The Modern Advertising Agency Selection Contest: A Case for Stipends to New Participants

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January 2016

Abstract: In the modern advertising agency selection contest, each participating agency specifies not only its proposed creative campaign, but also the budget required to purchase the agreed-upon media. The advertiser selects the agency that offers the best combination of creative quality and media cost, similar to conducting a score auction. To participate in the contest, each agency needs to incur an upfront bid-preparation cost arising from the development of a customized creative. Agency industry literature calls for the advertiser to fully reimburse such costs to all agencies that enter the contest. We analyze the optimal stipend policy of an advertiser facing agencies with asymmetric bid-preparation costs: the incumbent agency faces a lower bid-preparation cost than a competitor agency entering the contest. We show that reimbursing bid-preparation costs in full is never optimal and neither is reimbursing any part of the incumbent’s bid-preparation cost. However, a stipend strictly lower than the competitor’s bid-preparation cost can benefit the advertiser under certain conditions. We provide a sufficient condition (in terms of the distribution of agency values to the advertiser) for such a new-business stipend to benefit the advertiser.

Key Words: Advertising Agencies, Contests, Score Auctions

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Advertising is one of the most important and expensive marketing activities in which any firm engages. Worldwide, advertisers will spend about $592 billion on advertising in 2015, an increase of 6% over 2014 (emarketer 2014). The United States is the dominant advertising market — spending by U.S. firms accounts for about a third of the worldwide total. A majority of U.S. advertisers hire full-service advertising agencies to both develop and deliver their communication strategies (Horsky 2006). To select an agency, most advertisers periodically hold a contest among several candidate agencies. In this paper, we ask whether and when the advertiser looking to hire a full-service agency should offer stipends to the contest participants.

The advertising agency selection contest departs from many other procurement situations due to the fact that the participants incur a high cost preparing each bid. Each agency needs to customize its product to the advertiser’s specific business: to participate in a contest, an agency needs to assign a dedicated team to develop its pitch, conduct marketing research specific to the advertiser’s campaign goals, design several alternative creative approaches, and perform preliminary copy-testing. We ask whether the advertiser should offer stipends to help defray these bid-preparation costs and thus encourage more participation in the contest. A participation stipend is different from the winner’s compensation, because it is awarded whether or not the agency wins the contract. The advertising industry press continuously debates the stipend question. Naturally, the various agency associations argue that agencies should always be compensated for their upfront work, and that they should be compensated in full. For example, the Canadian association strongly protested the Bell Canada contest in which the advertiser did not cover agencies’ full bid-preparation costs (Brendan 1998). Gardner (1996, pp. 33) expresses this position eloquently: “If you insist that the finalist agencies demonstrate their creative abilities on your product, thinking like professionals and working like professionals, then you
should treat them like professionals. Pay them. You wouldn’t do less for your lawyers, bankers or accountants.” On the flipside, the advertisers may perceive such compensation as an added and unnecessary cost. In practice, recent surveys indicate that about half of major advertisers offer a stipend to cover some of the agencies’ bid-preparation costs (American Association of Advertising Agencies [AAAA] 2007, Parekh 2009). Moreover, about 30% of agencies say they will pitch only if provided with an upfront fee (Borgwardt 2010).

The following specific managerial questions emerge from the industry debate: Why are stipends offered only in some of today’s contests, and how can an advertising manager decide whether to offer such stipends in his particular contest? Why do real-world stipends tend to cover only a portion of the bid-preparation cost, and how should a manager determine the best possible stipend level to offer? Why are stipends, when offered, often offered only to new participants and not to incumbent agencies. Should the advertising firm ever offer a stipend to all agencies?

Historically, the industry practice fixed the compensation of the winning agency to 15% of the list price of media billings. As a result of the fixed compensation, the contest traditionally focused on selecting the agency with the best creative idea (Gross 1972). Nowadays, the media-buying aspect of the service is highly competitive, and only 5% of U.S. firms continue to compensate by a percentage of media billings based on the list price (Association of National Advertisers 2013). Instead, a modern full-service agency contest solicits each agency’s bid of a media price in addition to its creative idea, recognizing that different agencies face different media costs due to economies of scale and scope (Silk and Berndt 1993). The advertiser combines the creative and financial aspects of each pitch into an overall evaluation of each agency, often using a scorecard to keep track of the different aspects of those pitches (Medcalf 2006, Buccino 2009, Drum 2010, Argent 2014). The agency whose combination of creative
quality and media price delivers the highest profit to the advertiser wins the contract. The contemporary advertising contest has thus evolved to resemble a score auction—a mechanism often used in other procurement settings to facilitate competition among suppliers with different costs and qualities (Che 1993, Beil and Wein 2003).

The lack of a theoretical or practical industry consensus regarding the stipends calls for a careful analysis within a game-theoretic model that captures the essence of the contest and the entry game among invited agencies that precedes it. We propose to capture the essence of the modern advertising contest by a score auction with asymmetric bid-preparation costs and upfront participation stipends. We do not claim real-world contests exactly follow the rules of a score auction as, for example, a government procurement contest would. The advertiser does not actually announce the bidding and scoring rules, and can engage the contestants in additional price negotiations after the initial pitch. We thus use the score auction as a parsimonious model of both contract allocation and the price paid to the winning agency. The next paragraph gives an overview of our modeling assumptions.

To model the entry into the contest, we consider two kinds of agencies common on the advertising contest scene: (1) the incumbent agency, which is familiar with the advertiser and its industry and thus faces lower bid-preparation costs than (2) a competing agency trying to win the advertiser’s business.1 In addition to the difference in bid-preparation costs, the agencies also differ in their ability to increase the advertiser’s profit, which we call the “value” of an agency to the advertiser. The value of an agency arises from a combination of its expected creative quality and its media costs, the latter of which is private information of each agency. The model we

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1 “Incumbent” can be considered a mere label of the agency that faces a lower bid-preparation cost for another reason. To focus our analysis on the implication of asymmetry in bid-preparation costs, we assume that the two agencies are otherwise symmetric, i.e., their potential values to the advertiser are drawn from the same distribution. We thus abstract away from other advantages an actual incumbent might have.
propose starts with the advertiser publicly announcing the stipends available to each agency upon entry into the contest, with the stipend to the competitor usually called a “new-business stipend” in the industry. The agencies then consider each other’s incentives within an entry game, and enter when their equilibrium expected surplus from participation exceeds the part of their bid-preparation cost the stipend does not defray. In the final stage, the agencies that decided to enter bid in a score auction. Specifically, each agency reveals its creative quality to the advertiser during the pitch and also submits a media-price bid, allowing the score auction to rank all contestants in terms of their profitability to the advertiser.

Two technical questions underlie all of the above managerial questions: When is the higher advertiser profit from more contest participants worth the increased upfront cost of providing participation stipends? And how should these stipends depend on the agencies’ bid-preparation costs and on the distribution of agencies’ values to the advertiser? We find the asymmetry in bid-preparation costs between the incumbent and the competitor is necessary for stipends to benefit the advertiser: when the agencies face the same bid-preparation cost (e.g., when they are both bidding for new business and the incumbent does not participate either due to being terminated by the advertiser or resigning the account in order to service a competitor), the advertiser should offer no stipends. When one agency’s bid-preparation cost is lower than that of the other agency (we call the lower-cost agency an “incumbent”, see footnote 1), we obtain a general characterization of the optimal stipend scheme: first, we show the incumbent should not receive a stipend, but a new-business stipend strictly lower than the competitor’s bid-preparation cost can benefit the advertiser under certain conditions. Second, we provide a sufficient condition (in terms of the distribution of agency values) for a new-business stipend to benefit the advertiser.
To characterize when new-business stipends benefit the advertiser and to illustrate how one can apply our sufficient condition, we then consider several tractable distributional families of values-to-advertiser in the population of agencies. We find our sufficient condition is satisfied by a wide range of distributions, but we also find distributions that do not support any new-business stipends even when the incumbent has a lower bid-preparation cost than the competitor. Specifically, the advertiser should not offer any stipends when the population of agencies contains relatively many weak agencies that can deliver only a small profit to the advertiser.

We also explore how our results would generalize under alternative sets of assumptions, and find they are robust to adding more competitors, changing the amount of information each agency has about the quality of its creative idea before entering the contest, and to situations in which the advertiser uses a reserve price in addition to stipends.

**RELATED LITERATURE**

This paper contributes to the small quantitative marketing literature concerning advertising agencies (i.e., Gross 1972, Silk and Berndt 1993, Villas-Boas 1994, Horsky 2006). Within this literature, the most related study is by Gross (1972), who examines the selection of the best creative campaign while keeping constant the media budget and the prize to the contest winner. At the time of Gross’s writing, the advertising environment was different in that remunerations of agencies were standardized and the pitch did not include the media budget (see the previous section for a detailed discussion). He correctly identifies the importance of evaluating several creative campaigns and screening them accurately, but he does not consider the stipend as a strategic variable. This paper endogenizes the stipend decision and extends the analysis to the modern contests that solicit each agency’s bid of a media price in addition to a demonstration of
its creative quality. In the last section of the paper, we also extend our analysis “back in time” and briefly consider the incentives for stipends in 20th century quality-only contests with a fixed prize for the winner. We find new-business stipends can also be optimal in such a model, but stipends to the lower-cost “incumbent” agencies cannot be categorically ruled out.

Horsky (2006) is the first to address the bi-dimensionality of the modern advertising pitch. She documents the emergence of competition in the media-buying component and the appearance of the specialized media shops, which have the advantage of being media market makers, because the industry practice of not working with rival advertisers does not bind them. Unlike Horsky (2006), we restrict our work to the competition among the still-dominant full-service agencies and the process by which a specific one is chosen.

Although the agency selection process we study is usually called a “contest,” we do not analyze a typical contest as conceptualized in the contest theory literature starting with Tullock (1980). Instead, we argue the modern advertising agency selection “contest” resembles a scoring auction. Nevertheless, even in classic Tullock contests, the principal finds it optimal to level the playing field using “handicapping” policies that sometimes let a weaker player win with an inferior performance. We also find the principal (“advertiser” in our nomenclature) wants to level the playing field by subsidizing the entry of the bidder with a higher participation cost. Therefore, leveling the playing field by somehow favoring or helping the weaker player seems to be a general idea, and we contribute by characterizing how and when participation stipends can level the playing field in procurement auctions with endogenous entry and asymmetric bid-preparation costs. We discuss our contribution to the procurement-auction literature next, especially as it relates to leveling the playing field.

