is determined by small shareholders' competitive trading in the stock market. In a similar manner to the proof for the main model, we conjecture and verify that, for any given level of L's ownership Θ, the equilibrium stock price is linear in $\ln(-W_t^M)$, and has the form of:

$$\widehat{P}(t, \Theta, W_t^M) = \widehat{\Lambda}(t, \Theta) + \frac{1}{\gamma^M r} \ln(-W_t^M). \tag{A91}$$

Given the above stock price process, S makes its optimal consumption and portfolio choices to maximize:

$$W_t^S = \max_{c_t^S, \theta_t^S} E_t^{a^M} \left[\int_t^\infty e^{-\delta^S(\tau - t)} u^S(c_\tau^L) d\tau \right] = E_t^{a^M} \left[-\frac{1}{\gamma^S} \int_t^{t_{i+1}} e^{-\delta^S(\tau-t) - \gamma^S c_\tau^S} d\tau + W_{t_{i+1}}^S \right], \tag{A92}$$

subject to its total wealth process,

$$dY_t^S = (rY_t^S - c_t^S)dt + \theta_t^S(dX_t - c_t^M dt + d\widehat{P}_t - r\widehat{P}_t dt) = (rY_t^S - c_t^S)dt + \theta_t^S d\widehat{R}_t. \tag{A93}$$

In the above, $\widehat{R}_t$ is the excess dollar return of the stock that, given the guessed form of the stock price process (A91), follows

$$d\widehat{R}_t = \widehat{\mu}_R(t, \Theta)dt + \widehat{\sigma}_R(t, \Theta)dZ_t, \tag{A94}$$

where the drift term, $\widehat{\mu}_R(t, \Theta)$, and the diffusion term, $\widehat{\sigma}_R(t, \Theta)$, are given by

$$\widehat{\mu}_R(t, \Theta) = \mu(\widehat{a}_t^M) + \frac{1}{\gamma^M} \ln(\gamma^M r) + \frac{1}{\gamma^M r} \left(\widehat{\mu}_W - \frac{1}{2}\widehat{\sigma}_W^2 \right) - \Psi(\widehat{a}_t^M) + \widehat{\Lambda}_t(t, \Theta) - r\widehat{\Lambda}(t, \Theta), \tag{A95}$$

$$\widehat{\sigma}_R(t, \Theta) = \left(\sigma - \frac{\widehat{\sigma}_W}{\gamma^M r} \right) = \frac{\sigma}{\left(1 + \frac{\gamma^L}{\gamma^M}\Theta\right)} > 0. \tag{A96}$$

The second equation for $\widehat{\sigma}_R$ in (A96) follows from (A87).

We conjecture S's value function as a negative exponential form and solve for its optimal consumption and demand for the stock. We then impose the stock market clearing condition, $\int_S \theta_t^S dS = 1 - \Theta$, from which we obtain

$$\widehat{\mu}_R(\Theta) = (1 - \Theta)\, \gamma^S r \widehat{\sigma}_R^2(\Theta). \tag{A97}$$

By combining the above with (A95) and (A96), we obtain the following ODE for the time-deterministic function $\widehat{\Lambda}(t, \Theta)$ in the stock price process (A91):

$$\widehat{\Lambda}_t(t, \Theta) = r\widehat{\Lambda}(t, \Theta) - \widehat{k}(\Theta), \tag{A98}$$

where

$$\widehat{k}(\Theta)=\mu(\widehat{a}_t^M)-\left(-\frac{1}{\gamma^M}\ln(\gamma^M r)-\frac{1}{\gamma^M r}\left(\widehat{\mu}_W-\frac{1}{2}\widehat{\sigma}_W^2\right)+\Psi(\widehat{a}_t^M)\right)-\gamma^S r(1-\Theta)\left(\sigma-\frac{\widehat{\sigma}_W}{\gamma^M r}\right)^2. \tag{A99}$$

By integrating (A98) over period $[t,t_{i+1})$, we express $\widehat{\Lambda}(t,\Theta)$ recursively as follows.

$$\widehat{\Lambda}(t,\Theta)=\phi_i(t)\widehat{k}(\Theta)+(1-r\phi_i(t))\widehat{\Lambda}(t_{i+1},\Theta), \tag{A100}$$

where $\phi_i(t)=\frac{1}{r}\left(1-e^{-r(t_{i+1}-t)}\right)$.