Starting with McAfee and McMillan (1989), the procurement-auction literature shows
why a buyer facing asymmetric bidders may benefit from leveling the playing field using price-preference subsidies. Specifically, McAfee and McMillan (1989) analyzed government-procurement price-preference policies that subsidize bids of weaker (higher-cost) bidders in first-price sealed-bid auctions, and demonstrated that the buyer (i.e., the government) may benefit from such policies, relative to when the contract is simply awarded to the lowest bidder. The reason costly price-preference policies can reduce the buyer’s procurement cost is that they put pressure on the stronger bidders to lower their bids, and the resulting increase in competition can outweigh the inefficiency of not assigning the contract to the lowest-cost supplier. Flambard and Perrigne (2006) apply the theory to snow-removal contracts in Montreal, demonstrating real-world asymmetries can be sufficiently strong to make price-preference policies profitable for the buyer. Branco (2002) shows that protection of the weaker bidders may provide an incentive for them to adopt more efficient technologies, which will eventually lower their costs (in the long run). Krasnokutskaya and Seim (2011) extend the theory of price-preference policies in first-price sealed-bid procurement auctions to the more realistic situation of endogenous entry. Following the endogenous-entry paradigm of Levin and Smith (1994), Krasnokutskaya and Seim’s bidders need to incur a cost in order to learn their cost types and participate in the auction. No closed-form entry or bidding policies exist in the resulting model, so Krasnokutskaya and Seim use numerical methods to estimate equilibria under the model parameters calibrated on California highway procurement. They conclude endogenous entry plays an important role in determining the optimal price-preference policy and its potential benefit to the buyer.

Analogous to the price-preference literature analyzing an existing yet controversial institution for leveling an asymmetric playing field in government procurement, we analyze an
existing and controversial field-leveling practice in advertising agency selection contests, namely, participation stipends. The most closely related paper is Gal, Landsberger, and Nemirovski (2007), who consider stipends in a different setting without an incumbent and without the auctioneer having any knowledge about the realized asymmetry in bid-preparation costs before the game. Like Gal et al. (2007), we study the second-price sealed-bid auction. The key benefit of this pricing rule is that unlike Krasnokutskaya and Seim (2011), we can analyze a model with endogenous entry in closed form. In contrast to Gal et al., we allow the advertiser to know the extent of the entry-cost asymmetry between the incumbent and the competitor. Also in contrast to their results, we find distributions for which entry subsidies are not optimal.

Like Gal et al. (2007) and in contrast to Krasnokutskaya and Seim (2011), we endogenize entry by following the framework of Samuelson (1985) to capture the idea of the agencies’ costly bid preparation. He shows that when all bidders face the same bid-preparation cost and know their valuations of the auctioned object before they make their entry decisions, the entry game involves a selection of higher-value bidders. The endogenous distribution of participating bidders in turn influences the reserve price the auctioneer should use, and the auctioneer may want to limit the number of invitees. In contrast to Samuelson’s (1985) key assumption, we allow the bid-preparation costs to differ across the bidders (capturing an important advantage of incumbency), and consider participation stipends. We find this asymmetry is necessary for stipends to benefit the advertiser.

We also consider (in an extension of our main model) the possibility that agencies do not know their value to the advertiser when they enter the contest, and they need to incur a cost to learn it. This alternative assumption about endogenous entry is analogous to Levin and Smith’s (1994) model used by Krasnokutskaya and Seim (2011). We extend Levin and Smith’s (1994)
model to asymmetric entry costs, and confirm an asymmetry in bid-preparation costs is necessary
for stipends to benefit the advertiser, and the stipend should not reimburse any agency in full. In
another extension, we allow the advertiser to charge a strategic reserve price, and we show a
new-business stipend continues to benefit him, with the benefit increasing in the amount of bid-
preparation cost asymmetry.

**MODEL: AN AUCTION WITH ASYMMETRIC BID-PREPARATION COSTS**

To describe our model, we first introduce the players (advertiser and agencies), then define the
rules of the auction-based contest with potential participation stipends, and finally discuss some
of the key simplifying assumptions that make our analysis tractable. Table 1 summarizes the
notation we use in our main model. We now describe the different actors in our model, and their
motivations.

**TABLE 1: NOTATION**

- $i$: index of agencies, $i=0$ represents the incumbent and $i=1$ represents the competitor
- $x_i$: the value of agency $i$ to the advertiser (gain from trade between $i$ and the advertiser)
- $k_i$: agency $i$'s bid-preparation cost
- $V$: the maximum value $x_i$ in the population of agencies
- $F$: distribution of value $x_i$ in the population of agencies
- $r_i$: the stipend the advertiser offers to agency $i$
- $L_i$: the entry threshold value such that agency $i$ with $x_i \geq L_i$ enters
- $\Pi$: the advertiser’s expected profit
- $S_i$: agency $i$’s expected surplus from participating in the contest
- $R$: the reserve price (in Extension 3)
- $\alpha$: the mass point at zero (in Extension 5)
- $H$: the distribution of advertising qualities (in Section “A Model of The Past”)
- $P$: the fixed contest prize (in Section “A Model of The Past”)

*Advertiser:* A firm (hereafter called “the advertiser”) is soliciting a contract with a full-
service advertising agency to develop and deliver a fixed amount of advertising exposures (e.g.,
measured in gross rating points, GRPs). When the advertiser hires no agency, he receives an outside-option payoff, which we normalize to zero.

Agencies: Two agencies qualified to bid on the contract are indexed by \( i=0,1 \): the incumbent agency \( i=0 \) currently serves the advertiser, and the competitor agency \( i=1 \) is interested in competing for the advertiser’s business.\(^2\) Each agency has its own creative quality \( q_i \) and media cost \( c_i \). The quality \( q_i \) corresponds to the advertiser’s profit-lift from using agency \( i \)’s creative in the entire campaign, whereas the cost \( c_i \) is agency \( i \)’s cost of delivering the creative to consumers through the appropriate media (including any costs of servicing the advertiser, such as producing the final polished advertising copy).

We assume both \( q_i \) and \( c_i \) are private information of the agencies at the beginning of the game;\(^3\) \( q_i \) of contest entrants is revealed during the pitch, and \( c_i \) remains private information throughout the game. At the beginning of the game, the advertiser only knows \( x_i \equiv q_i - c_i \) is distributed \( iid \) according to some distribution \( F(x) \) in the population of agencies. \( F \) has full support on the \([0, V]\) interval. We only consider agencies with \( q_i \geq c_i \) because agencies with \( q_i < c_i \) cannot outperform the advertiser’s outside option profit (normalized to zero). Including them would introduce the possibility of no trade, but it would not change our qualitative results.

We call \( x_i \equiv q_i - c_i \) the value of agency \( i \) to the advertiser. More exactly, \( x_i \) is the expected increase in profits (relative to the outside option) the advertiser would get if agency \( i \) serviced the advertiser and delivered the contractual amount of advertising at its media cost \( c_i \).

Most of our results depend only on the distribution of \( x_i \), but its decomposition into two

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\(^2\) Focusing on a single competitor greatly simplifies the analysis while exposing intuition. In the Extensions section, we will generalize a special tractable case of the model to an arbitrary number of competitors.

\(^3\) To learn its quality, each agency analyzes its initial creative ideas, available artists, and the nature of its match with the advertiser. This assumption results in a selection of higher-value agencies during the entry stage, but this selection is not critical to most of our results. In the Extensions section, we consider agencies that know only as much as the advertiser about their values before the contest, and we show that most of our results continue to hold.
dimensions \((q,c)\) captures several diverse agency types that might be competing for the contract: a small creative boutique agency with little media clout is captured as \((\text{high } c, \text{high } q)\), whereas a pure media house with a lot of media clout but little creative ability is captured as \((\text{low } c, \text{low } q)\). See Figure 1 for an illustration of these representative agencies. Note that although different in terms of qualities and costs, the creative boutique can be quite similar to the media house in terms of value to the advertiser.

Figure 1: Distribution of advertising agencies in \((q,c)\) space

Note to Figure: The diagonally hatched parallelogram is the support \(\mathbf{P}\) of joint distribution \(G\). Uniform distribution on \(\mathbf{P}\) yields the Uniform\([0,V]\) distribution of scores \(x = q – c\). Uniform distribution on the shaded triangle yields “decreasing triangle” distribution \(F(x) = 1 – (V-x)^2/V^2\).

Figure 1 also illustrates how the value distribution \(F\) may arise from an underlying bivariate distribution \(G\) of agency types \((q_i,c_i)\): without loss of generality, we normalize the highest
possible \( c_i \) to 1, so \( 0 \leq c_i \leq 1 \), and allow an upper limit \( V \) on the added value \( x_i \) any agency can provide. As discussed above, we consider only agencies with \( q_i \geq c_i \). Therefore, we focus on \( G \) with support on a parallelogram defined by \( P = \{(c_i, q_i) \mid c_i \in [0,1] \text{ & } q_i \in [c_i, c_i + V]\} \). The bivariate distribution \( G \) then implies a univariate \( F(x) = \int_{(q,c) \in P} 1(q - c \leq x) dG(q,c) \) with support \([0,V]\), where 1 is the indicator function.

Several of our example \( F \) distributions arise naturally from a simple bivariate \( G \): a uniform \( G \) distribution with full support on \( P \) implies a uniform \( F \) on \([0,V]\). A uniform \( G \) distribution on \( P \) with an upper bound on quality such that \( q \leq V \leq 1 \) implies a decreasing triangle distribution \( F(x) = 1 - \frac{(V - x)^2}{V^2} \) with support on \([0,V]\).

In addition to its own value \( x_i \), each agency faces a fixed bid-preparation cost \( k_i \) of preparing a professional pitch specific to the advertiser’s products. The activities \( k_i \) covers include development and testing of creative ideas before the pitch, as well as building and maintaining a relationship with the advertiser. Unlike the media and servicing cost \( c_i \), which is incurred only by the winning agency, all agencies entering the contest need to spend their \( k_i \) to participate. The incumbent agency is already familiar with the advertiser and the industry, so its bid-preparation cost is lower than that of the competitor: \( 0 \leq k_0 \leq k_1 \). It is natural to assume everyone knows which agency is the incumbent and which is the competitor. Moreover, everyone also knows the approximate magnitude of these bid-preparation costs because agency executives discuss bid-preparation costs publicly (Medcalf 2006), and the costs mostly arise from easily estimated personnel hours needed to prepare a competitive pitch. To simplify

\[\text{For a sense of magnitude, we note that according to the CEO of M&C Saatchi, the cost of developing the average pitch is about$100,000 internally (mainly time) and$30,000 externally (Medcalf 2006).}\]
analysis, we let the bid-preparation costs be common knowledge in the beginning of the game. We believe our key results would continue to hold qualitatively even if the advertiser’s belief also included some uncertainty around each $k_i$.

A key difference between the cost of participation $k$ and the cost of production $c$ is that $k$ is verifiable. In the advertising example, the cost of creative development can be supported with invoices for materials and hours of employee time spent, whereas the cost of subsequent media buying remains private and inherently unverifiable.