Proof of Proposition 6

We now consider L's optimal ownership choice for the period $[t_i,t_{i+1})$ with $i\le N$, which is made at date t_i^-. By (A82) in the proof of Proposition 4, L's value function at t_i right after she chooses the ownership level Θ is

$$W_{t_i}^L=-\frac{1}{\gamma^L r}e^{-\gamma^L\left[r(B_{t_i}^L+\widehat{G}(t_i,\Theta))+\frac{\Theta}{\gamma^M}\ln(-W_{t_i}^M)+\frac{\delta^L-r}{\gamma^L r}\right]}. \tag{A101}$$

We denote L's money market account balance and shareholdings at t_i^- (before her new trade) by $B_{t_i^-}^L$ and $\Theta_{t_i^-}^L=\Theta_{t_{i-1}}^L$, respectively. L's optimal trading decision to choose the new ownership level Θ maximizes the value function above, more exactly, $B_{t_i}^L+\widehat{G}(t_i,\Theta)+\frac{\Theta}{\gamma^M r}\ln(-W_{t_i}^M)$ in the value function, given her current holdings $\Theta_{t_{i-1}}^L$. By considering the proceeds from the new trade, $(\Theta_{t_{i-1}}^L-\Theta)\widehat{P}(t_i,\Theta,W_{t_i}^M)$, it maximizes $B_{t_i^-}^L+(\Theta_{t_{i-1}}^L-\Theta)\widehat{P}(t_i,\Theta,W_{t_i}^M)+\widehat{G}(t_i,\Theta)+\frac{\Theta}{\gamma^M r}\ln(-W_{t_i}^M)$, which, along with (A91), derives the time-deterministic function $\widehat{G}$ in L's value function at t_i^- as below:

$$\begin{aligned}\widehat{G}(t_i^-,\Theta_{t_{i-1}}^L)&=\max_{\Theta}\,(\Theta_{t_{i-1}}^L-\Theta)\widehat{\Lambda}(t_i,\Theta)+\widehat{G}(t_i,\Theta)\\&=\max_{\Theta}\,(\Theta_{t_{i-1}}^L-\Theta)\widehat{\Lambda}(t_i,\Theta)+\phi_i(t_i)\widehat{V}(\Theta)+(1-r\phi_i(t_i))\widehat{G}(t_{i+1},\Theta),\end{aligned} \tag{A102}$$

where the second line follows from (A90). The FOC is then derived as

$$\text{FOC}:\ (\Theta_{t_{i-1}}^L-\Theta)\frac{\partial\widehat{\Lambda}(t_i,\Theta)}{\partial\Theta}-\widehat{\Lambda}(t_i,\Theta)+\phi_i(t_i)\widehat{V}'(\Theta)+(1-r\phi_i(t_i))\frac{\partial\widehat{G}(t_{i+1},\Theta)}{\partial\Theta}=0, \tag{A103}$$

where the last term is, due to the envelope theorem,

$$\frac{\partial\widehat{G}(t_{i+1},\Theta)}{\partial\Theta}=\widehat{\Lambda}(t_{i+1},\Theta_{t_{i+1}}^L). \tag{A104}$$

By (A100), we rewrite the FOC above as

$$\text{FOC}: \ (\Theta^L_{t_{i-1}} - \Theta)\frac{\partial \widehat{\Lambda}(t_i,\Theta)}{\partial \Theta} + \phi_i(t_i)\left[\widehat{V}'(\Theta) - \widehat{k}(\Theta)\right] = 0. \tag{A105}$$