**Figure 2: Timeline of the game**

<table>
<thead>
<tr>
<th>Invitation stage:</th>
<th>Entry stage: Agencies simultaneously decide whether to participate. Everyone observes outcome.</th>
<th>Contest: Score auction among participating agencies to assign contract, and determine price.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advertiser announces stipends available to each agency upon entry.</td>
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</table>

**Contest rules and timing of the game:** The contest game proceeds in three stages (see Figure 2 for the timeline): first, the advertiser announces the stipend $r_i$ payable to agency $i$ upon its entry into the contest. In the second stage, the agencies get their private signals about the potential value $x_i$ they can deliver to the advertiser, and they play a simultaneous-move entry game to decide whether to sink the $k_i$ and enter the contest. At this stage, each agency also submits receipts to substantiate the costs it incurred in preparing its bid, $k_i$. In the final stage, the entrants bid for the contract in an auction that determines the winner and the contract price the advertiser pays for the winner’s services. We discuss the score-auction mechanism next.

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5 The submission of receipts to substantiate ongoing labor and other costs is the common practice in the advertising agency industry, and limits the potential for a moral hazard problem whereby agencies enter unprepared merely to collect the stipend. Reputation concerns are another reason such behavior should not happen.
In the contest stage, the advertiser runs a second-score sealed-bid auction (Che 1993) with a reserve price of zero\(^6\) to both allocate the contract and determine the price. In the auction, each agency pitches its creative ideas, credibly revealing its quality \(q_i\) and bids an amount of money \(\text{bid}_i\) it is willing to accept for servicing the contract and delivering the resulting advertisements. The advertiser ranks bidders in terms of their proposed profit-lift \(q_i - \text{bid}_i\), awards the contract to the agency with the highest profit-lift, and pays the winning agency the difference between its proposed lift and either the reserve price (if only one agency enters the contest) or the lift of the runner-up agency (when both enter). This auction is known to give each agency the incentive to bid its true cost \(c_i\), so the auction is isomorphic to one in which agencies bid their values \(x_i\), and the highest-value bidder wins and receives the difference between the two values as his compensation. The weakly dominant strategy to bid value in a second-score auction with exogenous qualities (Che 1993, Vickrey 1961) shows that both the entry equilibrium and the auction price depend only on the agencies’ values \(x_i\), not on their underlying combinations of quality and media cost. It also follows that when only one agency enters the contest, it captures its entire value to the advertiser.

**DISCUSSION OF ASSUMPTIONS**

Having outlined our model, we now discuss its key assumptions. Our most important simplifying assumption is that the bid-preparation costs are known, but the advertiser cannot infer or influence the resulting quality. This assumption acknowledges the creative quality-production process is idiosyncratic and noisy in the advertising setting. An unpredictable element of luck exists, coupled with hard-to-predict agency-advertiser match shocks. For example, a creative boutique agency may be outspent by its rivals but still occasionally (i.e., not systematically)

\(^6\) See the Extensions section for a version of the model with a strategic reserve price.
produce a higher-quality creative (as was the case in the “Got Milk?” campaign by Goodby, Silverstein & Partners). We therefore abstract away from another potential reason for offering stipends: to improve advertising quality. Within our model, the stipends merely encourage participation in the contest, and they do not affect quality at the margin.

Mathematically speaking, suppose an agency $i$ can invest any amount $k$ into quality improvement and other costs related to participation in the contest, such that its resulting quality $q_i(k)$ increases in $k$ with diminishing returns. Only the agency knows its quality-production function, which is idiosyncratic to its match with each particular advertiser. Such a quality-setting agency solves $k_i^* = \arg \max_k \left[ S_i \left( q_i \left( k \right) \right) - k \right]$ in the beginning of the game, and enters whenever its expected surplus $S_i$ exceeds the participation costs net of the stipend $S_i \left( q_i \left( k_i^* \right) \right) - k_i^* + r_i > 0$. Crucially, note the optimal investment $k_i^*$ and its associated quality $q_i \left( k_i^* \right)$ are unrelated to the stipend amount $r_i$, because the fixed stipend does not influence quality production at the margin. In our model, we effectively assume $k_i \equiv k_i^*$ is common information (everyone knows how much the two agencies tend to spend on new pitches), each agency has a good sense of its idiosyncratic quality function $q_i(k)$, but the advertiser does not know these functions exactly and thus remains uncertain about $q_i$. We thus implicitly assume qualities are exogenous in the sense of Engelbrecht-Wiggans et al. (2007).

Another key simplifying assumption is the second-price rule in our auction. We rely on the second-price auction to approximate the payoff in the real-world contest with closed-form expressions, but we do not claim the real-world contest follows the rules of the second-score auction exactly. The auction rules are not as firmly codified as in classic industrial procurement auctions, and some back-and-forth price negotiation often occurs between the advertiser and the
agencies after the pitches are made. Another modeling idea borrowed from government contracting would be to assume a first-score sealed-bid auction whereby the agencies propose profit-lifts, and the winning agency gets paid its own bid as compensation for servicing the contract and delivering the advertising through media. Unfortunately, a closed-form analysis of this auction is not tractable given the Samuelson (1985) model of endogenous entry: the asymmetry in participation costs leads to an asymmetry in the distributions of entrants’ values to the advertiser. Analyzing asymmetric first-price auctions is notoriously difficult (Maskin and Riley 2000) in that the equilibrium bidding strategies are often intractable (Kaplan and Zamir 2012). Because our goal is to merely approximate payoffs in a somewhat informal and iterative real-world contest, we chose the second-score auction for its tractability.  

We have reason to believe our second-score pricing rule can actually be more realistic in the advertising agency selection contest than the first-price rule: consider a post-pitch renegotiation stage in which the agencies compete only on media costs: with the qualities revealed during the initial pitch, the advertiser can declare a temporary winner and invite the losing agency to drop its media-cost bid until the loser’s score matches that of the temporary winner. From our discussions with advertisers, we understand that some post-pitch price negotiations and adjustments do often occur. If such a back-and-forth negotiation over cost can be carried out quickly and costlessly, the resulting “ascending score” auction is revenue equivalent to a second-price sealed-bid auction because the highest-value agency wins and the second-highest-value agency drops out of the media-price bidding at its true media cost.

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7 First- and second-price rules in a sealed-bid auction are well known to be revenue equivalent when the values of all bidders are drawn from the same distribution (Vickrey 1961, Che 1993). Our setting involves an asymmetry between the value distribution of entering incumbents and that of the entering competitors, and so the advertiser profits depend on the pricing rule, and their relative order of profitability is ambiguous (Maskin and Riley 2000).
ENTRY GAME, ADVERTISER PROFITS, AND OPTIMAL NEW-BUSINESS STIPENDS

We will show that the difference in their bid-preparation costs makes the agencies use different entry thresholds in the entry game, resulting in an asymmetry between bidders. Despite this asymmetry, analyzing the subsequent second-value auction is easy because each agency has a weakly dominant strategy to bid its value (Vickrey 1961). When only one agency enters the contest, the second-value auction awards it the contract for the reserve price, that is, for the advertiser’s outside option. When both agencies enter, the weakly dominant strategies imply the auction awards the higher-value bidder the contract for the “price” of the lower value. In other words, the winner delivers its advertising as pitched, and the advertiser compensates him with the difference between the values. We now proceed to the entry stage by backward induction.

Agency $i$ enters the contest when its expected surplus from bidding in the auction (denoted by $S_i$) exceeds its bid-preparation cost $k_i$ minus its stipend $r_i$. Suppose the opponent agency $-i$ uses a “threshold strategy” of entering iff $x_{-i} \geq L_{-i}$, where $L_{-i}$ is the opponent threshold type indifferent between entering or not. The expected surplus of agency $i$ satisfies:

$$S_i(x_i) = x_i F(L_{-i}) + 1(x_i > L_{-i}) \int_{L_{-i}}^{x_i} (x_i - x_{-i}) dF(x_{-i})$$

The first part of $S_i$ arises when the opponent does not enter; therefore, agency $i$ pockets its value as surplus. The second part of $S_i$ captures the competitive payoff in the auction when the opponent does enter and when $x_i$ exceeds the opponent’s entry threshold. Agency $i$ enters when $k_i - r_i < S_i(x_i)$. Because $S_i$ increases in $x_i$, agency $i$ also uses a threshold entry strategy.

Because the opponent’s entry threshold $L_{-i}$ influences the surplus of the focal agency $i$ (and vice versa), the agencies are engaged in an entry game. Consider the two threshold types $L_i$
and suppose the incumbent faces a lower bid-preparation cost even net of the stipend; that is,
\[ k_0 - r_0 < k_1 - r_i. \]

Then the incumbent enters more often \((L_1 \geq L_0)\) and the thresholds must satisfy:

\[
\begin{align*}
    k_1 - r_i &= L_1 F \big( L_0 \big) + \int_{L_0}^{L_1} \big( L_1 - x_0 \big) dF \big( x_0 \big), \\
    k_0 - r_0 &= L_0 F \big( L_1 \big),
\end{align*}
\]

(2)

where the ordering of the cutoffs \((L_1 \geq L_0)\) implies the threshold incumbent \(L_0\) can only win when the competitor does not enter. By contrast, the threshold competitor \(L_1\) can also win over weak incumbent entrants. The following lemma describes sufficient conditions for a pure-strategy entry equilibrium to exist (please see Appendix for a proof):

**Lemma 1:** When \( k_1 - r_i \leq V - E \big( x \big) \), the entry game has a unique pure-strategy equilibrium with a pair of thresholds \( V \geq L_1 \geq L_0 > 0 \).

No general closed-form solution of (2) exists, but we can exploit the structure of the entry system to obtain a general characterization of the advertiser’s optimal stipend strategy. To analyze the advertiser’s problem, we now derive his expected profit \( \Pi \).

Figure 3 shows how the profit \( \Pi \) depends on the realized values of the two agencies: we can use (2) to express \( \Pi \) entirely in terms of the two entry thresholds by substituting for \( r_i \):

\[
\Pi \big( L_1, L_0 \big) = \Pr \big( x_0, x_1 > L_4 \big) E \big( \min \big( x_0, x_1 \big) \big| x_0, x_1 > L_4 \big) + \Pr \big( L_0 < x_0 < L_4 < x_1 \big) E \big( x_0 \big| L_0 < x_0 < L_4 \big)
\]

\[
- \Pr \big( \text{competitor enters} \big) \left[ k_1 - L_1 F \big( L_0 \big) - \int_{L_0}^{L_1} \big( L_1 - x_0 \big) dF \big( x_0 \big) \right] - \Pr \big( \text{incumbent loses and sets the price} \big) \left[ 1 - F \big( L_4 \big) \right] \left[ k_0 - L_0 F \big( L_1 \big) \right]
\]

(3)

where the first two terms correspond to the shaded regions in Figure 3 and the last two terms collect the expected stipend payments to the competitor and the incumbent, respectively.
The profit parametrization in (3) transforms the advertiser’s profit-maximization problem into a two-dimensional screening problem whereby a higher stipend \( r_i \) corresponds to a lower entry threshold \( L_i \). Note that whereas a stipend that covers the full bid-preparation cost \( (r_i=k_i) \) corresponds to \( L_i=0 \), no stipend \( (r_i=0) \) corresponds to some positive entry threshold \( L_i > 0 \).