Using the definitions of $\widehat{V}(\Theta)$ and $\widehat{k}(\Theta)$ in (A89) and (A99),

$$\begin{aligned}\widehat{V}(\Theta) &= \Theta\widehat{\Omega}_1(\Theta) + \frac{\Theta}{\gamma^M r}\widehat{\Omega}_2(\Theta) - \frac{1}{2}\gamma^L r\Theta^2\widehat{\Omega}_3(\Theta)\\ \widehat{k}(\Theta) &= \widehat{\Omega}_1(\Theta) + \frac{1}{\gamma^M r}\widehat{\Omega}_2(\Theta) - \gamma^S r(1-\Theta)\widehat{\Omega}_3(\Theta),\end{aligned}$$

where $\widehat{\Omega}_1(\Theta) \equiv (\mu(\widehat{a}^M_t) + \frac{1}{\gamma^M}\ln(\gamma^M r) - \Psi(\hat{a}^M_t))$, $\widehat{\Omega}_2(\Theta) \equiv \left(\widehat{\mu}_W - \frac{1}{2}\widehat{\sigma}^2_W\right)$, and $\widehat{\Omega}_3(\Theta) \equiv \left(\sigma - \frac{\widehat{\sigma}_W}{\gamma^M r}\right)^2$. The derivative of $\widehat{V}(\Theta)$ with respect to Θ is

$$\widehat{V}'(\Theta) = \widehat{\Omega}_1(\Theta) + \Theta\widehat{\Omega}'_1(\Theta) + \frac{1}{\gamma^M r}\left(\widehat{\Omega}_2(\Theta) + \Theta\widehat{\Omega}'_2(\Theta)\right) - \gamma^L r\Theta\widehat{\Omega}_3(\Theta) - \frac{1}{2}\gamma^L r\Theta^2\widehat{\Omega}'_3(\Theta) \tag{A106}$$

Note that, in the first-best case, the manager's optimal effort level $\widehat{a}^M_t$ and the drift term $\widehat{\mu}_W$ in M's promised payoff process, which are specified by (A85) and (A87), respectively, do not depend on Θ. Also, by the definition of $\widehat{\sigma}_W$ in (A87), we can easily see that the sum of the three terms $\Theta\widehat{\Omega}'_1(\Theta) + \frac{1}{\gamma^M r}\Theta\widehat{\Omega}'_2(\Theta) - \frac{1}{2}\gamma^L r\Theta^2\widehat{\Omega}'_3(\Theta)$ in (A106) is zero. As a result, the FOC (A105) further reduces to

$$\text{FOC}: \ (\Theta^L_{t_{i-1}} - \Theta)\frac{\partial \widehat{\Lambda}(t_i,\Theta)}{\partial \Theta} + \phi_i(t_i)\left[\gamma^S - (\gamma^L + \gamma^S)\Theta\right] r\widehat{\Omega}_3(\Theta) = 0, \tag{A107}$$

where note that $\widehat{\Omega}_3(\Theta) = \left(\sigma - \frac{\widehat{\sigma}_W}{\gamma^M r}\right)^2 = \widehat{\sigma}^2_R(\Theta) > 0$. From the above, it is evident that, in the steady-state equilibrium, L's optimal ownership level is

$$\widehat{\Theta}^L_{ss} = \gamma^S/(\gamma^L + \gamma^S).$$

Proof of Proposition 7

As we showed in the proof of Proposition 2, M's contractual parameters in the second-best case, α and β, follow

$$\alpha < \frac{\mu_1}{\psi}; \ \beta < \gamma_M r. \tag{A108}$$

As M's first-best effort level is $\hat{a}^M_t = \frac{\mu_1}{\psi}$, his second-best effort level is lower than the first-best level. Also, the drift of M's promised payoff process, $\mu_W = \delta^M - \frac{\beta}{\gamma^M}$, is higher in the second-best case than the corresponding term, $\widehat{\mu}_W = \delta^M - r$, in the first-best case.

Suppose that L's equity stake is Θ both in the first-best case and in the second-best case. By (A64) and (A87),

$$\sigma_W(\Theta) = \frac{\gamma_L r\Theta\sigma + \left[(\gamma_L r\Theta\sigma)^2 + 4\left(1+\frac{\gamma_L}{\gamma_M}\Theta\right)\left(r-\frac{\beta}{\gamma_M}\right)\right]^{\frac{1}{2}}}{2\left(1+\frac{\gamma_L}{\gamma_M}\Theta\right)} > \frac{\gamma^L r\Theta}{\left(1+\frac{\gamma^L}{\gamma^M}\Theta\right)}\sigma\left(=\hat{\sigma}_W(\Theta)\right). \tag{A109}$$

That is, the second-best volatility of M's promised payoff process, $\sigma_W(\Theta)$, is greater than the first-best volatility, $\hat{\sigma}_W(\Theta)$. It immediately follows from their definitions (30) and (38) that the two PPS measures are higher in the second-best case than in the first-best case. By comparing (A62) and (A96), we can see that the dollar stock return volatility is lower in the second-best case than in the first-best case. By (A65) and (A97), the expected excess dollar return of the stock is lower in the second-best case than in the first-best case.

A.2 Proofs for the Owner-Manager Benchmark Model

We obtain the equilibrium of the owner-manager benchmark model in a similar manner to that shown in the proofs for our main model. We first characterize the equilibrium value functions and policies for the last period $[T,\infty)$, and then proceed to earlier periods. For expositional convenience, we show the proofs for an arbitrary period $[t_i, t_{i+1})$ with $i \leq N$.

Proof of Proposition 8

Consider period $t \in [t_i, t_{i+1})$ with $i \leq N$ where L's ownership level is $\Theta_t^L = \Theta$. L's optimal choices of effort and consumption solve

$$W_t^L = \max_{c_t^L, a_t^L} E_t^{a^L}\left[\int_t^\infty e^{-\delta^L(\tau-t)}u^L(c_\tau^L, a_\tau^L)d\tau\right] = E_t^{a^L}\left[-\frac{1}{\gamma^L}\int_t^{t_{i+1}} e^{-\delta^L(\tau-t)-\gamma^L(c_\tau^L-\Psi(a_\tau^L))}d\tau + W_{t_{i+1}}^L\right], \tag{A110}$$

subject to the budget constraint that captures the evolution of L's money market account balance,

$$dB_t^L = (rB_t^L - c_t^L)dt + \Theta dX_t = \left[rB_t^L - c_t^L + \Theta\mu(a_t^L)\right]dt + \Theta\sigma dZ_t. \tag{A111}$$

In (A110), $W_{t_{i+1}}^L = 0$ if $i = N$ (that is, $t_{i+1} = t_{N+1} = \infty$). Note that, in the owner-manager case, L exerts effort a_t^L by incurring the effort cost $\Psi(a_t^L)$, which is included in her utility function. We conjecture that L's value function has the following exponential form:

$$W_t^L = F(t, B_t^L) = -\frac{1}{\gamma^L r}e^{-\gamma^L\left[r(B_t^L+\widetilde{G}(t,\Theta))+\frac{\delta^L-r}{\gamma^L r}\right]}. \tag{A112}$$

The HJB equation associated with the dynamic programming problem defined by (A110) and (A111) is then given by

$$F_t + \max_{c_t^L, a_t^L}\left[F_B(rB_t^L - c_t^L + \Theta\mu(a_t^L)) + \frac{1}{2}F_{BB}\Theta^2\sigma^2 + u^L(c_t^L, a_t^L) - \delta^L F\right] = 0. \tag{A113}$$

The FOCs with respect to (c_t^L, a_t^L) are then

$$\begin{aligned} c_t^L: &\quad -F_B + u_c^L(c_t^L, a_t^L) = 0, \\ a_t^L: &\quad F_B \Theta \mu_1 + u_a^L(c_t^L, a_t^L) = 0. \end{aligned} \tag{A114}$$

The above conditions, along with the derivatives of $F(t, B_t^L)$, lead to the following optimal policies:

$$\widetilde{c}_t^L = r(B_t^L + \widetilde{G}(t,\Theta)) + \frac{\delta^L - r}{\gamma^L r} + \Psi(\widetilde{a}_t^L), \tag{A115}$$

$$\widetilde{a}_t^L = \frac{\mu_1}{\psi}\Theta. \tag{A116}$$

Substituting the optimal policies above and the derivatives of $F(t, B_t^L)$ into the HJB equation (A113) yields

$$\widetilde{G}_t(t,\Theta) = r\widetilde{G}(t,\Theta) - \widetilde{V}(\Theta), \tag{A117}$$

where $\widetilde{V}(\Theta)$ is given by

$$\widetilde{V}(\Theta) = \Theta\mu(\widetilde{a}_t^L) - \Psi(\widetilde{a}_t^L) - \frac{1}{2}\gamma_L r \Theta^2 \sigma^2 = \mu_0 \Theta + \frac{1}{2}\left[\frac{\mu_1^2}{\psi} - \gamma_L r \sigma^2\right]\Theta^2. \tag{A118}$$