Several terms cancel each other out in the profit function. Most notably, the second profit term in (3) generated by a competitor entering against a “weak” incumbent (i.e., \( x_0 < L_1 \)) is effectively returned to the competitor as part of his stipend (see online technical appendix for a detailed derivation):
\[ \Pi(L_1, L_0) = \int_{L_1}^V z f(z) \left[1 - F(z)\right] dz - \left[1 - F(L_1)\right] \left[k_1 - L_1 F(L_1)\right] - \left[1 - F(L_0)\right] \left[k_0 - L_0 F(L_0)\right] \] (4)

The first term in equation (4) is the net added profit from the auction competition; the second term is the expected payment to the competitor \textit{net} of the increase in profits when the incumbent enters but is weak (below \(L_1\)); and the third term is the expected payment to the incumbent.

Casting the stipend-optimization problem as a screening problem yields our main result:

**Proposition 1:** For any continuous value distribution \(F[0, V]\) and any bid-preparation costs \(0 \leq k_0 \leq k_1\), no positive stipend for the incumbent agency \(r_0 > 0\) can benefit the advertiser.

A positive stipend for the competitor agency \(r_1 > 0\) can benefit the advertiser only when \(r_1 < k_1\) and \(k_0 < k_1\). The optimal stipend for the competitor agency \(r_1\) is positive for every \(0 < k_1 < V - E(x)\) when \(F(L_1) \left[1 - F(L_1)\right] - f(L_1) \int_0^{L_1} 1 - F(x) dx < 0\) for every \(L_1\) in \([0, V]\).

Proposition 1 shows stipends can only benefit the advertiser when an incumbent agency is present, that is, when one agency has a strictly lower bid-preparation cost. Moreover, the advertiser should only offer a stipend to the competitor and ensure the stipend does not reimburse the competitor’s bid-preparation cost in full. The sufficient condition shows that even offering the competitor a small compensation for his cost disadvantage is not always beneficial. Instead, the advertiser must consider whether the benefit of attracting a marginal competitor exceeds the marginal increase of the stipend.

The incumbent should not receive a stipend, because (4) is increasing in \(L_0\); a reduction in \(L_0\) is clearly costly to the advertiser because it involves paying more incumbents more money, but the advertiser receives no benefit, because any increase in profit from the auction is paid to the competitor in the form of an increased stipend. In other words, increasing the “competitor
wins” region of Figure 3 by lowering \( L_0 \) results in no net increase in profits after the competitor receives his stipend. Therefore, reducing \( L_0 \) via an increase in \( r_0 \) from 0 is pointless.

To gain intuition into why a bid-preparation-cost asymmetry is necessary for a stipend to be optimal, suppose \( k_0 = k_1 \) and consider the marginal entrant \( x_i = L_i \). The entry game in (2) becomes symmetric with a common threshold \( k (1 - r) = LF (L) \), so the marginal entrant pockets his entire value whenever he wins; therefore, subsidizing his presence conditional on him winning is unnecessary. When he loses, his presence raises the auction price to \( L \), but losing implies the other agency has also entered and received its stipend. To make both agencies enter if \( x_i > L \), the reimbursement policy must compensate them for any expected effects of their mutual competition, so the advertiser cannot benefit solely from a threshold agency losing the auction. Another intuition for the necessity of a cost asymmetry follows from Samuelson’s (1985) result that the advertiser would actually want to charge a reserve price above its opportunity cost in the \( k_0 = k_1 \) case. With \( k_0 = k_1 \), a reserve price is isomorphic to a negative stipend because both instruments simply manipulate the common entry threshold.

The second part of Proposition 1 shows competitor (aka “new-business”) stipends can be optimal when the two agencies face different bid-preparation costs, but only for some value distributions \( F \), and only when the stipends are smaller than the competitor’s bid-preparation cost \( k_1 \). To see why reimbursements in full are not optimal for the advertiser, consider the marginal entrant eliminated by a small increase in \( L_1 \) from \( L_1 = 0 \) (i.e., a small reduction of the stipend from \( r_1 = k_1 \)). The entrant eliminated on the margin near \( L_1 = 0 \) has no gains from trade with the advertiser, so he loses the auction almost surely and does not put any competitive pressure on the incumbent. Naturally, subsidizing his entry cannot benefit the advertiser.
To gain intuition for the sufficient condition, consider the first partial derivative of the profit (4) with \( r_0 = 0 \):

\[
\frac{\partial \Pi(L_0, L_1 | r_0 = 0)}{\partial L_1} = F(L_1)\left[1 - F(L_1)\right] + (k_1 - L_1) f(L_1)
\]  

(5)

Note that thanks to the “weak incumbent” term cancelling out in equation (3), offering no stipend to the incumbent makes the advertiser’s profit independent of the incumbent’s cost \( k_0 \). In other words, the profit depends only on the marginal competitor \( L_1 \) and his bid-preparation cost \( k_1 \).

When \( L_1 > 0 \), the entrant eliminated on the margin by a small increase in \( L_1 \) can potentially win the subsequent auction against the incumbent, so whether his entry should be subsidized is not obvious from (5). To provide a sufficient condition for a subsidy, consider the \( L_1 \) corresponding to no reimbursement \((r=0)\): if the profit is decreasing at that \( L_1 \), some partial reimbursement (i.e., a reduction in \( L_1 \)) is more profitable than no reimbursement at all. The sufficient condition of Proposition 1 gives this derivative in terms of \( F \) alone.

**PROPOSITION 1 UNDER SPECIFIC DISTRIBUTIONAL ASSUMPTIONS: EXAMPLES**

We now turn to several specific distributional families to illustrate Proposition 1, provide tractable examples of positive optimal competitor stipends, and give examples of distributions that do not support any competitor stipends even when \( k_0 = 0 \). Figure 4 illustrates all distributions considered in our examples. Before examining our analysis of the examples, we invite the reader to glance at Figure 4 and guess which distributions support positive competitor stipends. Another interesting question to ask is which distributions suggest larger stipends and which suggest smaller ones (keeping \( k_i \) fixed). We did not have strong intuitions prior to discovering Proposition 1, and one of the main contributions of this paper is to develop such intuition.
Figure 4: Specific agency-value distributions analyzed to illustrate Proposition 1

Note to Figure: Each graph shows a probability density function. The graphs are merely illustrations, not necessarily drawn to comparable scale.

Examples of Distributions That Support Positive New-Business Stipends

**Example 1:** When $F$ is uniform on $[0,V]$, i.e., $F(x) = x/V$, and $0 \leq k_0 < k_1 < V$, the advertiser should offer no stipend to the incumbent agency and a stipend of $\frac{k_1^2 - k_0^2}{2k_1}$ to the competitor.

We work out the uniform example in the main text because of its simplicity. The sufficient condition of Proposition 1 reduces to $\frac{-L_1^2}{2V^2} < 0$, so the advertiser should offer a new-business stipend at least for $k_1 \leq V/2$. The entry equations become:

$$2V(k_1 - r_1) = L_0^2 + L_1^2$$
$$V(k_0 - r_0) = L_0L_1$$

(2U)

The advertiser profit function with no incumbent stipend ($r_0=0$) simplifies to a cubic:
\begin{align*}
3V^2\Pi(L_1, L_0 \mid r_0 = 0) &= (V - L_1)(L_1^2 + V^2 + L_1V - 3k_1V) 
\end{align*} 

(6)

where the lack of dependence on \(L_0\) holds for any \(F\), as can be seen from (4). Finally, equation (5) implies \(\frac{\partial \Pi(L_0, L_1 \mid r_0 = 0)}{\partial L_1} = \frac{k_1V - L_1^2}{V^2}\). Therefore, the first-order condition suggests \(L_1^* = k_1V\) as the optimal threshold, and it characterizes a maximum because the profit function is concave.

The sufficient condition of Proposition 1 is therefore loose in this particular example of \(F\) because the optimal \(L_1\) is within the support of \(F\) for all \(k_1 \leq V\), and not only for \(k_1 \leq V/2\).\(^8\)

To derive the optimal stipend for any \(k_1 \leq V\), we plug the optimal \(L_1\) into the second equation in (2U) with \(r_0 = 0\), solve for the incumbent threshold, and solve for \(r_1\) using the first equation in (2U). As one would expect from Proposition 1, the stipend is less than \(k_1\) (in fact, less than half of \(k_1\)), increasing in the difference between the two bid-preparation costs, and vanishes when the costs become equal. Interestingly, the stipend is also independent of \(V\)—a simplification special to the uniform \(F\).

Comparing the optimal stipend policy with a no-stipend status quo (\(r_i = 0\)) is interesting, and results in \(L_1^* = k_1V + V\sqrt{k_1^2 - k_0^2}\). Therefore, relative to the optimal stipend policy, the status quo involves less entry by the competitor and more entry by the incumbent.

**Example 2:** When \(F\) is the decreasing triangle distribution on \([0, V]\), that is, \(F(x) = 1 - \frac{(V - x)^2}{V^2}\) and \(0 = k_0 < k_1 < V\), the advertiser should offer no stipend to the incumbent agency and a stipend of \(r_1 = k_1/3\) to the competitor.

\(^8\) When \(k_1 > V\), even the highest-value competitor \(x_1 = V\) does not bring enough to the table to cover the bid-preparation cost, so no amount of competitive entry is beneficial to the advertiser. The optimal \(L_1 = V\) can then be implemented by any stipend small enough that \(r_1 \leq V/2\).
The proof of Example 2 is analogous to that of Example 1, except the entry system does not have a simple solution tractable to us when \( k_0 > 0 \) beyond a bound \( r_1 \leq \frac{k_1}{3} \). Setting \( k_0 = 0 \) ensures the incumbent always enters and makes the optimal stipend simple.

**Example 3**: When \( F \) is the increasing triangle distribution on \([0, 1]\), that is, \( F(x) = x^2 \), and \( 0 = k_0 < k_1 < V \), the advertiser should offer no stipend to the incumbent agency and a stipend of

\[
 r_1 = \frac{k}{3} + \frac{z}{9} - \frac{1}{3z}, \quad \text{where} \quad z = \sqrt[3]{3(9k_1 + 3 + 81k_1^2)}.
\]

It can be shown that \( r_1 > \frac{k}{2} \).

Example 3 demonstrates that simple distributional assumptions do not necessarily lead to simple optimal stipends, even when \( k_0 = 0 \). Comparing the three distributions in examples 1 through 3, we conjecture an intuitive comparative static in the relative probability of more creative agencies: when relatively more high-value agencies (with \( x \) close to \( V \)) than low-value agencies (with \( x \) close to 0) are present, the optimal reimbursement proportion increases.

**Examples of Distributions That Do Not Support Positive Competitor Stipends**

Our next example allows us to begin describing the kinds of value distributions that satisfy Proposition 1’s sufficient condition for positive competitor stipends. We consider the polynomial family \( F(x) = x^a \) for \( a > 0 \) with support on the unit interval. A large subset of the polynomial family does not admit any stipends:

**Example 4**: When \( F(x) = x^a \), \( a < 1 \), Proposition 1’s sufficient condition for a positive competitor stipend does not hold for small-enough \( k_1 \). When \( 0 = k_0 < k_1 < (1-a)\left(1-a^2\right)^\frac{1}{a} \), no \( r_1 > 0 \) benefits the advertiser.
The first part of the example shows the sufficient condition can fail, and the second part focuses on a tractable special case in which all possible stipends can be ruled out. To understand what is special about \( a < 1 \), note the restriction singles out the distributions that put a lot of probability mass near zero. To gain intuition into why mass near zero makes competitor stipends unprofitable, note that a lot of mass at the bottom of the support encourages competitive entry by increasing the chance that a weak incumbent is present. When the marginal competitor is itself weak, that is, when \( k_1 \) is small, subsidizing his entry cannot be profitable: either he wins and pockets almost his entire contribution to the social surplus or he loses and puts little pressure on a high-value incumbent. Thus, despite the asymmetry in bid-preparation costs \( (k_0 < k_1) \), the intuition behind Example 4 is analogous to the reason why stipends are not profitable under general \( F \) when \( k_0 = k_1 \).

Confirming this intuition, it can be shown that the sufficient condition of Proposition 1 fails for small-enough \( k_1 \) whenever \( F \) is a mixture of a mass point at 0 and a continuous distribution on \([0, V]\) regardless of the shape of \( F \) on the rest of its support (please contact authors for a detailed proof). Our last example provides a closed-form illustration, including the possibility that no stipends benefit the advertiser:

**Example 5:** When \( F \) is a mixture of a Uniform\([0,1]\) and an \( \alpha > 0 \) mass-point at zero, then for 

\[
0 = k_0 < k_1, \text{ the optimal competitor stipend is } r = \max \left( 0, \frac{k_1 - \alpha}{2} \right).
\]

Example 5 returns full circle to the Uniform distribution of Example 1, in which \( k_0 = 0 \) implied the competitor’s stipend should be half of his bid-preparation cost. Mixing in a mass of “bad” agencies at the low boundary of the support clearly reduces the stipend, and sometimes even
cancels out its profitability to the advertiser. As discussed above, the $k_1$ costs that do not support positive stipends are the lower ones, because a low $k_1$ implies a low-value marginal competitor.

These last two examples show clearly that new-business stipends should not be universal. One explanation for the difference in new-business-stipend popularity between the United States and Europe could be the difference in the underlying distributions of agency values to the advertiser: the American agency landscape may be more competitive, with many marginal agencies as in examples 4 (with $a<1$) and 5.

**EXTENSIONS: TIMING OF ENTRY, MULTIPLE COMPETITORS, RESERVE PRICES**

Extension 1: Agencies Do Not Know Their Values To Advertiser Before Entry

In our main model, we have assumed each agency knows its expected value to the advertiser in the beginning of the game. This “private information” assumption is plausible if each agency can predict the quality of its pitched ad before developing it, perhaps based on the available artists or other stable characteristics of the agency, such as a match between it and the advertiser. Alternatively, the agencies might be “shooting in the dark” at the time of entry, not learning the value of their work until after the pitch. In our first extension, we consider this alternative situation and show our qualitative results continue to hold.

Suppose agency $i=0,1$ faces a publicly known bid-preparation cost $k_i$ as in the main model but does not know its value $x_i$ at the time of entry. Instead, both agencies only know their values will be drawn $iid$ from some distribution $F$ at the time of the contest. For example, this assumption captures the possibility that the ad quality is in the eye of the beholder (advertiser). When $k_0=k_1$, the model reduces to Levin and Smith (1994), who find a symmetric mixed-strategy equilibrium whereby each agency enters with the same probability $\rho$ that makes it indifferent
between entering and not. Suppose \(0 < k_0 < k_1\) and let agency \(i\) enter with probability \(\rho_i\). Then agency \(i\) enters when

\[
k_i - r_i \leq (1 - \rho_i) E(x_i) + \rho_i E_x \left[ \int_0^x (x_i - x_{-i}) dF(x_{-i}) \right]
\]

where the LHS is the cost of entry, and the RHS is the expected benefit of entry. The first term on the RHS corresponds to \(i\) capturing its entire value in the event of \(-i\) not entering, and the second term corresponds to the expected payoff from the auction between two entrants. Contrast equation 7 with equation 1, and note the additional expectation operator on the RHS in the former – a consequence of the additional uncertainty about own \(x\).

We focus on the tractable uniform case \(F = \text{Uniform}[0,V]\), and begin our solution of the modified contest game with an analysis of the entry stage. The uniform assumption implies

\[
E(x_i) = \frac{V}{2} > E_x \left[ \int_0^x (x_i - x_{-i}) dF(x_{-i}) \right] = \frac{V}{6}.
\]

When the two bid-preparation costs net of stipends are similar such that \((k_i - r_i) \in \left[ \frac{V}{6}, \frac{V}{2} \right]\), each agency would prefer the other not enter the contest, and multiple equilibria result: a mixed-strategy equilibrium in which both agencies are indifferent between entering and not entering \(\{\rho_0, \rho_1\} = \left\{ \frac{3}{2} - \frac{3(k_0 - r_0)}{V}, \frac{3}{2} - \frac{3(k_1 - r_1)}{V} \right\}\) and two pure-strategy “pre-emption” equilibria \(\{\rho_0, \rho_1\} = \{0,1\}, \{\rho_0, \rho_1\} = \{1,0\}\). We assume the agencies play the mixed-strategy equilibrium because it approaches a symmetric equilibrium as \(k_0 - r_0 \to k_1 - r_1\). When, on the other hand, \(k_0 - r_0 \leq \frac{V}{6} < k_1 - r_1 \leq \frac{V}{2}\), the incumbent agency always enters and the competitor does not: \(\{\rho_0, \rho_1\} = \{1,0\}\). Finally, when \(k_i - r_i \leq V/6\), both agencies always enter \(\{\rho_0, \rho_1\} = \{1,1\}\). The following proposition gives the optimal policy:
Proposition 2: When $F$ is Uniform$[0, V]$ and the agencies do not know their value to the advertiser at the entry stage, the advertiser should offer no stipend to the incumbent agency. The competitor agency should only receive a positive stipend $r_i = k_i - \frac{V}{6}$ when $0 \leq k_0 \leq \frac{V}{6} < k_i \leq \frac{V}{2}$.

Proposition 2 shows the selection of higher-value agencies at the entry stage is not necessary for the qualitative results of Proposition 1 to hold. Instead, the critical piece is indeed pricing pressure on an incumbent whenever the incumbent has a low-enough bid-preparation cost to enter regardless of competition, but the competitor’s cost is high enough that the competitor would stay out of the contest without a stipend. Also echoing the result of Proposition 1, the optimal competitor stipend does not fully cover its bid-preparation cost, and equal bid-preparation costs make stipends suboptimal.

Extension 2: More Than One Competitor

In our second extension, we relax the assumption of a single competitor. For simplicity, suppose $k_0=0$ and let $N$ competitors face the same bid-preparation cost $k>0$. We need this equal-cost assumption to make the entry game tractable, relying on the results of Samuelson (1985) to guarantee a simple pure-strategy threshold equilibrium. Also for tractability, assume the competitors’ values $x_i$ are drawn iid from $F$ uniform on $[0, V]$.

One would expect that with more competitors, the advertiser would have to become stingier with the new-business stipends: although each entrant collects $r$, the more bidders that are already participating, the smaller the incremental decrease in the auction price from adding one more bidder. This intuition is incomplete because the presence of additional competitors also makes entry less likely by reducing the expected surplus given entry. Thus, the entry threshold rises and the agencies that decide to enter drive the profit up faster than the same number of
exogenous entrants would. Interestingly, these effects cancel each other out, and \( r = k/2 \) remains the optimal reimbursement policy regardless of the number of potential competitors, as in Example 1 with a single competitor and \( k_0 = 0 \):

**Proposition 3:** When \( F \) is uniform and \( N \geq 2 \) potential competitors exist with participation cost \( k \) in addition to the incumbent with no participation cost \( (k_0 = 0) \), the optimal new-business stipend is the same as when only one competitor exists, namely, \( r = k/2 \). The expected advertiser profit increases in \( N \).

A key simplifying aspect of Proposition 3 is the lack of dependence of the optimal reimbursement proportion on the number of potential competitors.

**Figure 5: Increased competition increases advertiser profits**

Note to Figure: Each line represents a different level of \( k \). The \( N \) does not count the incumbent.
Figure 5 illustrates how profits increase in $N$. The fact that profits are increasing in $N$ differs from the case of optimal reserve prices: the received intuition from auctions with bid-preparation costs is that increasing $N$ can reduce the auctioneer’s profit by reducing entry. For example, Samuelson (1985) concludes that “expected procurement costs need not decline with increases in the number of potential bidders.”

**Extension 3: Strategic Reserve Price**

So far, we have assumed the advertiser cannot use reserve price above its outside option. This assumption is realistic for settings in which the advertiser does not have enough commitment to reject positive (i.e., above outside option) offers. Without such commitment, he will be tempted to drop the reserve price and re-auction the contract in case all current bids are below its reserve. When such re-auctioning is instantaneous, the result is an instance of the Coase conjecture – the advertiser cannot credibly use a reserve above its outside option (McAfee and Vincent 1997).

Suppose instead that the advertiser can announce a public reserve $R > 0$ and commit not to re-auction the contract when no bids exceed $R$. To investigate the impact of a reserve price on optimal reimbursements and expected costs, we focus on the single-competitor case. In the online technical appendix, we show in full generality that the main qualitative conclusions of Proposition 1 continue to hold: incumbent stipends are never optimal, and offering a stipend to the competitor agency can be profitable for the advertiser.

A general characterization of the optimal reserve-stipend strategy $\{R^*, r_1^*\}$ is not tractable for an arbitrary $F$, but we show that when $F$ is Uniform[0,1], the optimal reserve price is $R^* = \frac{1}{2} - \frac{k_0 \sqrt{2}}{d}$ and the optimal competitor stipend is $r_1^* = \frac{k_1}{2} - \frac{k_0 (4k_0 + d \sqrt{2})}{2d^2}$, where
\[ d = \sqrt{1 + 4k_1 + (1 + 4k_1)^2 - (4k_0)^2} . \] For any \( k_0 > 0 \), both the optimal reserve and the optimal competitor stipend increase with \( k_1 \) (see online technical appendix for a proof).