By integrating (A117) over the period $[t, t_{i+1})$, we obtain

$$\widetilde{G}(t,\Theta) = \phi_i(t)\widetilde{V}(\Theta) + (1 - r\phi_i(t))\widetilde{G}(t_{i+1},\Theta), \tag{A119}$$

where $\phi_i(t) = \frac{1}{r}\left(1 - e^{-r(t_{i+1}-t)}\right)$. If it is the last period ($i = N$), then $\widetilde{G}(t,\Theta) = \frac{1}{r}\widetilde{V}(\Theta)$, which follows from the assumption on the terminal payoff to L, $\widetilde{G}(t_{N+1}) = \lim_{t\to\infty}\widetilde{G}(t,\Theta) = 0$. We can verify that the above W_t^L process, (A112), satisfies the conditions for the dynamic programming verification theorem so that it is, indeed, L's value function.

Proof of Proposition 9

The equilibrium stock price is determined by small shareholders' competitive trading in the stock market. We conjecture and verify that, for any given level of L's ownership Θ, the equilibrium stock price in the owner-manager case is a function of time and L's ownership level, $\widetilde{P}(t,\Theta)$. Given the stock price process, S makes its optimal consumption and portfolio choices to maximize:

$$W_t^S = \max_{c_t^S, \theta_t^S} E_t^{a^L}\left[\int_t^\infty e^{-\delta^S(\tau-t)} u^S(c_\tau^L) d\tau\right] = E_t^{a^L}\left[-\frac{1}{\gamma^S}\int_t^{t_{i+1}} e^{-\delta^S(\tau-t)-\gamma^S c_\tau^S} d\tau + W_{t_{i+1}}^S\right], \tag{A120}$$

where $W_{t_{i+1}}^S = 0$ if $i = N$ (that is, $t_{i+1} = t_{N+1} = \infty$), subject to the budget constraint that captures the evolution of S's stock and money market account balances. Specifically, S's money market balance evolves according to

$$dB_t^S = (rB_t^S - c_t^S)dt + \theta_t^S dX_t - \widetilde{P}_t d\theta_t^S. \tag{A121}$$

S's total wealth Y_t^S from his portfolio holdings is $Y_t^S = B_t^S + \theta_t^S \widetilde{P}_t$ and evolves according to

$$\begin{aligned} dY_t^S &= (rY_t^S - c_t^S)dt + \theta_t^S(dX_t + d\widetilde{P}_t - r\widetilde{P}_t dt). && \text{(A122)} \\ &= (rY_t^S - c_t^S)dt + \theta_t^S d\widetilde{R}_t, && \text{(A123)} \end{aligned}$$

where we use the definition of the excess dollar return of the stock, $d\widetilde{R}_t = dX_t + d\widetilde{P}_t - r\widetilde{P}_t dt$. Given the guessed form of the stock price process $\widetilde{P}(t,\Theta)$, the excess dollar return follows

$$d\widetilde{R}_t = \widetilde{\mu}_R(t,\Theta)dt + \widetilde{\sigma}_R(t,\Theta)dZ_t, \tag{A124}$$

where the drift term, $\widetilde{\mu}_R(t,\Theta)$, and the diffusion term, $\widetilde{\sigma}_R(t,\Theta)$, are given by

$$\begin{aligned} \widetilde{\mu}_R(t,\Theta) &= \widetilde{\mu}(\widetilde{a}_t^L) + \widetilde{P}_t(t,\Theta) - r\widetilde{P}(t,\Theta), && \text{(A125)} \\ \widetilde{\sigma}_R(t,\Theta) &= \sigma. && \text{(A126)} \end{aligned}$$