**Figure 6: Advertiser profit with reserve prices and/or competitor stipends**

![Graph showing advertiser profit with reserve prices and/or competitor stipends.](image)

**Note to Figure:** The baseline uses neither the reserve price nor stipends. The solid line indicates the presence of the optimal competitor stipend. The circles indicate the presence of the optimal reserve price. \( F \) Uniform[0,1] and \( k_0 = 0 \) throughout.

When \( F \) is Uniform[0,1] and \( k_0 = 0 \), the formulae simplify dramatically, and we obtain the optimal reserve \( R^* = \frac{1}{2} \) familiar from textbooks, along with now-familiar expression for the optimal
incumbent’s stipend: \( r_i^* = \frac{k_i}{2} \) (same as in Example 1). This special case \((F \text{ is Uniform}[0,1] \text{ and } k_0=0)\) also makes it tractable to solve for the optimal reserve in the absence of stipends. We find that as \( k_1 \) rises, the optimal reserve price drops but remains above \( 2/5 \). Importantly, the advertiser is better off with a new-business stipend and its corresponding optimal reserve than with an optimal reserve alone, and the profit difference increases as \( k_1 \) (and hence the difference between \( k_1 \) and \( k_0 \)) increases (see Figure 6 for an illustration). Therefore, a new-business stipend is a step in the right direction from a mechanism optimal without asymmetries in bid-preparation costs to a mechanism optimal under that asymmetry. Figure 6 illustrates the advertiser profits with and without a strategic reserve, and with and without the optimal competitor stipend.

**A MODEL OF THE PAST: A QUALITY CONTEST WITH A FIXED PRIZE**

Contrasting the modern auction-based contest with the traditional 20\(^{th}\)-century contest whereby agencies competed for the best creative quality and the winner received a fixed prize (traditionally 15\% of the media billings) is useful. Consider two agencies \( i=0,1 \), and assume each agency has a different privately known creative quality \( q_i \) and a publicly known bid-preparation cost \( k_i \) such that \( 0 \leq k_0 \leq k_1 \). Let qualities be distributed iid according to a distribution \( H \) on \([0,V]\), and suppose the agency with the higher quality wins the contest and receives a fixed prize \( P \).

The entry game is different from the auction contest because the winner’s payoff does not depend on the quality of the loser. As a result, the incumbent always enters, but the competitor stays out of the contest when his quality is low (see the proof of Proposition 4). Proposition 4 provides a sufficient condition for the advertiser to use a new-business stipend to encourage more competitor entry:
**Proposition 4:** For any continuous quality distribution $H[0,V]$ and any bid-preparation costs $0 \leq k_0 \leq k_1$, let $L_1$ satisfy $k_1 = PH(L_1)$. A positive stipend for the competitor agency $r_1 > 0$ benefits the advertiser when $P\left[1-H(L_1)\right] < \int_0^L H(x) dx$.

The sufficient condition in Proposition 4 is derived analogously to that in Proposition 1, but Proposition 4 is weaker than Proposition 1 because positive incumbent stipends cannot be categorically ruled out: when the bid-preparation costs of both agencies are high enough and similar enough, the advertiser can benefit from offering both agencies a stipend. Nevertheless, incumbent stipends can be ruled out whenever $k_0$ is sufficiently smaller than $k_1$ such that the optimal $r_1$ is such that $k_0 \leq k_1 - r_1$. Then the incumbent always enters, and offering him a stipend cannot benefit the advertiser. A uniform-distribution example illustrates both possibilities:

**Example 7:** When $H$ is Uniform$[0,1]$ and $0 \leq k_0 < -2P^2 + \sqrt{P^4 + 2P^3} < k_1 < P$, the advertiser maximizes his profit by offering no stipend to the incumbent and offering the following positive stipend to the competitor: $r_1 = k_1 + 2P^2 - P\sqrt{2(k_1 + P + 2P^2)}$. When $k_0 = k_1$ and $2\left(\sqrt{P^3 + P^4} - P^2\right) < k_1 < P$, positive stipends to both agencies benefit the advertiser.

Please see the Appendix for details of Example 7. The contrast with Example 1 is striking: the seemingly simpler contest with a fixed prize involves a much more complicated new-business stipend scheme that depends on the prize. Interestingly, $r_1$ is not even monotonic in $P$. Also in contrast to Example 1 that rules out stipends under $k_0 = k_1$, Example 7 gives a sufficient condition for the advertiser to offer both agencies a stipend under symmetric bid-preparation costs.
DISCUSSION

Advertisers commonly hold contests to select an advertising agency, and the level of contest activity seems to have picked up recently (Finneran 2009, Parekh 2010). The compensation method of advertising agencies has been in flux since the demise of the standard compensation contract based on 15% of the media billings. This paper focuses on the fact that the advertising agency selection contest has become similar to a procurement score auction in which the pitch involves not only the creative idea, but also a proposed price for buying the media. One of the consequences of the modern contest is increased competition whereby the compensation for an occasional contest victory does not provide enough profit to cover upfront bid-preparation costs for all the contests in which the agency participates (Rice 2006). To cover such costs, advertising agency associations worldwide recommend reimbursing the losing agencies for contest-preparation expenses (Brendan 1998, Gardner 1996), but the advertisers predictably resist the idea because it seems like an added cost. Two recent surveys indicate that about half of today’s advertising contests involve some form of stipend to help defray the costs (AAAA 2007, Parekh 2009). The industry has not reached a consensus as advertising practitioners continue debating the pros and cons of reimbursements, as well as the details of optimal policies.

We analyzed the score-auction model with endogenous entry and asymmetric bid-preparation costs to account for the difference between the incumbent agency that currently serves the advertiser and a competitor agency that is bidding against the incumbent for new business. Our analysis indicates the advertisers are right to resist demands for reimbursements in full, and right to resist offering stipends to the incumbent agency. However, we find offering a stipend to agencies competing with the incumbent for new business can increase the advertiser’s overall profit. The optimality of a new-business stipend depends on (1) the asymmetry in bid-preparation costs and (2) the distribution of value-to-Advertiser in the population of agencies.
First, when the incumbent and the competitor face the same bid-preparation cost, the advertiser should not offer any stipends. Or, more commonly, when the incumbent does not participate in the contest and only new-to-the-account agencies compete, no stipends should be offered. Therefore, the business reason for providing reimbursements is not necessarily to increase the competition in general, but to increase pricing pressure on the incumbent who enters the contest more often. Second, we provide and analyze a simple sufficient condition on the distribution of agency values to the advertiser for new-business stipends to benefit the advertiser. We find new-business stipends only benefit the advertiser when the population distribution of agencies is not too concentrated near the bottom of its support.

The fact that about half the real-world contests offer no reimbursement may thus be partially explained by contests in which the incumbent agency does not participate for whatever reason. Further, it can be explained by some contests attracting agencies with values-to-advertiser concentrated at the bottom of their support.

Regarding the details of the optimal new-business-stipend policy, we provide a set of assumptions (uniform distribution of agency profitability to the advertiser) under which the optimal reimbursement policy is simple—a fixed proportion of the creative development costs regardless of the costs’ magnitude, the number of potential competitors, or the presence of a strategic reserve. However, we additionally illustrate that seemingly simple assumptions (e.g. a triangle distribution) can also imply fairly complex relationships between the magnitude of the bid-preparation costs and the optimal new-business policy.

The entry game into our auction-driven score contest involves a systematic selection of higher-valued agencies because the lower-valued agencies are less likely to win and are hence less likely to cover their bid-preparation costs. We find this selection at the entry stage is not
necessary for the qualitative results described above: even when the agencies do not know their values at entry time, the advertiser should not offer a stipend to the incumbent, and new-business stipends can benefit the advertiser only when the incumbent faces a lower bid-preparation cost than the competitor. The critical force for the optimality of new-business stipends is thus the pricing pressure on an incumbent whenever the incumbent has a lower bid-preparation cost than the competitor.

Many of our qualitative results extend to contests based solely on quality that award the winner a fixed prize. Such contests were often used in advertising agency selection during the 20th century, with the prize being 15% of the list price of the media billings. We provide a sufficient condition for new-business stipends in that context, as well. One result that does not extend is the sub-optimality of incumbent stipends: we describe a situation in which both agencies should receive a stipend.

In our modeling, we have assumed the incumbent agency has an advantage in lower bid-preparation costs but has no cost advantage in executing the creative and media buying tasks after the contest. In other words, in our setup, the incumbent and the competitor are asymmetric only in the bid-preparation costs before the auction, and ex-ante symmetric in parameters active during the auction. We did not formally examine the generalization to incumbent advantages at both stages of the auction, but we speculate that the case for new-business stipends would strengthen under such assumptions. The literature reviewed earlier in this paper (e.g., McAfee and McMillan 1989, Branco 2002) found that the auctioneer should give an advantage to the “weaker” bidder during the auction by allowing that bidder to win the auction even if he quoted a higher price. Given these results, it would stand to reason that in our scenario, if the incumbent also continued to have an advantage during the auction (and not just during entry), the case for
giving a stipend to the new participants would strengthen in order to induce them to take part in this contest that is already biased against them for an additional reason. We conjecture that the same reasoning would likely apply if the advertiser’s familiarity with the incumbent makes him more uncertain about the profit-lift expected from the new entrant.

Our paper is the first to highlight the role of an incumbent in a score-auction contest environment. Any client firm wishing to hire a service provider in a context in which incumbents might exist could apply our model, for example, in choosing a new outside accounting/auditing or legal office, or an outside consulting firm. However, for our reimbursement strategy to apply, differences in bid-preparation costs as well as pre-contest quality differences among bidders need to exist. The advertising contest is also theoretically analogous to a contest among architects to design an extension or renovate an existing city museum and then supervise its construction. A more appealing design will please residents and increase attendance, donations, and tourism to the city. The most creative architect might not necessarily be an efficient construction supervisor. In the architect-selection context, our results imply the contest organizers should not offer a reimbursement of design costs unless they have a clear “incumbent” who will participate in the context no matter what (e.g., the architect who designed the original building, or one who already did a preliminary study and already offered a possible design).
REFERENCES


APPENDIX: PROOFS OF PROPOSITIONS AND DETAILS OF EXAMPLES

Proof of Lemma 1: Integration by parts yields the following RHS of the first equation in (2):

\[ L_0 F(L_0) + \int_{L_0}^{L_1} F(z) \, dz. \]

Plugging in \( L_0 = \frac{k_0 - r_0}{F(L_1)} \) that solves the second equation yields the first equation in terms of only \( L_1: k_1 - r_i = \frac{k_0 - r_0}{F(L_1)} F\left(\frac{k_0 - r_0}{F(L_1)}\right) + \int_{L_0}^{L_1} F(z) \, dz. \)

The RHS is obviously continuous and increasing in \( L_1 \) large enough that \( L_1 F(L_1) \geq k_0 - r_0 \), so the intermediate value theorem implies a unique \( L_1 \geq L_0 \) exists that satisfies the equation as long as

\[ k_1 - r_i \leq \int_0^V F(z) \, dz = V - E(x). \]

Because the solution thus involves a \( L_0 = \frac{k_0 - r_0}{F(L_1)} \leq L_1 \leq V \), the condition is sufficient to guarantee a pair of thresholds \((L_0, L_1)\) is between 0 and \( V \).