We conjecture S's value function as a negative exponential form and solve for its optimal consumption and demand for the stock. We then impose the stock market clearing condition, $\int_S \theta_t^S dS = 1 - \Theta$, from which we obtain

$$\widetilde{\mu}_R(\Theta) = (1-\Theta)\,\gamma^S r \widetilde{\sigma}_R^2(\Theta) = (1-\Theta)\,\gamma^S r\sigma^2. \tag{A127}$$

By combining the above with (A125) and (A126), we obtain the following ODE for the stock price process $\widetilde{P}(t,\Theta)$:

$$\widetilde{P}_t(t,\Theta) = r\widetilde{P}(t,\Theta) - \widetilde{k}(\Theta), \tag{A128}$$

where

$$\widetilde{k}(\Theta) = \mu(\widetilde{a}_t^L) - \gamma^S r(1-\Theta)\sigma^2 = \mu_0 + \frac{\mu_1^2}{\psi}\Theta - \gamma^S r(1-\Theta)\sigma^2, \tag{A129}$$

which is linear in L's equity stake Θ. By integrating (A128) over period $[t, t_{i+1})$, we express $\widetilde{P}(t,\Theta)$ recursively as follows.

$$\widetilde{P}(t,\Theta) = \phi_i(t)\widetilde{k}(\Theta) + (1 - r\phi_i(t))\widetilde{P}(t_{i+1},\Theta), \tag{A130}$$

where $\phi_i(t) = \frac{1}{r}\left(1 - e^{-r(t_{i+1}-t)}\right)$. From the terminal condition on the stock price, $\lim_{t\to\infty}\widetilde{P}(t,\Theta) = 0$, the stock price is constant (time-invariant) during the last period $t \in [t_i, t_{i+1})$ with $i = N$: $\widetilde{P}(t,\Theta) = \frac{1}{r}\widetilde{k}(\Theta)$.

Proof of Corollary 4

By (A127), the expected excess dollar return of the stock declines with L's ownership level Θ. Its Sharpe ratio, $\widetilde{\Sigma}_R(\Theta) = \frac{\widetilde{\mu}_R(\Theta)}{\widetilde{\sigma}_R(\Theta)} = (1-\Theta)\,\gamma^S r\sigma$, also declines with Θ.

Proof of Proposition 10

We now consider L's optimal ownership choice at each trading date t_i^- given her equity stake before the new trade, $\Theta^L_{t_i^-} = \Theta^L_{t_{i-1}}$. By (A112), L's value function at t_i right after she chooses the new ownership level Θ is

$$W^L_{t_i} = -\frac{1}{\gamma^L r} e^{-\gamma^L \left[r(B^L_{t_i} + \widetilde{G}(t_i,\Theta)) + \frac{\delta^L - r}{\gamma^L r} \right]}. \tag{A131}$$

The new ownership level Θ at t_i maximizes $B^L_{t_i} + \widetilde{G}(t_i,\Theta)$ in the value function above. By considering the proceeds from the new trade, $(\Theta^L_{t_{i-1}} - \Theta)\widetilde{P}(t_i,\Theta)$, it maximizes $B^L_{t_i^-} + (\Theta^L_{t_{i-1}} - \Theta)\widetilde{P}(t_i,\Theta) + \widetilde{G}(t_i,\Theta)$, which, along with (A119), derives the time-deterministic function $\widetilde{G}$ in L's value function at t_i^- as below:

$$\begin{aligned} \widetilde{G}(t_i^-, \Theta^L_{t_{i-1}}) &= \max_{\Theta} \, (\Theta^L_{t_{i-1}} - \Theta)\widetilde{P}(t_i,\Theta) + \widetilde{G}(t_i,\Theta) \\ &= \max_{\Theta} \, (\Theta^L_{t_{i-1}} - \Theta)\widetilde{P}(t_i,\Theta) + \phi_i(t_i)\widetilde{V}(\Theta) + (1 - r\phi_i(t_i))\widetilde{G}(t_{i+1},\Theta). \end{aligned} \tag{A132}$$