It remains to be checked that \( L_0 \leq V \), i.e., that \( \frac{k_0 - r_0}{F(L_1)} \leq V \). QED Lemma 1

Proof of Proposition 1: In the profit (4), substitute for \( L_0 \) using the second equation in (2):

\[ \Pi(L_1, r_0) = \int_{L_1}^{V} z f(z) \left[ 1 - F(z) \right] \, dz - \left[ 1 - F(L_1) \right] \left[ k_1 - L_1 F(L_1) \right] - r_0 \left[ 1 - F\left( \frac{k_0 - r_0}{F(L_1)} \right) \right]. \]

Note that for every \( L_0 < L_1 \) such that \( r_0 \geq 0 \), the profit is decreasing in \( r_0 \):

\[ \frac{\partial \Pi(L_1, r_0)}{\partial r_0} = -f\left( \frac{k_0 - r_0}{F(L_1)} \right) \frac{r_0}{F(L_1)} - \left[ 1 - F\left( \frac{k_0 - r_0}{F(L_1)} \right) \right] < 0. \]

Fix any \( r_0 > 0 \) and let \( \Pi^*(r_0) \) be the optimal profit achieved by manipulating \( L_1 \) in \( \Pi(L_1, r_0) \) given the fixed \( r_0 \). By the envelope theorem, \( \Pi^*(r_0) \) is decreasing in \( r_0 \):

\[ \frac{d \Pi^*(r_0)}{dr_0} = \frac{\partial \Pi^{*}}{\partial r_0} \bigg|_{(r_0, L_1)} < 0. \]

Therefore, no \( r_0 > 0 \) can benefit the advertiser more than \( r_0 = 0 \).

When \( k_0 = k_1 \) and \( r_i = 0 \), the entry thresholds are the same: \( L_0 = L_1 \). Offering a stipend to the
competitor agency would result in $L_0 > L_1$, and the competitor agency would play the role of agency 0 in the profit function (4). The above argument shows the advertiser would be better off reducing the competitor stipend back to zero. When $k_0 = k_1$, the advertiser does not benefit from offering a stipend to both agencies either: let $L = L_0 = L_1$ in (4) and note

$$\frac{d\Pi(L)}{dL} = 2 f(L) \left[ k - LF(L) \right] + 2 \left[ 1 - F(L) \right] F(L) > 0.$$  

The second term combines the marginal decrease in profit of $L f(L)$ per entrant and the marginal savings in the per-capita stipend amount of $L f(L) + F(L)$. Because the latter exceeds the former, the advertiser always benefits from raising $L$.

Now consider the $0 \leq k_0 < k_1$ case and let $r_0 = 0$. The sub-optimality of reimbursing the competitor in full follows from equation (5) with $L_i = 0$:

$$\frac{\partial \Pi(L_0, L_1 | r_0 = 0)}{\partial L_1} \bigg|_{L_i = 0} = k_i f(0) > 0.$$  

The sufficient condition for the optimality of a positive stipend ensures $\frac{\partial \Pi(L_0, L_1 | r_0 = 0)}{\partial L_1} \bigg|_{L_i = \overline{L}_i} < 0$, where $\overline{L}_i$ is the entry threshold corresponding to $r_1 = 0: k_i = \int_0^{\overline{L}_i} \left[ \overline{L}_i - x \right] dF(x) = \int_0^{\overline{L}_i} F(x) dx$, where the second equality follows from integration by parts. Expressing the cost $k_1$ in terms of the $\overline{L}_i$ threshold in equation (5) yields the condition. Setting $\overline{L}_1 = V$ yields the upper bound on $k_1$ that admits an entry threshold interior to the support of $F$. QED Proposition 1.

Details of Example 2: The sufficient condition of Proposition 1 simplifies to

$$\frac{L'(1 - a^2 - L_i^a)}{1 + a} < 0,$$

which is violated for $a < 1$ and for small-enough $L_1$ such that $L_i^a < 1 - a^2$. Recall the condition is the derivative at $L_1$ corresponding to $r_1 = 0$, so the entry equation (2) satisfies
\[ k_i = \frac{aL_i^{\alpha+1} + L_i^{\alpha+1}}{1+a}, \quad k_0 = L_0L_i^2. \] Setting \( k_0 = 0 \) makes the entry game tractable with \( k_i = \frac{L_i^{\alpha+1}}{a+1} \).

Therefore, \( L_i^2 < 1 - a^2 \Leftrightarrow k_i = \frac{L_i^{\alpha+1}}{a+1} < (1-a)^{\frac{1}{\alpha+1}}. \) The fact that the condition is violated shows the profit function is increasing in \( L_1 \) at \( L_{\text{max}} = \left[ (a+1)k_1 \right]^{\frac{1}{\alpha+1}} \) corresponding to \( r_1=0 \). It remains to be shown that the profit function is also increasing for all \( L \) between 0 \( (r_1=k_1) \) and \( L_{\text{max}} \) \( (r_1=0) \). The derivative of the profit function is:

\[
\frac{\partial \Pi}{\partial L_1} = \frac{ak_i + L_1 - aL_1 - L_1^{\alpha+1}}{L_1^{\alpha+1}} > 0 \Leftrightarrow ak_i + L_1 \left( 1-a - L_i^a \right) > 0.
\]

Expressing \( k_1 \) in terms of \( L_{\text{max}} \):

\[
ak_i + L_1 \left( 1-a - L_i^a \right) = \frac{al_i^{\alpha+1}}{a+1} + L_1 \left( 1-a - L_i^a \right) > 0
\]

\[ \Leftrightarrow a \left( L_{\text{max}}^{\alpha+1} - L_1^{\alpha+1} \right) + L_1 \left( 1-a^2 - L_i^a^a \right) > 0, \] which holds because \( L_1<L_{\text{max}} \) and \( 1-a^2 - L_i^a > 0 \).

Therefore, the profit is indeed increasing on the entire \([0, L_{\text{max}}]\) range. QED

**Example 2**

**Details of Example 5:** With \( k_0 = 0 \), the incumbent always enters, and the marginal competitor entrant satisfies: \( 2(k_1 - r_1) = 2\alpha L_1 + (1-\alpha) L_i^2 \). The basic structure of advertiser profit remains as in equation (4) with \( F(x) = \alpha + (1-\alpha) x \) and \( f(x) = (1-\alpha) \) for any \( x>0 \). The profit with \( r_0=0 \) simplifies to \( \Pi \left( L_1, L_0 \mid r_0 = 0 \right) = \left( 1-\alpha \right)^2 \Pi \left( L_1, L_0 \mid r_0 = 0, \alpha = 0 \right) - \alpha \left( 1-\alpha \right) \left( 1-L_1 \right) \left( k_1 - L_1 \right) \), where the second term captures the possibility that the competitor enters with \( x_1>L_1 \) and the incumbent is either weak or does not enter. With the help of equation (5), the derivative can be rearranged as:

\[
\frac{\partial \Pi \left( L_1, L_0 \mid r_0 = 0 \right)}{\partial L_1} \propto \frac{\partial \Pi \left( L_1, L_0 \mid r_0 = 0, \alpha = 0 \right)}{\partial L_1} + \alpha \left( 1-L_1 \right)^2,
\]

where the derivative with \( \alpha=0 \) is \( k_1 - L_i^2 \) as shown in the derivation of Example 1. Therefore, the first-order condition (FOC) is:

\[ k_1 + \alpha = 2\alpha L_1 + (1-\alpha) L_i^2 \].

The second-order condition implies the profit function \( \Pi \left( L_1, L_0 \mid r_0 = 0 \right) \) is concave in \( L_1 \):

\[
\frac{\partial^2 \Pi \left( L_1, L_0 \mid r_0 = 0 \right)}{\partial L_1^2} \propto -2L_1 - 2\alpha \left( 1-L_1 \right) < 0.
\]

Therefore, the FOC characterizes
the maximum, and \( \frac{\partial \Pi(L_1, L_0| r_0 = 0)}{\partial L_1} < 0 \) for all \( L_1 \) smaller than the solution to the FOC. Noting the RHS of the entry equation is the same as the RHS of the FOC, it is immediate that both equations hold at \( r_1 = \frac{k_1 - \alpha}{2} \). When \( r_1 < 0 \), the advertiser would want to charge a participation fee instead of awarding a stipend. Equivalently, the positive root of the FOC is too large to support a stipend. From concavity of the profit function in \( L_1 \), the best non-negative stipend to use in this situation is thus \( r_1 = 0 \). \textit{QED Example 5}

**Proof of Proposition 2:** The advertiser only makes a positive profit when both agencies enter, but pays the stipend to each entrant independently:

\[
\Pi(\rho_0, \rho_1) = \rho_0 \rho_1 \left( \frac{V}{3} \right) - \rho_0 \left( k_0 - \left( \frac{3 - 2 \rho_1}{6} \right) V \right)_{\min(\nu_i)} - \rho_1 \left( k_1 - \left( \frac{3 - 2 \rho_0}{6} \right) V \right)_{\nu_i}.
\]

Suppose first that both bid-preparation costs exceed the competitive contest payoff \( k_i \in \left[ \frac{V}{6}, \frac{V}{2} \right] \) and consider a stipend policy that results in \( (k_i - r_i) \in \left[ \frac{V}{6}, \frac{V}{2} \right] \) and the associated mixed-strategy equilibrium \( \{\rho_0, \rho_1\} = \left\{ \frac{3}{2} - \frac{3(k_i - r_i)}{V}, \frac{3}{2} - \frac{3(k_0 - r_0)}{V} \right\} \). Expressing the profit function in terms of the stipends makes it clear the advertiser cannot benefit from positive stipends: \( \Pi(\rho_0, \rho_1) = \frac{3}{4} \left[ V + \frac{4k_0 k_1}{V} - 2(k_0 + k_1) \right] = \frac{3r_0 r_1}{V} \). Therefore, the advertiser maximizes his within-equilibrium payoff by setting stipends to zero (setting only one stipend to zero is revenue neutral). Alternatively, the advertiser can set \( k_i - r_i = \frac{V}{6} \) to induce guaranteed entry by both agencies and obtain a profit of \( \frac{2V}{3} - k_0 - k_1 \), which is lower than \( \Pi(0, 0) \) as long as \( k_i > \frac{V}{6} \).