The FOC is then derived as

$$\text{FOC}: \; (\Theta^L_{t_{i-1}} - \Theta)\frac{\partial \widetilde{P}(t_i,\Theta)}{\partial \Theta} - \widetilde{P}(t_i,\Theta) + \phi_i(t_i)\widetilde{V}'(\Theta) + (1 - r\phi_i(t_i))\frac{\partial \widetilde{G}(t_{i+1},\Theta)}{\partial \Theta} = 0, \tag{A133}$$

where the last term is, due to the envelope theorem,

$$\frac{\partial \widetilde{G}(t_{i+1},\Theta)}{\partial \Theta} = \widetilde{P}(t_{i+1}, \Theta^L_{t_{i+1}}). \tag{A134}$$

By (A130), we rewrite the FOC above as

$$\text{FOC}: \; (\Theta^L_{t_{i-1}} - \Theta)\frac{\partial \widetilde{P}(t_i,\Theta)}{\partial \Theta} + \phi_i(t_i)\left[\widetilde{V}'(\Theta) - \widetilde{k}(\Theta)\right] = 0. \tag{A135}$$

First, consider L's last trading date, t_N^-. As shown in the end of the proof of Proposition 9, the stock price during the last period $t \in [t_N, t_{N+1} = \infty)$ is constant, $\widetilde{P}(t,\Theta) = \frac{1}{r}\widetilde{k}(\Theta)$, which is linear in L's ownership level Θ. By (A118) and (A129), we can derive L's optimal equity stake for the last period from the FOC above:

$$\begin{aligned} &\text{FOC}: (\Theta^L_{t_{N-1}} - \Theta)\frac{\partial \widetilde{P}(t_N,\Theta)}{\partial \Theta} + \phi_N(t_N)\left[\widetilde{V}'(\Theta) - \widetilde{k}(\Theta)\right] = \frac{1}{r}\left[(\Theta^L_{t_{N-1}} - \Theta)\widetilde{k}'(\Theta) + \widetilde{V}'(\Theta) - \widetilde{k}(\Theta)\right] = 0 \\ &\quad \Rightarrow \widetilde{\Theta}^L_{t_N}(\Theta^L_{t_{N-1}}) = \frac{\nu\Theta^L_{t_{N-1}} + \gamma^S r\sigma^2}{\nu + (\gamma^L + \gamma^S)r\sigma^2}, \end{aligned} \tag{A136}$$

where $\nu = \frac{\mu_1^2}{\psi} + \gamma^S r\sigma^2 > 0$. Accordingly, L's new equity stake at t_N^-, $\Theta^L_{t_N}(\Theta^L_{t_{N-1}})$, is linear in her current equity stake, $\Theta^L_{t_{N-1}}$. We then see that the stock price at t_{N-1} is also a linear function of L's equity stake:

$$\widetilde{P}(t_N, \Theta^L_{t_{N-1}}) = \widetilde{P}(t_N, \widetilde{\Theta}^L_{t_N}(\Theta^L_{t_{N-1}})) = \frac{1}{r}\widetilde{k}(\widetilde{\Theta}^L_{t_N}(\Theta^L_{t_{N-1}})) = \eta_{0N} + \eta_{1N}\widetilde{\Theta}^L_{t_N}(\Theta^L_{t_{N-1}}), \quad \text{(A137)}$$

$$\eta_{0N} = \frac{1}{r}(\mu_0 - \gamma^S r\sigma^2);\ \eta_{1N} = \frac{1}{r}\nu = \frac{1}{r}\left(\frac{\mu_1^2}{\psi} + \gamma^S r\sigma^2\right). \quad \text{(A138)}$$

Next, we further derive L's optimal ownership choice at any earlier trading date t_i^- from the FOC (A135). To do so, we first show that the equilibrium stock price in the owner-manager case is, in general, linear in the large shareholder's equity stake,

$$\widetilde{P}(t, \Theta) = \eta_{0i} + \eta_{1i}\Theta, \quad \text{(A139)}$$

which we showed for the last period in (A137). Specifically, we assume that (A139) holds during the period $t \in [t_{i+1}, t_{i+2})$ and verify that it also holds during the earlier period $t \in [t_i, t_{i+1})$. By the assumption, (A139) holds at t_{i+1} so that $\frac{\partial \widetilde{P}(t_{i+1},\Theta)}{\partial\Theta} = \eta_{1i+1}$, which, in turn, determines L's optimal equity stake from the FOC (A135) at t_{i+1}^- as a function of her current holdings, $\Theta^L_{t_i}$,

$$\widetilde{\Theta}^L_{t_{i+1}}(\Theta^L_{t_i}) = \frac{\eta_{1i+1}\Theta^L_{t_i} + \phi_{i+1}(t_{i+1})\gamma^S r\sigma^2}{\eta_{1i+1} + \phi_{i+1}(t_{i+1})(\gamma^S + \gamma^L)r\sigma^2}. \quad \text{(A140)}$$