Finally, the advertiser cannot benefit from setting only one \( k_i - r_i = \frac{V}{6} \) to induce guaranteed
entry by agency $i$, because the best response to $\rho_i=1$ is $\rho_{-i}=0$ as long as $k_i \in \left[\frac{V}{6}, \frac{V}{2}\right]$, and the advertiser only makes a positive profit when both agencies enter. Therefore, the advertiser should not offer stipends when $k_i \in \left[\frac{V}{6}, \frac{V}{2}\right]$.

Now suppose only the competitor’s cost exceeds the competitive contest payoff: $0 \leq k_0 \leq \frac{V}{6} < k_1 \leq \frac{V}{2}$. Offering any stipend to the incumbent who always enters even with $r_0=0$ is obviously not beneficial. The minimum stipend that induces the competitor to enter is $r_i = k_i - \frac{V}{6}$, and the resulting profit exceeds the (zero) profit without competitor entry when

$$\Pi(\text{comp entry}) = \left(\frac{V}{3}\right) - \left(\frac{k_i - V}{6}\right) = \frac{V}{2} - k_i > 0 = \Pi(\text{no comp entry}) \Leftrightarrow k_i < \frac{V}{2}.$$

**Proof of Proposition 3:** A competitor with $x_i = L_1$ knows he can only win if all other competitors stay out of the auction and the incumbent’s $x_0$ is below $L_1$. Therefore, the entry threshold satisfies: $\Pr(N-1 \text{ } x_i's < L_1) \Pr(x_0 < L_1) E(L_1 - x_0 | x_0 < L_1) = \left(\frac{L_1}{V}\right)^{N-1} \frac{L_1^2}{2} = k - r$.

Fix $k$ for clarity and consider the expected price the advertiser obtains. Let $\pi_n$ be the advertiser’s profit when $n \geq 2$ agencies (including both the incumbent and/or the competitors) exist with $x_i$ above the $L_1$ cutoff. The uniform assumption yields $\pi_n$ in a closed form: $\pi_n$ is the expected second-highest draw from $n \geq 2$ draws $x_i$ distributed iid on $[L, V]$ according to the uniform distribution $\Pr(x < z) = \frac{z-L}{V-L}$. When $n$ draws are distributed iid according to $F(x)$, the density of the second-smallest order statistic is $n(n-1)f(x)[1-F(x)]F^{n-2}(x)$. For $F(z) = \frac{z-L}{V-L}$, which implies $\pi_n = \int_L^V n(n-1)\left(\frac{1}{V-L}\right)^n\left(\frac{V-z}{V-L}\right)^{n-2} dz = \frac{2L + (n-1)V}{n+1}$. 

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To keep notation compact, define \( \pi_1 \) as the expected profit with one competitor (by definition above \( L_1 \)) and the incumbent below \( L_1 \): \( \pi_1 \equiv \frac{L_1}{2} \).

The advertiser averages over all possible numbers \( n=0,1,2,\ldots,N \) of entering competitors. Denoting the probability that a single entrant enters, \( V \equiv \frac{V-L}{V} \), the expected profit is

\[
\Pi_N(s) = \sum_{n=1}^{N} \binom{N}{n} s^n (1-s)^{N-n} \left[ s \pi_{n+1} + (1-s) \pi_n - nr \right] = -rsN + \sum_{n=1}^{N} \binom{N}{n} s^n (1-s)^{N-n} \left[ s \pi_{n+1} + (1-s) \pi_n \right]
\]

where the second equality follows from \( \sum_{n=1}^{N} \binom{N}{n} s^n (1-s)^{N-n} = E\left[n \mid n \sim \text{Binomial}(s,N)\right] = sN \).

Adding all the probabilities that a given \( \pi_n \) occurs yields

\[
\sum_{n=1}^{N} \binom{N}{n} s^n (1-s)^{N-n} \pi_{n+1} + \sum_{n=1}^{N} \binom{N}{n} s^n (1-s)^{N-n} \pi_n =
= Ns (1-s)^N \pi_1 + s^{N+1} \pi_{N+1} + \sum_{m=2}^{N} \left[ \binom{N}{m-1} + \binom{N}{m} \right] s^m (1-s)^{N-m+1} \pi_m.
\]

Finally, re-parametrizing in terms of the entry probability \( s \) uses the uniform assumption

\[
V(1-s)^{N+1} = 2(k-r) \Rightarrow r = k - \frac{V(1-s)^{N+1}}{2}. \]

The profits \( \pi_m \) follow from \( L = (1-s)V \)

\[
\Rightarrow \pi_1 = \frac{(1-s)V}{2}, \pi_m = V - 2s \left( \frac{V}{m+1} \right) \text{ for } m \geq 2. \]

The above four transformations yield

\[
\Pi_N(s) = sN \left[ V(1-s)^{N+1-k} \right] + \left\{ s^{N+1} \pi_{N+1} + \sum_{m=2}^{N} \left[ \binom{N}{m-1} + \binom{N}{m} \right] s^m (1-s)^{N-m+1} \left[ V - 2s \left( \frac{V}{m+1} \right) \right] \right\}.
\]

Although the part in the curly brackets does not easily simplify further, its derivative in \( s \) does:

\[
\frac{d}{ds} \left\{ \ldots \right\} = sNV \left(N+1\right)(1-s)^N. \]

Therefore, the first derivative of the profit in \( s \) takes a remarkably simple form:

\[
\frac{d}{ds} \Pi_N = N \left[ V(1-s)^{N+1-k} \right].\]

By \( \frac{d^2 E(\Pi)}{ds^2} = -N(\left(N+1\right)(V-v)(1-s)^N < 0, \) the first-order condition characterizes the optimal entry probability:
At the optimal \( s^* \),
\[
\Pi_N(s^*) = \frac{s^*(1-s^*)^{N+2}}{2+N}
\]
, which can be shown to be increasing in \( N \). \( QED \) Proposition 3

**Proof of Proposition 4**: In the entry game, it is natural to look for an equilibrium in threshold strategies because a higher \( q_i \) implies a higher probability of winning the contest. Suppose the opponent \(-i\) enters when \( q_{-i} \geq L_{-i} \). The expected entry surplus of agency \( i \) is
\[
S_i(q_i) = P \frac{H(L_{-i})}{Pr(-i \text{ not enter})} + P(1-H(L_{-i})) \max \left\{ 0, \frac{H(q_i) - H(L_{-i})}{1-H(L_{-i})} \right\}
\]
\[
\begin{cases}
q_i < L_{-i} : PH(L_{-i}) \\
q_i \geq L_{-i} : PH(q_i)
\end{cases}
\]
Suppose \( k_0 - r_0 \leq k_1 - r_1 \), and let \( L_0 = 0 \) (i.e., assume the incumbent enters regardless of its quality). In response, the competitor plays a threshold strategy that satisfies \( k_1 - r_1 = PH(L_1) \).

Closing the loop, always entering \((L_0 = 0)\) is the incumbent’s best response to such a competitor: either the \( q_0 \) is low \((q_0 < L_1)\), in which case \( S_0(q_0) = PH(L_1) = k_1 - r_1 > k_0 - r_0 \), or \( q_0 \) is high, in which case \( S_0(q_0) = PH(q_0) > PH(L_1) = k_1 - r_1 > k_0 - r_0 \). Therefore, \( \{ L_0 = 0, k_1 - r_1 = PH(L_1) \} \)
is a Nash equilibrium as long as the stipends are such that \( k_0 - r_0 \leq k_1 - r_1 \).

Now suppose \( k_0 < k_1 \) and consider stipends that maintain \( k_0 - r_0 \leq k_1 - r_1 \). Express the advertiser profit in terms of \( L_1 \):
\[
\Pi(L_1) = H(L_1) \int_{L_1}^{V} z dH(z) + 2 \int_{L_1}^{V} z[H(z) - H(L_1)] dH(z) + H(L_1) \int_{0}^{L_1} zdH(z) - \left[ 1 - H(L_1) \right] \left[ k_1 - PH(L_1) \right] =
\]
\[
= 2 \int_{L_1}^{V} zH(z) dH(z) + H(L_1) \int_{0}^{L_1} zdH(z) - \left[ 1 - H(L_1) \right] \left[ k_1 - PH(L_1) \right]
\]
The first derivative of the profit functions is
\[
\frac{d\Pi}{dL_1} = h(L_1) \left[ k_1 + P - 2PH(L_1) - \int_{0}^{L_1} (L_1 - z) dH(z) \right].
\]
see the \( \frac{d\Pi}{dL_1} \) at \( r_1=0 \), substitute \( k_1 = PH(L_1) \). Differentiation by parts yields that \( \frac{d\Pi}{dL_1} \bigg|_{r_1=0} < 0 \) iff

\[ P \left[ 1 - H \left( L_1 \right) \right] < \int_0^{L_1} H \left( x \right) \, dx \]

at \( L_1 \) that satisfies \( k_1 = PH(L_1) \). QED Proposition 4

Details of Example 7: Letting \( H \left( L_1 \right) = L_1 \) simplifies the first-order condition to a quadratic:

\[ 0 = \frac{d\Pi}{dL_1} = k_1 + P - 2PL_1 - \frac{L_1^2}{2} \]. The second-order condition obviously implies \( \Pi \) is concave. The positive root of the quadratic FOC equation is \( L_1^* = -2P + \sqrt{2(k_1 + P + 2P^2)} \), implying the optimal stipend via the entry-threshold condition \( k_1 - r_i = PL_1 \). Marginal analysis only makes sense when the highest-quality competitor enters even with \( r_1=0 \), that is, when \( k_1 < P \). The stipend implied by \( L_1^* \) is positive whenever \( L_1^* \frac{k_1}{P} \) corresponding to \( r_1=0 \). Substituting for \( L_1^* \) yields \( -2P^2 + \sqrt{P^4 + 2P^3} < k_1 \). The stipend implied by \( L_1^* \) is small enough that \( k_0 \leq k_1 - r_i \) as long as \( PL_1^* > k_0 \iff -2P^2 + \sqrt{P^4 + 2P^3} > k_0 \). Hence the first constraint. Now consider the \( k_0 = k_1 \equiv k \) case in which the advertiser must pay both agencies to lower the entry threshold \( L_1 \). The general profit function becomes \( \Pi(L_1) = \int_{L_1}^v zH(z) \, dz + H(L_1) \int_0^{L_1} zdH(z) - 2 \left[ 1 - H(L_1) \right] \left[ k - PH(L_1) \right] \),

and the first derivative under the uniform assumption \( H \left( L_1 \right) = L_1 \) is \( \frac{d\Pi}{dL_1} = 2k + 2P - 4PL_1 - \frac{L_1^2}{2} \).

Substituting the \( k = PL_1 \) yields the first derivative at \( r_i=0 \): \( \frac{d\Pi}{dL_1} \bigg|_{r_i=0} = -2k + 2P - \frac{k^2}{2P} \). Positive stipends to both agencies benefit the advertiser whenever \( \frac{d\Pi}{dL_1} \bigg|_{r_i=0} < 0 \), yielding the second inequality of the Example. QED Example 7.