Plugging (A129) into the stock price process (A130) at $t \in [t_i, t_{i+1})$ when L's equity stake is $\Theta^L_{t_i}$ leads to

$$\begin{aligned}\widetilde{P}(t, \Theta^L_{t_i}) &= \phi_i(t)(\mu_0 - \gamma^S r\sigma^2 + \nu\Theta^L_{t_i}) + (1 - r\phi_i(t))\widetilde{P}(t_{i+1}, \Theta^L_{t_i}) \\ &= \phi_i(t)(\mu_0 - \gamma^S r\sigma^2 + \nu\Theta^L_{t_i}) + (1 - r\phi_i(t))\widetilde{P}(t_{i+1}, \widetilde{\Theta}^L_{t_{i+1}}(\Theta^L_{t_i})). \quad \text{(A141)}\end{aligned}$$

By assumption, (A139) holds during the period $t \in [t_{i+1}, t_{i+2})$ so that $\widetilde{P}(t_{i+1}, \widetilde{\Theta}^L_{t_{i+1}}(\Theta^L_{t_i})) = \eta_{0i+1} + \eta_{1i+1}\widetilde{\Theta}^L_{t_{i+1}}(\Theta^L_{t_i})$. By (A140), it is clear that the linearity of the stock price in L's equity stake $\Theta^L_{t_i}$ also holds during the period $t \in [t_i, t_{i+1})$. Using (A141), we can obtain the recursive relations for the price coefficients as follows:

$$\begin{aligned}\eta_{1i} &= \phi_i(t_i)\nu + (1 - r\phi_i(t_i))\frac{\eta^2_{1i+1}}{\eta_{1i+1} + \phi_{i+1}(t_{i+1})(\gamma^S + \gamma^L)r\sigma^2}, \\ \eta_{0i} &= \phi_i(t_i)(\mu_0 - \gamma^S r\sigma^2) + (1 - r\phi_i(t_i))\left[\eta_{0i+1} + \frac{\eta_{1i+1}\phi_{i+1}(t_{i+1})\gamma^S r\sigma^2}{\eta_{1i+1} + \phi_{i+1}(t_{i+1})(\gamma^S + \gamma^L)r\sigma^2}\right]. \quad \text{(A142)}\end{aligned}$$

Proof of Corollary 5

By comparing (A62) and (A126), it is clear that the dollar return volatility in the agency model, $\sigma_R(\Theta) = \left(\sigma - \frac{\sigma_W}{\gamma^M r}\right)$, is lower than the volatility in the benchamrk owner-manager model, $\widetilde{\sigma}_R(\Theta) = \sigma$, becuase $\sigma_W > 0$. By (A65) and (A127), both the expected excess dollar return and the Sharp ratio are also lower in the agency model than in the benchmark owner-manager model when L's ownership stake is the same in both cases.

Proof of Corollary 6

L's steady state ownership is identical in the three models. By comparing (A96) and (A126), the dollar return volatility in the first-best case, $\hat{\sigma}_R(\Theta) = \left(\sigma - \frac{\hat{\sigma}_W}{\gamma^M r}\right)$, is lower than the volatility in the owner-manager case, $\widetilde{\sigma}_R(\Theta) = \sigma$, becuase $\hat{\sigma}_W > 0$. By (A97) and (A127), both the expected excess dollar return and Sharp ratio are lower in the first-best benchmark case than in the owner-manager case. By Proposition 7, the expected excess dollar return, dollar return volatility, and Sharpe ratio of the stock in the steady state are lower in the second-best agency case than in the first-best case.